Coherence properties of synchrotron light sources: application to x-ray microscopes

by

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Lecture 4 of the series

COHERENT X-RAYS AND THEIR APPLICATIONS

A series of tutorial–level lectures edited by Malcolm Howells*

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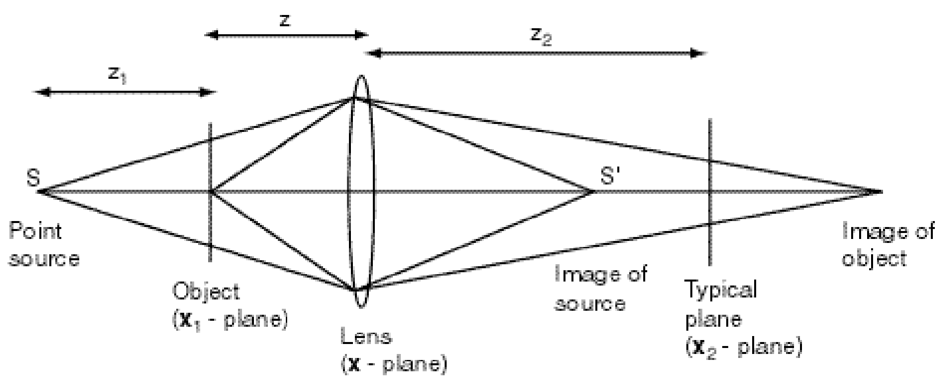
SOME FILES THAT ARE NOW ON THE WEBSITE

1. M. R. Howells, C. J. Jacobsen, A. Warwick, Principles and applications of zone plate x-ray microscopes, in “The Science of Microscopy”, P. Hawkes, J. Spence eds., Springer, 2007 (referred to as Howells et al SoM)
2. M. R. Howells, Theory of the production of the Fourier transform of an object in planes other than the back focal plane of a lens, Advanced Light Source Report, June 2000
3. M. R. Howells, B. M. Kincaid, The properties of Undulator Radiation. "New Directions in Research using Third Generation Synchrotron Sources". F. S. Schlachter and F. Wuilleumier, Kluwer Academic Publishers, 315-358, (1994)
4. M. R. Howells, ERRATA for Howells and Kincaid “The properties of undulator radiation” page 25

A MORE DIFFICULT PROBLEM IN COHERENT OPTICS

- Background: it is often shown in text books that the wave field at the back focal plane of a lens is proportional to the Fourier transform of the object [Goodman 1968, Fig. 5.5 and equation 5-19 for example]
- Problem: the trouble is that the proof given usually assumes that the object is illuminated with parallel light
- Question: can the Fourier transform of the object be obtained when the object is illuminated by an axial point source? - Assume the source is at a distance where the Fresnel approximation can be used.



- The solution is in the document "Theory of the production of the Fourier transform of an object in planes other than the back focal plane of a lens"
- The problem is also discussed by [Goodman 2005, section 5.4.2] using an operator method

APPICATION OF SCHELL'S THEOREM TO THE DIFFRACTION PATTERN OF A CIRCULAR APERTURE WITH PARTIALLY COHERENT ILLUMINATION

Shore, Whitney and Thompson 1966

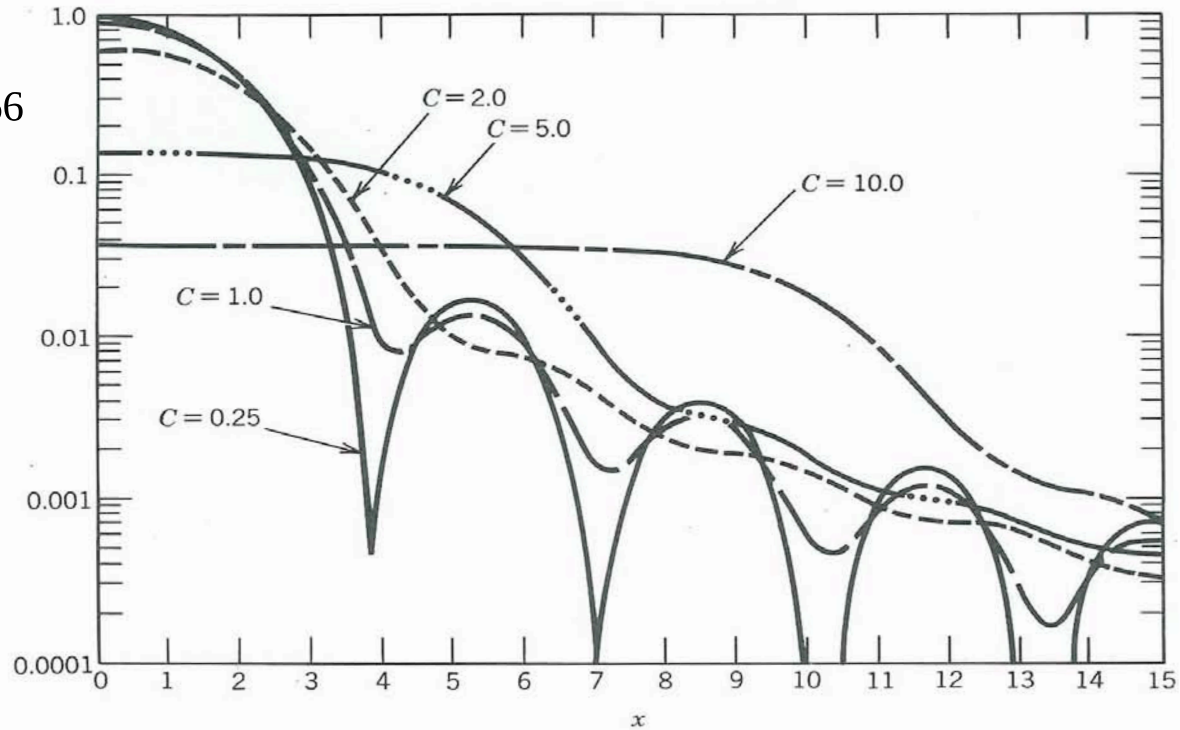
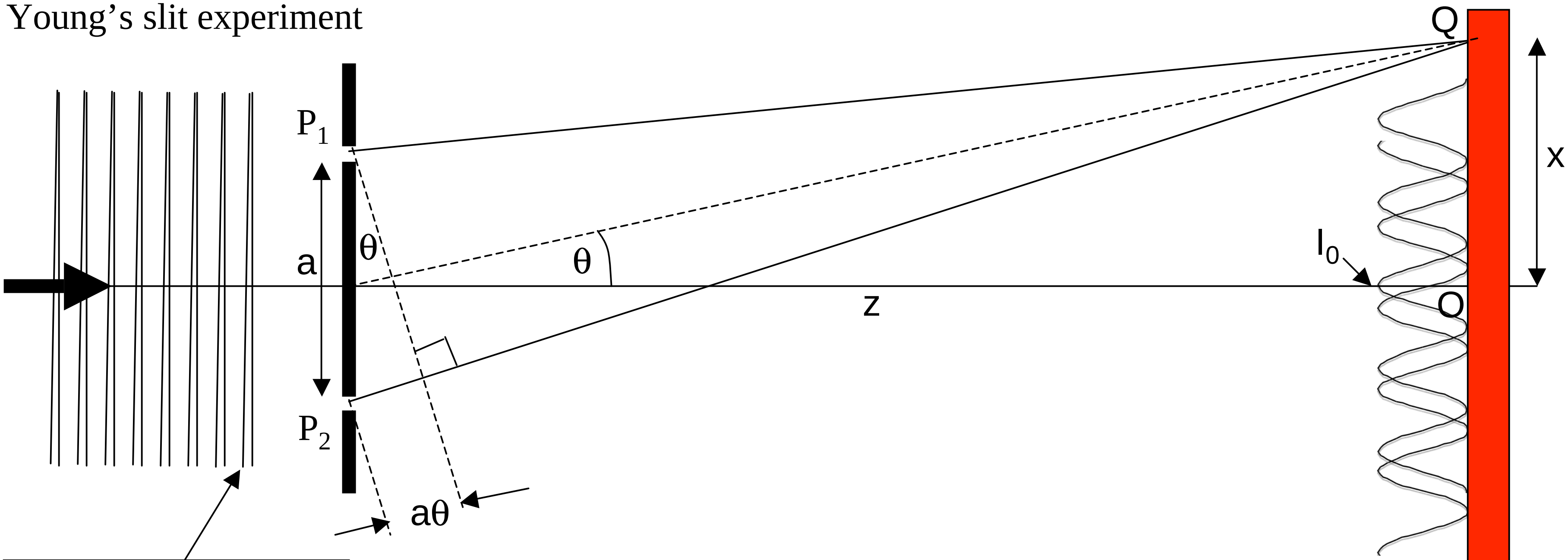


Figure 5-23. Diffraction pattern of a circular aperture for various states of transverse coherence. The parameter C represents the ratio of the area of the circular aperture to a coherence area. A circular incoherent source was assumed. The variable x has been normalized. (Ref. 5-34). (Courtesy of B. J. Thompson and the Optical Society of America.)

- The regular ring structure from the circular aperture is formed in the same way as the irregular speckles from an irregular structure
- The main message is that partial coherence blurs the features of the pattern
- This gives some idea why coherent diffraction preserves information better than incoherent

THE DEGREE OF SPATIAL COHERENCE IS DETERMINED BY THE DEGREE OF COLLIMATION

Young's slit experiment

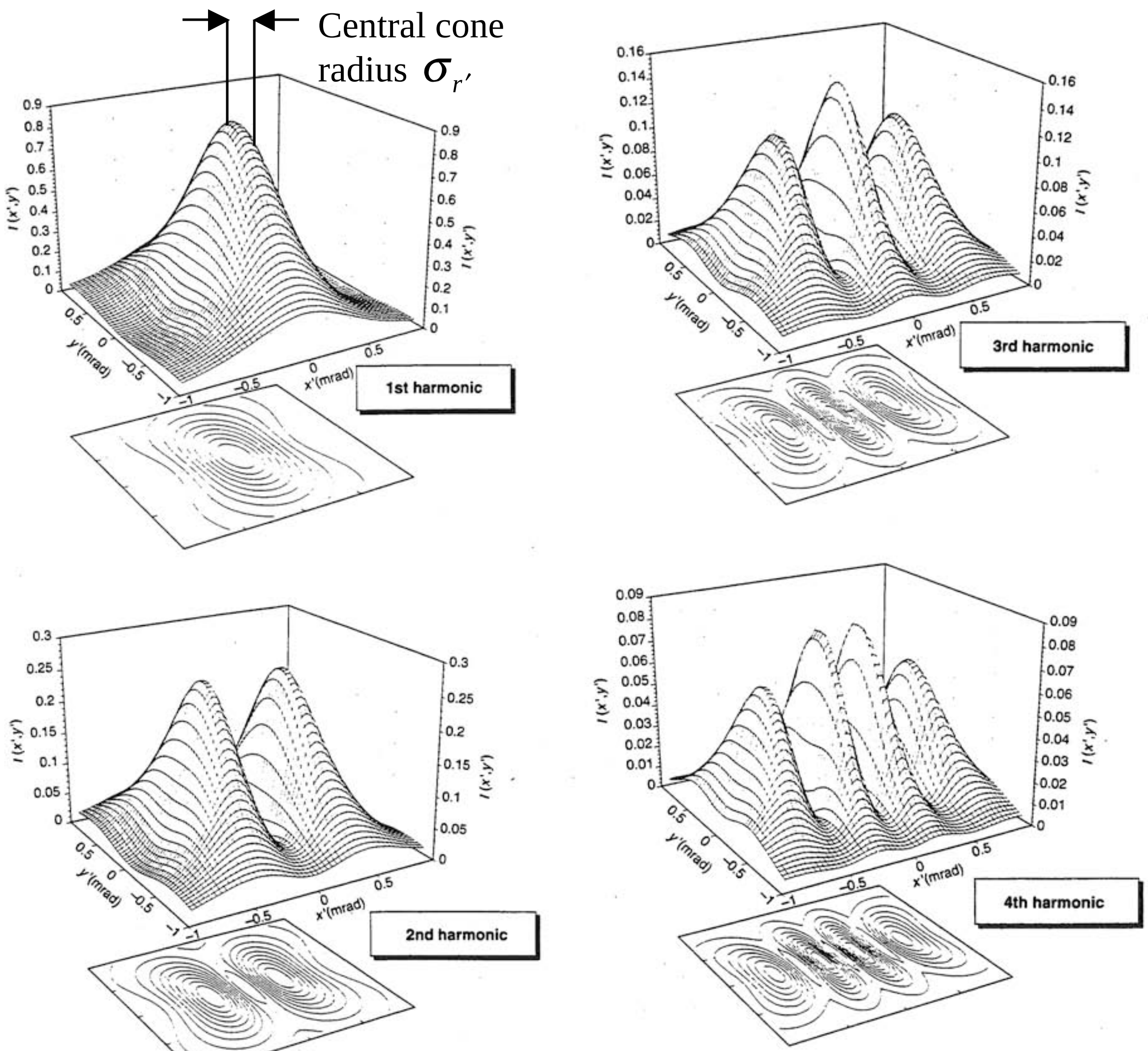


Second wave tilted by $\varepsilon=\lambda/(4a)$ giving an additional path lag of $\lambda/4$ of the signal from P_2 relative to that from P_1

- The fringe blurring caused by $\pm\lambda/4$ path change is considered tolerable so we say that P_1 and P_2 are "coherently" illuminated
- If the beam spread FULL angle is A (equals $\pm\varepsilon$) then the coherence width a is given by the $aA\approx\lambda/2$
- The equation $aA\approx\lambda/2$ is important and *defines* a spatially coherent beam

THE UNDULATOR ONE-ELECTRON PATTERN

Howells, Kincaid 1994



$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left[1 + \frac{K^2}{2} + \gamma^2\theta^2\right]$$

- The central cone of the undulator output is the most useful part and can be selected with a pinhole or slits
- It is emitted in the axial direction ($\theta = 0$) and has RMS angular width equal to $\sigma_{r'}$ which is also about $1/\left(\gamma\sqrt{nN}\right)$ for low K: much narrower than a bend magnet or the overall power emission of an undulator
- It is also has the angular width at which the red shift $\Delta\lambda_n/\lambda_n$ is equal to $1/nN$
- We will model the central cone of the undulator beam as a Gaussian laser mode

SO WHAT IS A GAUSSIAN LASER MODE?

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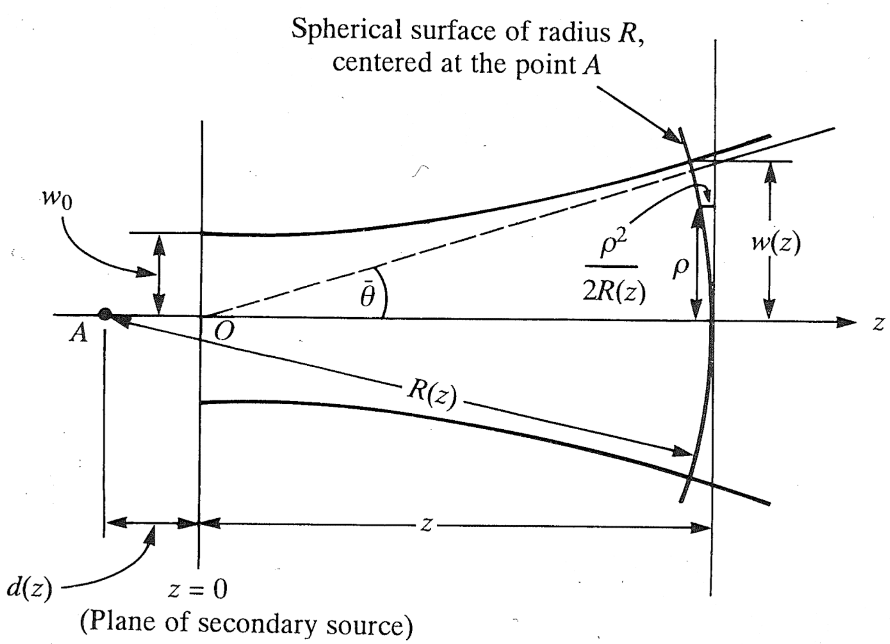


In the laser literature the width-angle product of a Gaussian mode are represented by the following expression [Yariv equation 2.5-18], [Mandel and Wolf equation 5.6-40]

We will recast this into "storage ring" notation:

At axial distance z from the beam waist, $w(z)$ is the off-axis distance at which the electric field E falls to $1/e$ of its on-axis value. At the waist, w_0 equals $w(0)$, while the beam angle, $\left[w(z)/z\right]_{\text{FAR FIELD}}$, is represented by $\bar{\theta}$.

Now suppose that the laser beam intensity depends only on the distance r from the axis, and thus

$I(r) = \mathrm{e}^{-r^2/2\sigma_I^2}$ or $E(r) = \mathrm{e}^{-r^2/4\sigma_I^2}$

so that $\sigma_E = \sqrt{2}\ \sigma_I$. Now E falls to $1/e$ when $r^2/2\sigma_E^2 = 1$ so that $r_{1/e}^{\text{field}} = \sqrt{2}\ \sigma_E = 2\sigma_I$ whence $w_0 = 2\sigma_I$. Similarly $\bar{\theta} = 2\sigma_{I'}$ where the prime implies the beam *angle*.

Substituting into the “laser” expression

$$\bar{\theta} = \frac{\lambda}{\pi w_0} \longrightarrow \boxed{\sigma_I \sigma_{I'} = \frac{\lambda}{4\pi}}$$

THE UNDULATOR ONE-ELECTRON PATTERN

- The on-axis monochromatic one-electron pattern emitted by an undulator is a spatially-coherent beam - also known as a diffraction-limited beam or a wave mode
- We will model it as a Gaussian laser mode with RMS intensity width and angular width equal to σ_r and $\sigma_{r'}$ - so that the ***width-angle product or emittance*** is given by

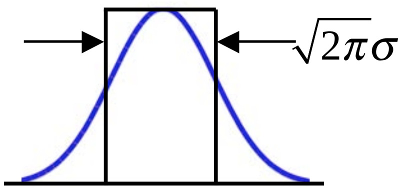
$$\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi}$$

- We will rearrange this using the fact that a rectangle of width $\sqrt{2\pi}\sigma$ and height 1 has equal area to a Gaussian of RMS width σ and height 1 - thus we get

$$\left(\sqrt{2\pi}\sigma_r\right)\left(\sqrt{2\pi}\sigma_{r'}\right) = \frac{\lambda}{2}$$

$$\boxed{\Delta_c \Delta_c' = \frac{\lambda}{2}}$$

Worth remembering this



- Where $\Delta_c = \sqrt{2\pi}\sigma_r$ and $\Delta_c' = \sqrt{2\pi}\sigma_{r'}$ - this is the relation you use to choose beam-line slit widths to get a coherent beam
- This is now the same as our earlier representation of a spatially coherent beam

$$aA = \frac{\lambda}{2}$$

THE UNDULATOR ONE-ELECTRON PATTERN:CONTINUED

Reference: X-ray Data Booklet article by K.-J. Kim

- The width and anglular width are related to the undulator length L as follows

$$\boxed{\sigma_r = \frac{\sqrt{\lambda_n L}}{4\pi}, \quad \sigma_{r'} = \sqrt{\frac{\lambda_n}{L}}, \quad \varepsilon_c = \sigma_r \sigma_{r'} = \frac{\lambda_n}{4\pi}}$$

- For example, for L = 2m, λ = 1Å we obtain $\boxed{\sigma_r = 1.1\ \mu\text{m and } \sigma_{r'} = 7.1\ \mu\text{radian}}$
- The width-angle product in general can also be called the ***phase-space area*** or in 2D the ***phase-space volume - a coherent beam has the minimum possible phase-space volume***
- We characterise the source by the ***brightness* (B)** - it is invariant with respect to beam propagation and focusing - therefore it is an *intrinsic descriptor* of the source strength measured in ph/sec/mm^2/mr^2/0.1%BW - we can now see that the brightness could be called the photon flux per unit phase-space volume per unit fractional bandwidth per unit time
- Since the phase-space volume *of a coherent beam* is $(\lambda/2)^2$ we can conclude that the coherent flux is given by $\mathbf{B}(\lambda/2)^2\, \Delta\lambda/\lambda$
- This wavelength scaling implies that you can get more flux by going to longer wavelength - (provided the available beam is not already single mode)

UNDULATOR WITH A REAL ELECTRON BEAM: SPATIAL MODES

- In an undulator in a normal storage ring the electrons emitting their one-electron patterns have an RMS spread in position of σ_x and σ_y and in angle of $\sigma_{x'}$ and $\sigma_{y'}$
- The combined RMS widths due to the one-electron pattern and the electron beam are

$$\Sigma_x = \sqrt{\sigma_x^2 + \frac{\lambda_n L}{4\pi}} \qquad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \frac{\lambda_n}{L}}$$

$$\Sigma_y = \sqrt{\sigma_y^2 + \frac{\lambda_n L}{4\pi}} \qquad \Sigma_{y'} = \sqrt{\sigma_{y'}^2 + \frac{\lambda_n}{L}}$$

ACTUAL UNAPERTURED BEAM

- Similarly we can define the full widths as before $\Delta_{\Sigma x} = \sqrt{2\pi}\,\Sigma_x$ and $\Delta_{\Sigma x'} = \sqrt{2\pi}\,\Sigma_{x'}$ etc
- The phase space areas in x and y are now $\Delta_{\Sigma x}\Delta_{\Sigma x'}$ and $\Delta_{\Sigma y}\Delta_{\Sigma y'}$
- Of course we could define the beam width and angle by slits - then we could write the phase space areas in x and y as $\Delta_x^{\mathrm{SLIT}}\Delta_{x'}^{\mathrm{SLIT}}$ and $\Delta_y^{\mathrm{SLIT}}\Delta_{y'}^{\mathrm{SLIT}}$
- If the phase space area in x is say m_x times bigger than the coherent phase space area then we say that the beam has m_x modes in x and similarly for y - therefore for the unapertured beam we would have

$$m_x = \frac{\Delta_{\Sigma x}\Delta_{\Sigma x'}}{\lambda/2} \quad \text{and} \quad m_y = \frac{\Delta_{\Sigma y}\Delta_{\Sigma y'}}{\lambda/2}$$

FOR MORE ACCURATE CALCULATIONS

W. Joho - SLS report 1997

RMS angular spread:

$$\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \mu_\lambda k \frac{\lambda_n}{L}}$$

$$\Sigma_{y'} = \sqrt{\sigma_{y'}^2 + \mu_\lambda k \frac{\lambda_n}{L}}$$

where

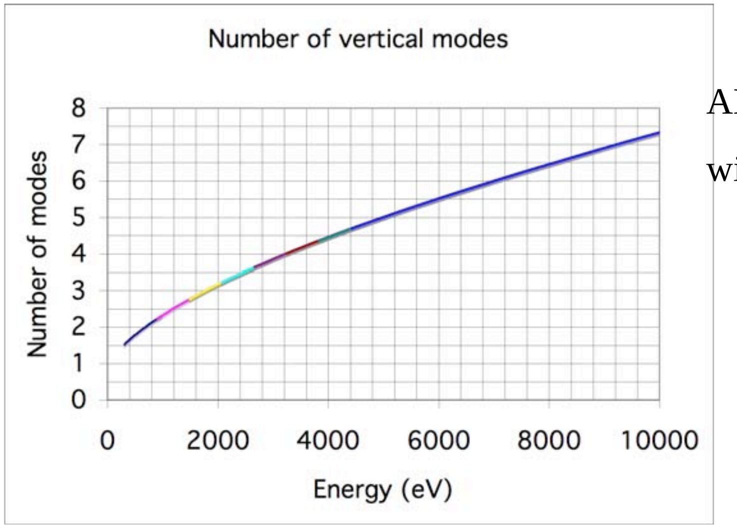
$$\mu_\lambda^2 = 1 + \left(\frac{2\sigma_E nN}{k}\right)^2 \qquad \sigma_E = \frac{\Delta\gamma(\mathrm{RMS})}{\gamma} \qquad k = 0.36$$

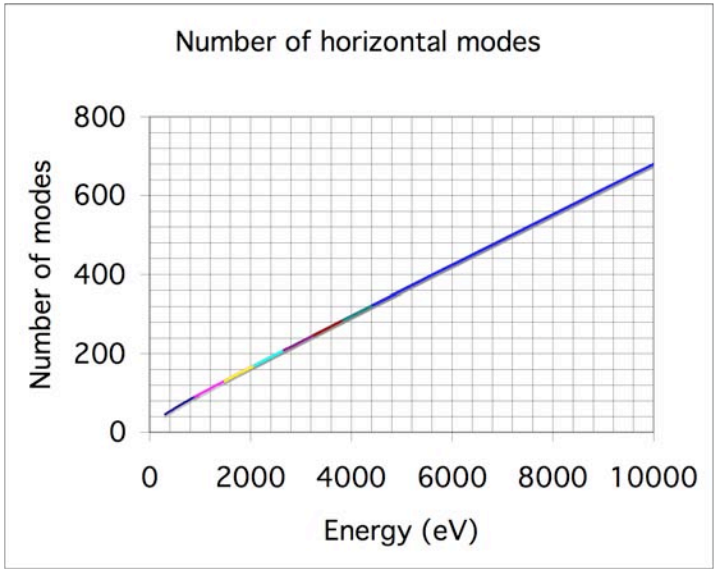
Total RMS bandwidth:

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{\mathrm{RMS}} = \frac{\lambda_u}{2n\lambda}\left[\sigma_{x'}^2 + \sigma_{y'}^2 + 2\mu_\lambda k \frac{\lambda_n}{L}\right]$$

- μ_λ is a correction for the finite energy spread of the electron beam
- γ is the electron energy in units of its rest mass
- k is a constant coming from equating the area under the central peak of the sinc^2 function in the expression for the undulator spectrum with the area under a Gaussian

NUMBER OF MODES: ALS EXAMPLE





Parameters used are for the ALS COSMIC project undulator with upgraded ALS

$$m_x = \frac{\Delta_{\Sigma x}\Delta_{\Sigma x'}}{\lambda/2}$$

$$m_y = \frac{\Delta_{\Sigma y}\Delta_{\Sigma y'}}{\lambda/2}$$

	A	B
5	E(GeV)=	1.9
6	Length(m)=	2
7	SR AMPS=	0.5
8	Und lamu(cm)=	3.2
9	N=	62
10	deltag/g =	0.00097
11	betax (m)=	13.6
12	betay (m)=	2.25
13	emit_x (m.radian)	6.3E-09
14	emit_y (m.radian)	3E-11

	C	D
11	sigx(μm)=	292.71
12	sigy(μm)=	8.22
13	sigx'(μr)=	21.52
14	sigy'(μr)=	3.65

- Note the number of modes expressed as a function of energy is independent of the harmonic number
- The e-beam tends to dominate compared to diffraction except at the lowest energies
- Which one dominates determines the scaling of the brightness with *N* in an interesting way - see [Howells, Kincaid 1994 equations 58-60]

QUESTIONS:

Question1: what are the units when we say the coherent phase-space area is $\lambda/2$

Answer: if lamda is in Å then the phase-space area is in Å.radians - similarly for any other length unit - for example one Å.radian = 10^{-4} mm.mr

Question 2: how many modes does a coherence experiment use and what happens to the unused modes?

Answer: a coherence experiment uses only one mode - the rest must be (unavoidably) wasted

Question 3: are there ways to turn the wasting of modes to advantage?

Answer: yes - you can arrange to over illuminate the beam-line slits which makes the beam line less sensitive to drifts and vibrations

COHERENT FRACTION AT THE "AS-BUILT" ALS

Howells, Kincaid 1994

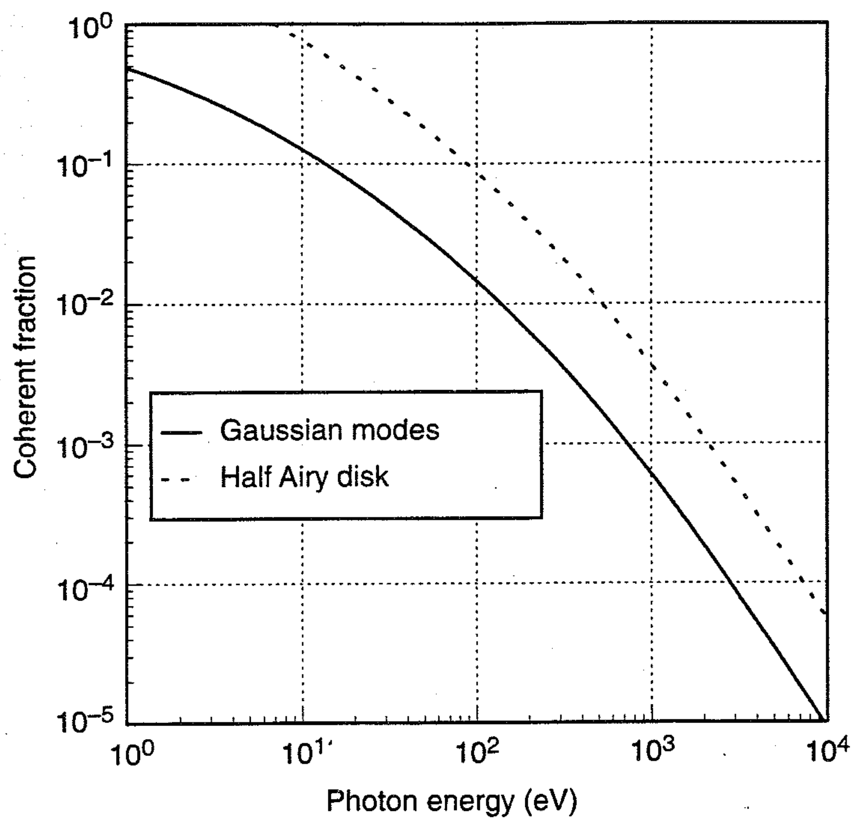
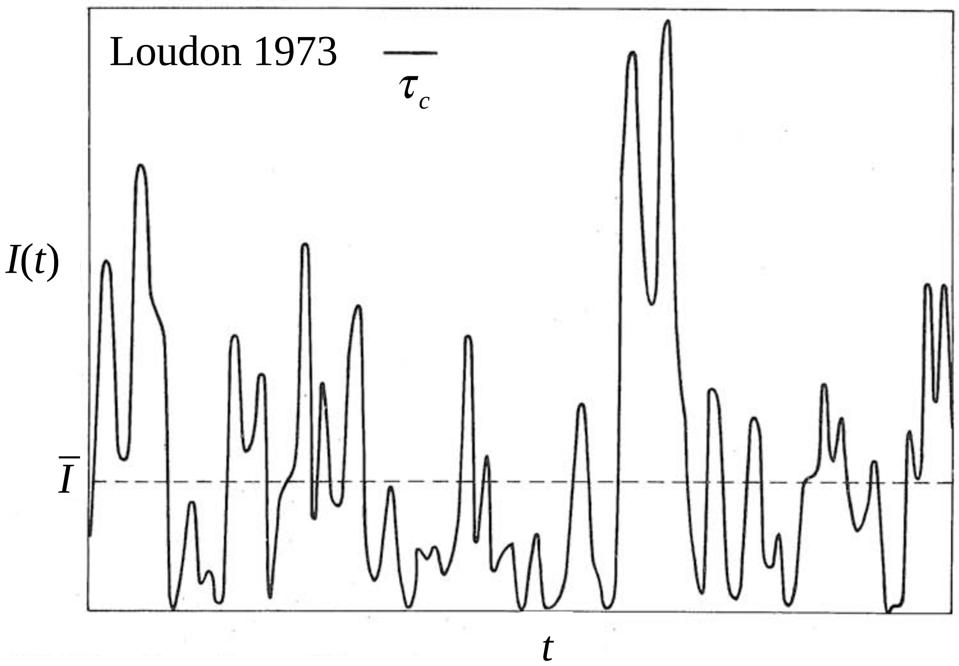


Figure 10. Coherent fraction of the central cone radiation from ALS undulators for the two definitions of coherent phase space discussed in the text.

- So how much do you have to throw away?
- The coherent fraction of the central-cone radiation at the original ALS

PHYSICAL PICTURE OF AN UNDULATOR AS A PSEUDO-THERMAL SOURCE



Negative exponential probability distribution

with mean $\overline{I}$ and standard deviation $\overline{I}$

$$p_I(I) = \begin{cases} \dfrac{1}{\overline{I}} \exp\left(\dfrac{-I}{\overline{I}}\right) & I \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- The instantaneous electric field in a beam of light arriving from a source consisting of a large population of independent emitters (such as an undulator) is the sum of many signals with random amplitudes and phases. The resultant magnitude of such a field has Gaussian probability density by the Central Limit theorem

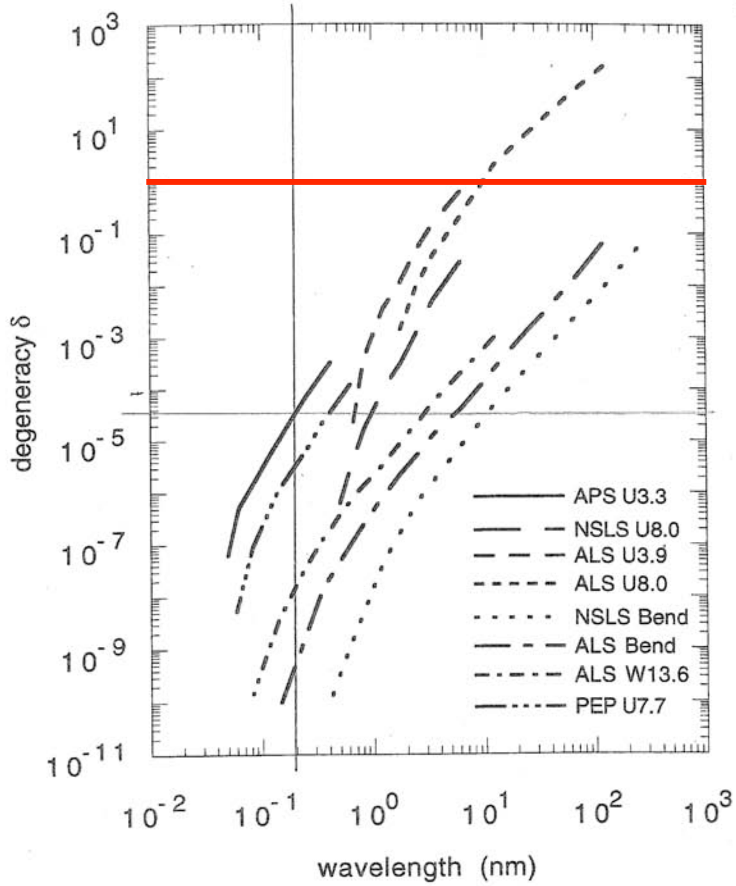
$$p_E(E) = \frac{1}{\sigma_E \sqrt{2\pi}} \exp\left\{\frac{-E^2}{2\sigma_E^2}\right\}$$

- Because the wave trains are emitted at random times the observed intensity fluctuates randomly with a characteristic time scale equal to the wave-train length (the coherence time (τ_c))
- The instantaneous intensity can be shown to have a negative exponential probability distribution [Goodman 1985, equation 4.2-9]
- The integrated intensity W (integration time large compared to τ_c) has a Gaussian probability density

$$p_W(W) = \frac{1}{\sigma_W \sqrt{2\pi}} \exp\left\{\frac{-W^2}{2\sigma_W^2}\right\}$$

THE DEGENERACY PARAMETER

Gluskin et al 1992



δ_W for some US storage rings calculated from

$\delta_W = 8.33\times10^{-25}\,D\mathbf{B}(\text{usual units})\left[\lambda\left(\text{Å}\right)\right]^3$

- The brightness is the number of photons per unit phase-space volume per unit fractional band width per unit time
- The degeneracy parameter δ_W is the number of photons per coherent phase-space volume per coherence time (number of photons per mode) - using *D* for the "on time" fraction of the storage ring

$$\delta_W = D\mathbf{B}\left(\frac{\lambda}{2}\right)^2\left(\frac{\lambda^2}{\Delta\lambda}\frac{1}{c}\right)\left(\frac{\Delta\lambda}{\lambda}\right) = \frac{D\mathbf{B}\lambda^3}{4c}$$

- δ_W is the most fundamental measure of source strength and is the average number of wave trains that overlap in one spatial-temporal mode [Goodman 1985 sect 9.3]
- It determines the performance of experiments like Hanbury-Brown-Twiss interferometry and phenomena such as photo-electron bunching [Goodman 1985, ch 9]
- X-rays are bosons so values of δ_W greater than one are possible - for electrons δW<1 always but nevertheless electron sources currently achieve higher brightness values [Spence and Howells 2002]

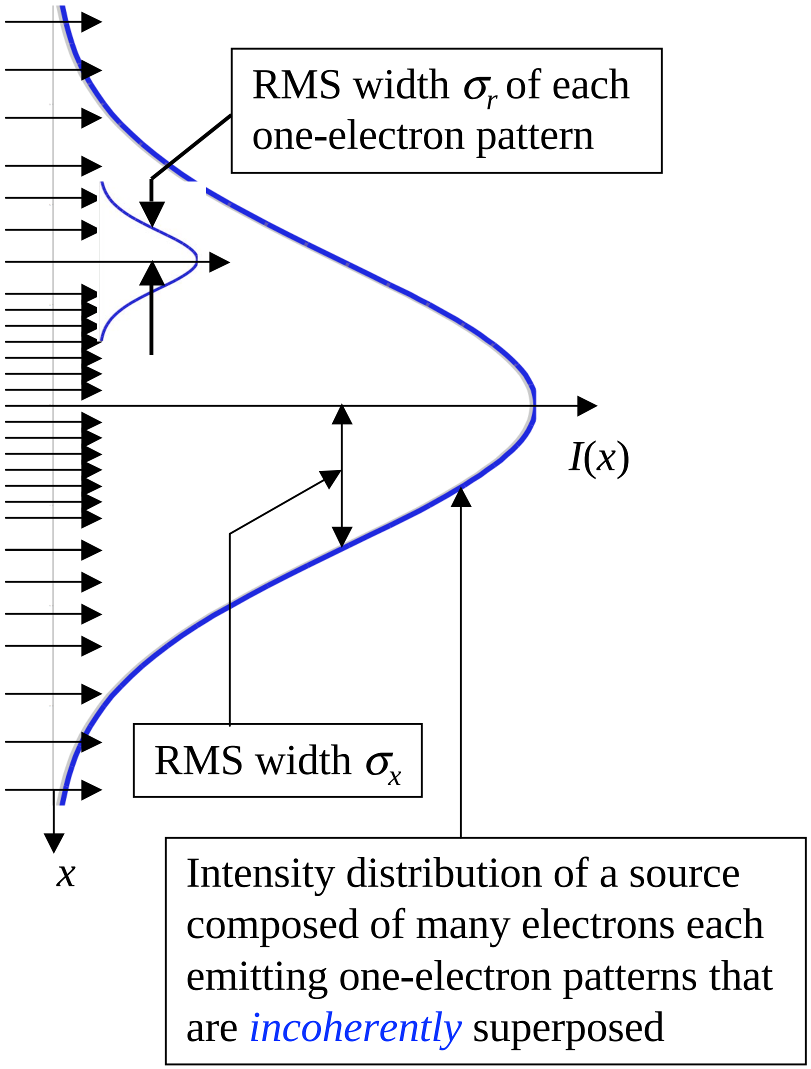
COHERENCE PROPERTIES OF VARIOUS SOURCES

Source type	*E* field instantaneous magnitude $p_E(E)$	Instantaneous intensity (I) $p_I(I)$	Integrated intensity (W) (time >> τ_c) $p_W(W)$	Band width	Degeneracy parameter (δ_W)	Character
Single-mode laser well above threshold	~constant	~constant	~constant	narrow	>>>1	orderly
Thermal light	Gaussian	Negative exponential	Gaussian	Wide	<<1	Chaotic
Pseudothermal light*	Gaussian	Negative exponential	Gaussian	Narrow	SXR: < or ~ 1 HXR: <1	Chaotic

***Monochromatic synchrotron radiation or laser light passed through a rotating diffuser**

- Laser and undulator radiation have certain similarities: chiefly that their phase space volumes are very small compared to competing sources
- In fact, as shown here, the physics of the emission and the statistical properties of the resulting radiation are very different

DIGRESSION: COHERENCE PROPERTIES OF A REAL UNDULATOR BEAM (advanced topic)



- A source which consists of a wide distribution of small independent emitters is known as a quasi-homogeneous source [Goodman 1985 equation 5.6-27]. Its mutual intensity (*in one dimension for simplicity*) is given by

$$J_{12}\left(x+\frac{\Delta x}{2},x-\frac{\Delta x}{2}\right)=\sqrt{I\left(x+\frac{\Delta x}{2}\right)}\sqrt{I\left(x-\frac{\Delta x}{2}\right)}\;\mu_{12}(\Delta x)$$

- A single one-electron pattern is ideally coherent so its complex coherence factor or normalized correlation function (with RMS coherence width $\sigma_{\Delta x}\approx\sigma_r$) is modeled as [Howells, Kincaid 1994]

$$\mu_{12}(\Delta x)=\exp\left\{-\frac{-\Delta x^2}{2\sigma_{\Delta x}^2}\right\}$$

- Therefore J_{12} is

$$J_{12}\left(x+\frac{\Delta x}{2},x-\frac{\Delta x}{2}\right)=\exp\left\{-\frac{-x^2}{2\sigma_x^2}\right\}\exp\left\{-\frac{-\Delta x^2}{2\sigma_{\Delta x}^2}\right\}$$

DIGRESSION: COHERENCE PROPERTIES OF A REAL UNDULATOR BEAM (continued)

- So now we have the mutual intensity J_{12} at the source (x plane) - to use the undulator beam in a coherence experiment we need to know J_{12} at a downstream plane (x_1 plane) where we may put an optic or a sample - so we need to know how to PROPAGATE J_{12}
- To do this we use a trick introduced by K.-J. Kim and explained in [Howells, Kincaid 1994] (for another approach see [Goodman 1985] section 5.4 especially eq 5.4-8)
- Kim defined the Fourier transform of the mutual intensity as the "brightness function (**B)**" (also known as the generalized radiance [Mandel and Wolf 1995])
- The trick is that the form of the brightness function is conserved during propagation by distance z and its value after propagation it is determined simply by a linear transformation of the coordinates

$$\mathbf{B}\left(x_1,\, x_1';z\right) = \mathbf{B}\left(x - x'z,\, x';0\right)$$

- So to get J_{12} after propagation the procedure is (1) Fourier transform J_{12} at the source to get **B** at the source (2) apply the linear transform to propagate **B** to the x_1 plane at distance z, (3) inverse Fourier transform **B** to get the propagated J_{12}
- This is done in [Howells, Kincaid 1994] and the errata to that with the following result

DIGRESSION: COHERENCE PROPERTIES OF A REAL UNDULATOR BEAM (result)

Defining

$$\frac{1}{\sigma_{\Delta x}^2} = \frac{1}{4\sigma_x^2} + \frac{1}{\sigma_r^2} \approx \frac{1}{\sigma_r^2}, \qquad \sigma_{x_1}^2 = \frac{z^2}{k^2\sigma_{\Delta x}^2}, \qquad \sigma_{\Delta x_1}^2 = \sigma_{\Delta x}^2 + \frac{z^2}{k^2\sigma_x^2} \approx \frac{z^2}{k^2\sigma_x^2}, \qquad \psi = \frac{x_1 \Delta x_1 \lambda z}{2\pi\sigma_x^2\sigma_{\Delta x_1}^2}$$

Noting that $\sigma_{x_1}^2$ refers to the width of the beam while $\sigma_{\Delta x_1}^2$ refers to coherence width, we have

$$J_{12}\left(x_1 + \frac{\Delta x_1}{2},\ x_1 - \frac{\Delta x_1}{2}\right) = \frac{2\pi\sigma_{\Delta x}}{\sigma_{\Delta x_1}} \underbrace{\exp\left\{-\frac{-(x_1)^2}{2\sigma_{x_1}^2}\right\}}_{\text{Beam intensity distribution}} \underbrace{\exp\left\{-\frac{-(\Delta x_1)^2}{2\sigma_{\Delta x_1}^2}\right\}}_{\text{Complex coherence factor } \mu_{12}} \exp(i\psi)$$

- Set $\Delta x_1 = 0$ (the two typical points coincide) - we then get the suitably scaled beam intensity distribution of RMS width σ_{x_1}
- Set $x_1 = 0$ (the two typical points are symmetrically disposed about the axis) - the phase $\psi = 0$ and we get the correlation function between the two points at spacing Δx_1 with RMS width $\sigma_{\Delta x_1}$
- In the approximation $k\sigma_r << z/\sigma_x$ which is usually valid for synchrotron undulators the result for the rms coherence width is the same as that given by the van-Cittert-Zernike theorem

DIGRESSION: COHERENCE PROPERTIES OF A REAL UNDULATOR BEAM (summary in words)

The beam intensity distribution:

- The width of the delivered beam intensity depends on the coherence width of the x-ray emitters in the source
- The relationship is reciprocal - the beam width gets smaller when the coherence width gets larger
- Question: then why are undulator x-ray beams narrow? Answer: because the coherence width of about one micron is still large when expressed in wavelengths

The beam coherence width:

- The coherence width of the delivered beam depends on the source size
- The relationship is reciprocal - the coherence width gets smaller when the source size gets larger (the "bad-collimation" argument expressed in the van-Cittert-Zernike theorem)
- The van-Cittert-Zernike theorem is valid in the approximation $k\sigma_r << z/\sigma_x$ which is usually true for synchrotron undulators

THE BEAM EMITTANCE MUST BE AS SMALL OR SMALLER THAN THE ACCEPTANCE OF THE EXPERIMENT OR SOME LIGHT MUST BE LOST

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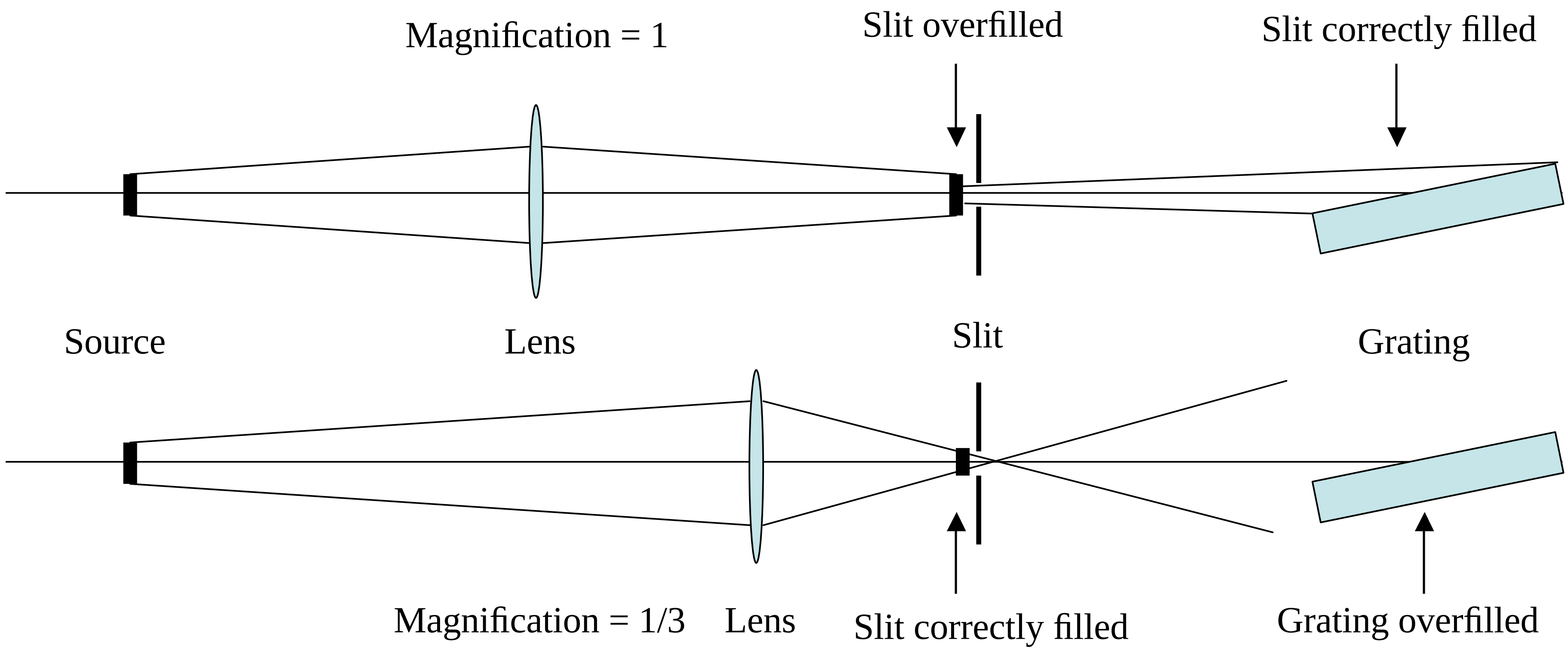


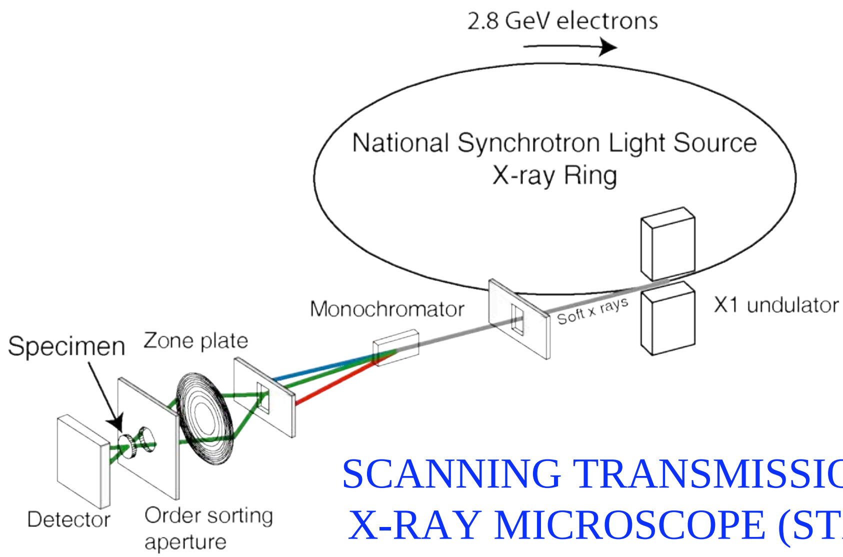
- Light from the source is focused on the slit - initially the angle is right but the width is too great
- We change the magnification to reduce the size but that causes the angle to increase
- The width-angle product is conserved during focusing - it can be reduced only by losing light
- The beam emittance is greater than the monochromator acceptance - loss of light cannot be avoided

EXAMPLES OF PHASE-SPACE ACCEPTANCE OF OPTICAL COMPONENTS AND EXPERIMENTS

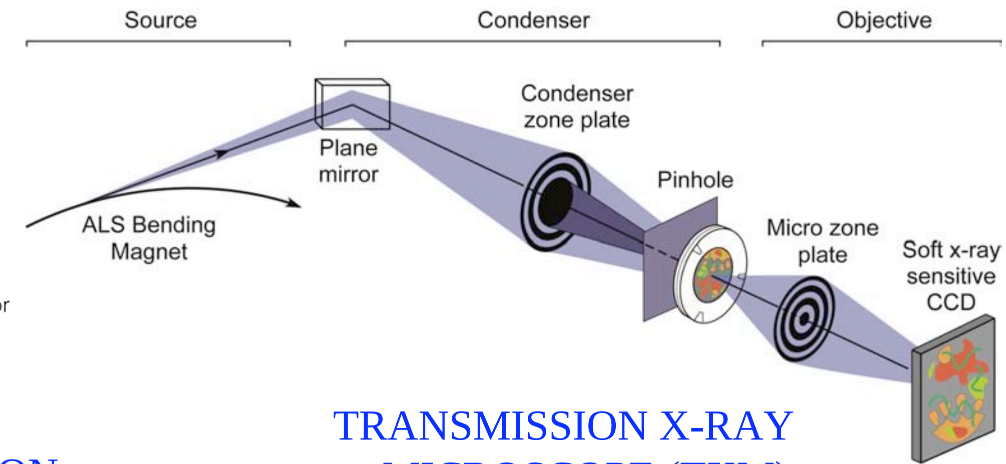
Optical component or experiment	Limitation to beam width	Limitation to beam angular spread	Comments
Crystallography experiment	Size of crystal	Clean separation of neighboring orders	
Plane grazing angle mirror	Optical aperture of the mirror	75% say of the critical angle	
Grating monochromator with entrance slit	Entrance slit set for required resolution	Angle subtended at the slit by the grating aperture	Equal to the slit-width-limited wavelength resolution times the number of grooves
Full field transmission x-ray microscope in bright field (TXM)	Sample width	Twice the numerical aperture of the objective lens	At synchrotrons the beams normally have a phase-space area that is too small and has to be artificially increased by "wobbling"
Scanning transmission x-ray microscope (STXM)	Secondary source (pinhole) size	Angle subtended at secondary source by the zone plate	The real requirement is coherent illumination – that is the width-angular-spread product ≤ lamda/2
Double crystal monochromator	Crystal size	Rocking curve width	
Compound refractive lens	Transmission of the lens diminishes with distance from axis	Lens angular psf due to imperfections	

- If the phase-space acceptance of the experiment is smaller than the phase-space area of the x-ray beam in the *x* plane (say) then light must be lost - similarly in *y* - in this case we say it is a brightness experiment
- If the acceptance of the experiment is greater than the beam emittance then 100% of the beam can be accepted - this is a flux experiment
- If the acceptance of the experiment is for a beam of phase-space area $\lambda/2$ then this is a coherence experiment and for current storage rings light must always be lost

SCANNING-TRANSMISSION (STXMs) AND TRANSMISSION X-RAY MICROSCOPES (TXMs)



SCANNING TRANSMISSION X-RAY MICROSCOPE (STXM)



TRANSMISSION X-RAY MICROSCOPE (TXM)

	STXM	TXM
Usual source	Undulator	Bend magnet
Required beam	Coherent	Maximum incoherence
Principal imaging mode	Incoherent bright field	Incoherent bright field
Experiment class	Coherence experiment	Flux experiment
Data taking style	Serial	Parallel

Table 3: Zone plate microscopes

Microscope/ location	Light source	Illumination/ monochromator	Focusing, imaging	Contrast mechanisms	Techniques, x-ray energy	Citation
MES STXM	ALS undulator	grating	STXM	absorption, magnetization	NEXAFS, MCD 100 to 2000eV	[Tyliszczak 2004, Warwick 2004]
Polymer STXM	ALS bend magnet	grating	STXM	absorption	NEXAFS 250 to 750 eV	[Warwick 2002, Kilcoyne 2003]
XM-1 TXM	ALS bend magnet	zone plate condenser/mono	TXM	absorption, magn-etization, phase	Tomog, MCD 200 to 1800eV	[Meyer-Ilse 2000a]
XM-2 TXM	ALS bend magnet	zone plate condenser/mono	TXM	absorption, phase	Tomog 200 to 7000eV	
2-ID-B	APS undulator	multilayer-coated grating	STXM	abs, fluor, phase XANES, tomog	tomography 600 to 4000eV	[McNulty 2003a, McNulty 2003b]
2-ID-D	APS undulator	Crystal/multilayer	STXM	diffraction	strain mapping 6 to 20keV	[Cai 2003, McNulty 2003a]
2-ID-E	APS undulator	Crystal/multilayer	STXM	fluor, XANES diff, microdiff	5 to 35keV	[McNulty 2003a]
26-ID	APS undulator	Crystal/multilayer	STXM TXM	abs, fluor, diff XANES	3-30keV	[McNulty 2003a]
BL20B2	Spring8 bend magnet	crystal	STXM	absorption	4 to 113keV opt testing, tomog	[Suzuki 2003, Takano 2003]
BL47XU	Spring8 undulator	crystal	TXM	absorption	5-37.7keV tomography	[Suzuki 2003, Uesugi 2003]
BL20XU	Spring8 undulator	crystal – 250 m beam line	STXM	absorption	8-37.3, 24-113keV, μbeams	[Suzuki 2003]
BL24XU	Spring8 undulator	crystal	TXM	phase contrast	8.77-12.85, 12.4-18.17 keV	[Tsusaka 2001, Kagoshima 2003]
BL12	Ritsumeikan bend magnet	zone plate condenser/mono	TXM	absorption	water window,	[Takemoto 2003]
8A1 U7 SPEM	Pohang undulator	grating	STXM	photoemission	nanoXPS 100 to 1000eV	[Shin 2003, Yi 2005]
1B2 hard xray	Pohang b. magnet	crystal	TXM	absorption	6.95keV	[Youn 2005]
U41TXM	BESSYII undulator	zone plate condenser/mono	TXM	absorption, phase	water window, 2D, 3D imaging	[Guttman 2003, Wiesemann 2003]
UE46TXM	BESSYII undulator	zone plate condenser	TXM	absorption, magnetization	MCD 0.2 to 2keV	[Eimüller 2003]
TWINMIC	ELETTRA Undulator	grating	STXM & TXM	absorption phase contrast	NEXAFS 250 to 2000eV	[Kaulich 2003]
BL2.2 ESCA	ELETTRA undulator	grating	STXM	absorption photoemission	nanoXPS 200 to 1400eV	[Casalis 1995, Kiskinova 2003]
KINGS STXM	laser plasma	gas filtered spectrum	STXM	absorption	water window	[Michette 2000]
X1A STXMs	NSLS undulator	grating	STXM	absorption diffraction phase contrast	NEXAFS, cryomicroscopy 250 to 1000eV	[Jacobsen 2000a]
ID21 microscopes	ESRF undulator	grating, crystal	STXM and TXM	absorption fluorescence diffraction phase contrast	NEXAFS 200 to 7000eV	[Susini 2000]
ID22 imaging	ESRF undulator	crystal	STXM	absorption fluorescence phase contrast	5 to 70keV	[Weitkamp 2000]
Aarhus TXM	ASTRID bend magnet	zone plate condenser/mono	TXM	absorption	typically 517eV	[Uggerhøj 2000]
XRADIA	Chromium anode	reflective condenser	TXM	absorption, phase	tomography 5.4 keV	[Scott 2004]

- From: Howells et al, SoM
- 14 STXM's
- 12 TXM's
- 2 Combined STXM/TXM
- 28 Total almost all on synchrotrons

ZONE PLATE LENSES

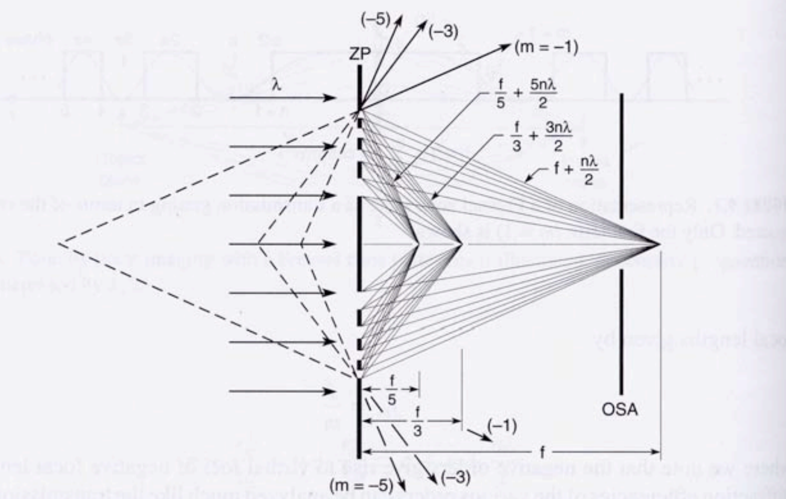
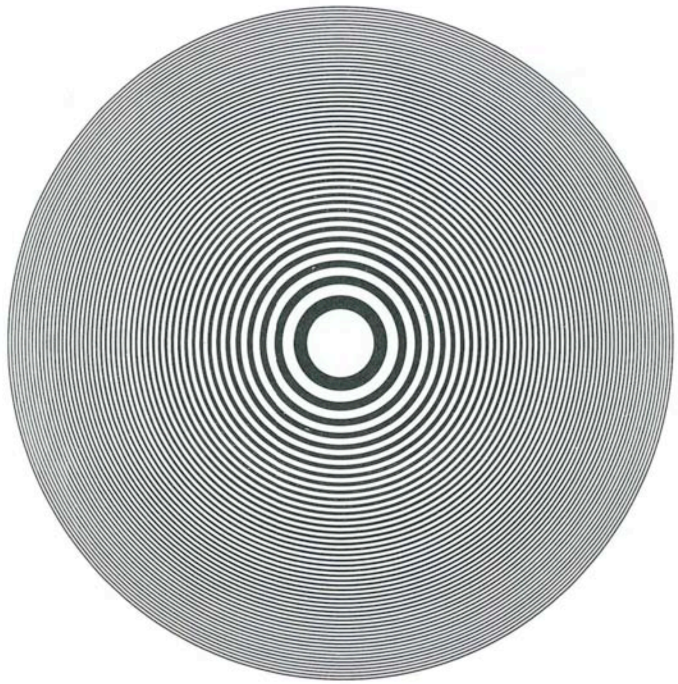
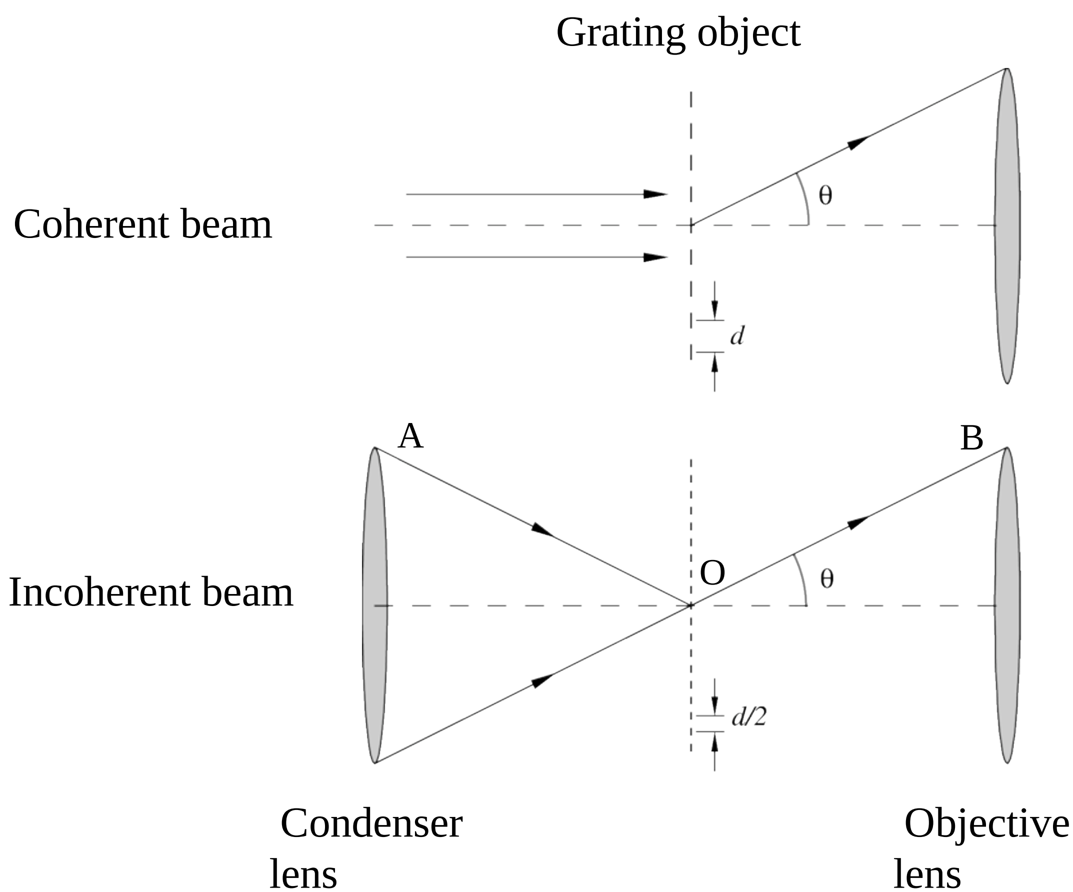


FIGURE 9.8. Zone plate diffractive focusing is illustrated for the first three positive orders. An order sorting aperture (OSA), of the type that would be used to block all but the first order, is also shown. Negative orders ($m = -1, -3, -5$) are shown as solid lines diverging from the optical axis, and projected backward to virtual foci (behind the lens) by dashed lines.



- The STXMs and TXMs we are discussing here both use zone plate objective lenses - these are small around 100 μm and resolution determining
- The condenser system for STXM is a beam line monochromator that focuses x-rays to a pinhole that acts as a secondary source of coherent illumination of the zone plate
- The condenser for a TXM has traditionally been a zone plate that also acts as a monochromator - traditionally such zone plates are large (around a centimeter) and difficult to make - they have been the Achilles heel of the TXM
- A new generation of single reflection monocapillaries promises a much better solution see [Howells et al SoM]

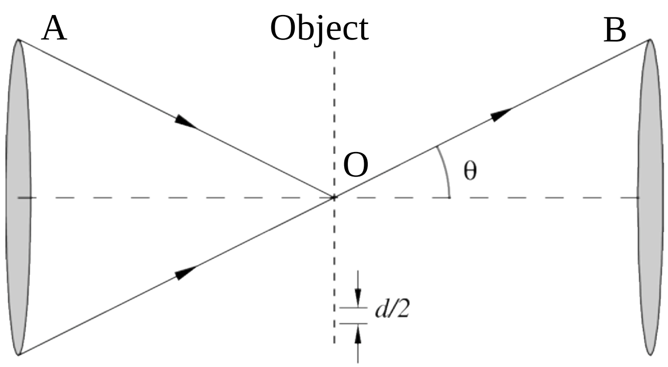
TXM DESIGN: COHERENCE IS BAD FOR YOU???



- Note that the largest deviation angle in the top diagram is θ while in the bottom diagram it is 2θ (angle AOB)
- The grating deviates the incoming ray according to $\lambda = D \sin\theta$ where D is the period of the grating (or $1/D$ is the frequency)
- So the highest resolvable frequency is achieved by accepting the largest deviation angle - AOB
- So the best configuration is to have a wide-angle beam on both the inward and outward side of the sample (matching of the condenser NA to the objective NA)
- So is coherence bad for you? - well yes if you used a coherent beam to illuminate a TXM - but no one ever does that
- However they do tend toward doing something like that because for fabrication reasons the NA of condenser zone plates is always at least a factor 2 smaller than a state-of-the-art objective zone plate

RECIPROCITY

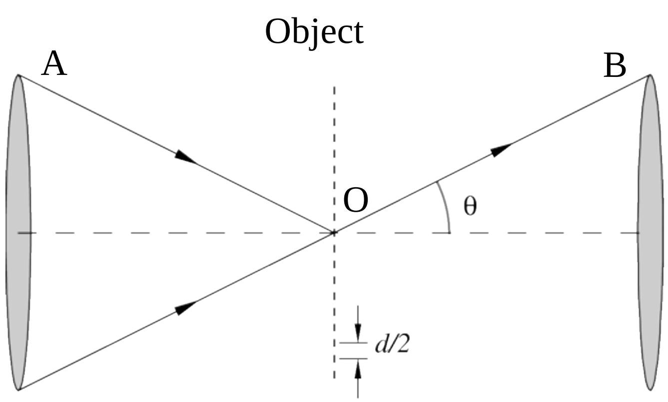
TXM



Condenser lens

Objective lens

STXM



Objective lens

Large-area Detector

- Note that for the STXM I just relabelled the TXM diagram - there is a fundamental reason why this is possible - reciprocity [Howells et al, SoM]
- The optics theory including the contrast transfer and resolution is mathematically the same for both the TXM and STXM provided both are imaging in incoherent bright field mode and you put the optical components in the opposite order - the understanding is that the detector in STXM plays the same role as the condenser in TXM - that is determining the beam angle on one side of the object
- The result is that for similarly large condenser and detector and the same objective zone plate you get the same resolution in STXM and TXM
- Well almost - actually the STXM has a slight advantage because it's much easier to get a large area detector than a large area condenser zone plate

EFFECT OF NA MATCHING ON A MICROSCOPE RESOLUTION

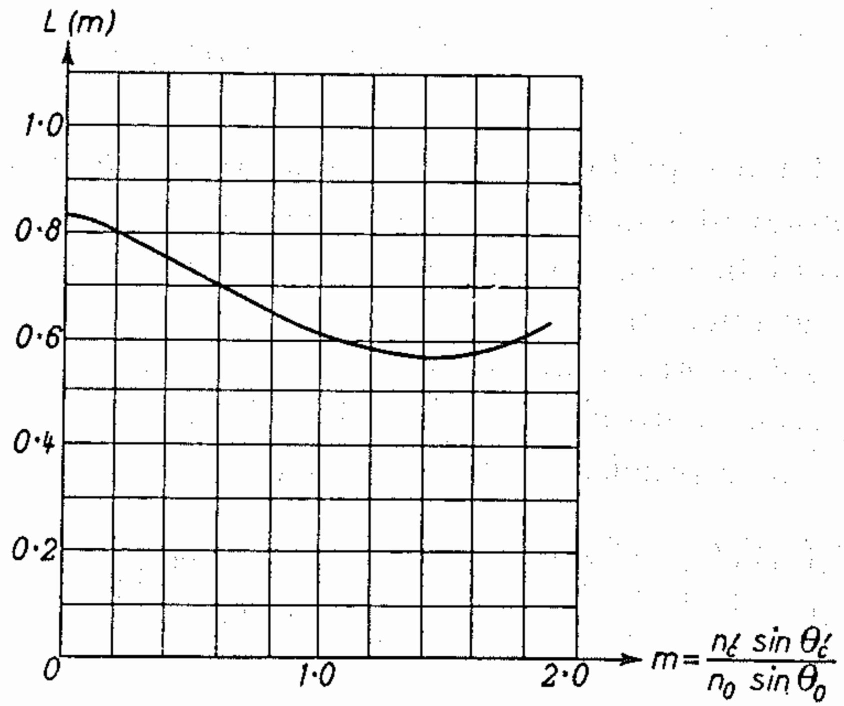


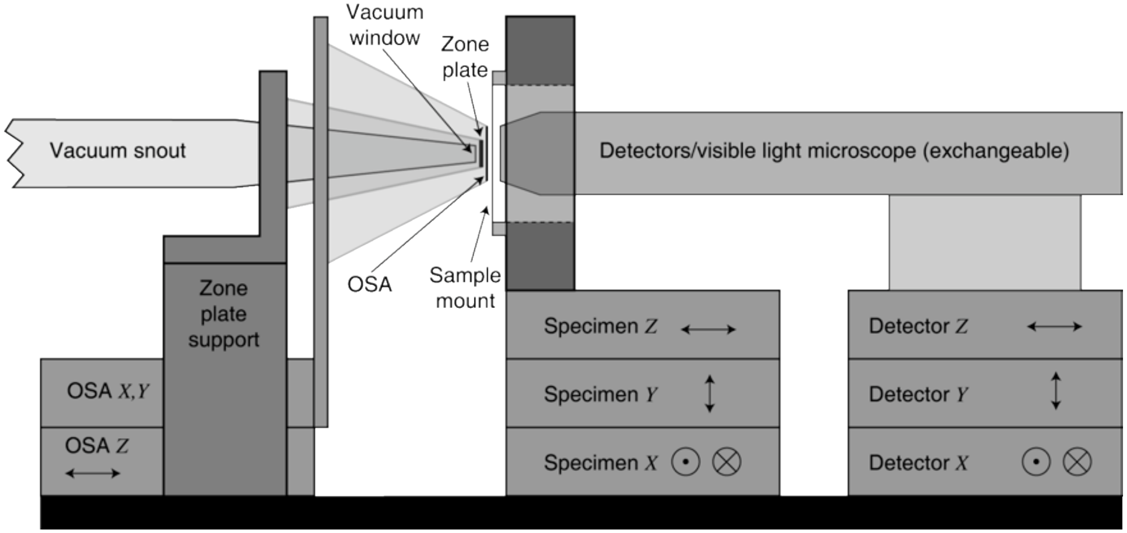
Fig. 10.13. Effect of the condenser aperture on the resolution of two pinholes of equal brightness. (After H. H. Hopkins and P. M. Barham, *Proc. Phys. Soc.*, **63** (1950), 72.)

Two-point resolution $L(m)$ in units of $\lambda/\mathrm{NA}_{obj}$ as a function of the ratio $(\mathrm{NA}_{cond})/(\mathrm{NA}_{obj})$

CHALLENGE: OVERCROWDING AROUND THE SAMPLE, SOLUTION: HARDER X-RAYS

ESRF

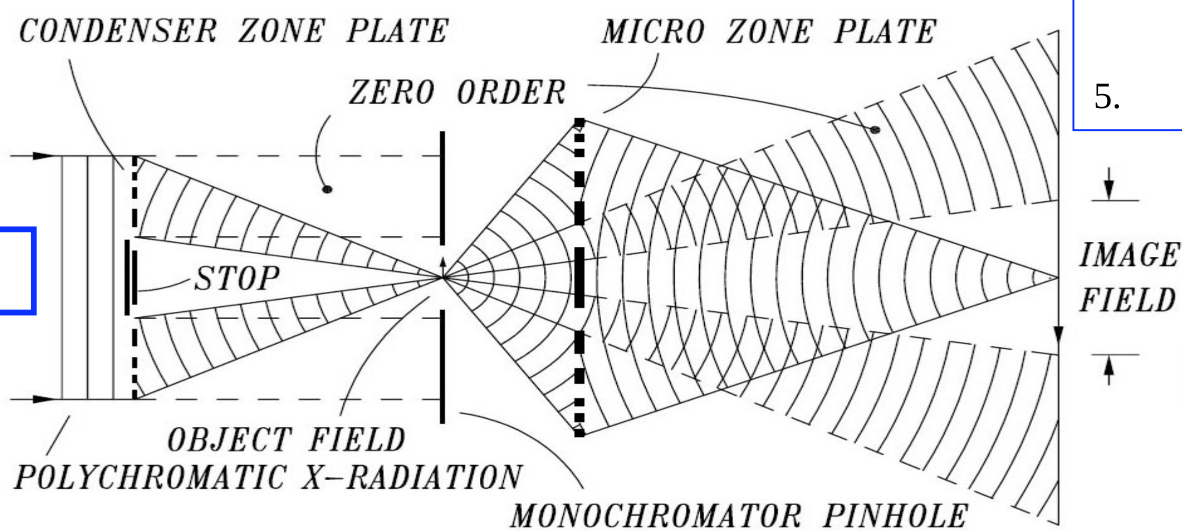
STXM



Problems:

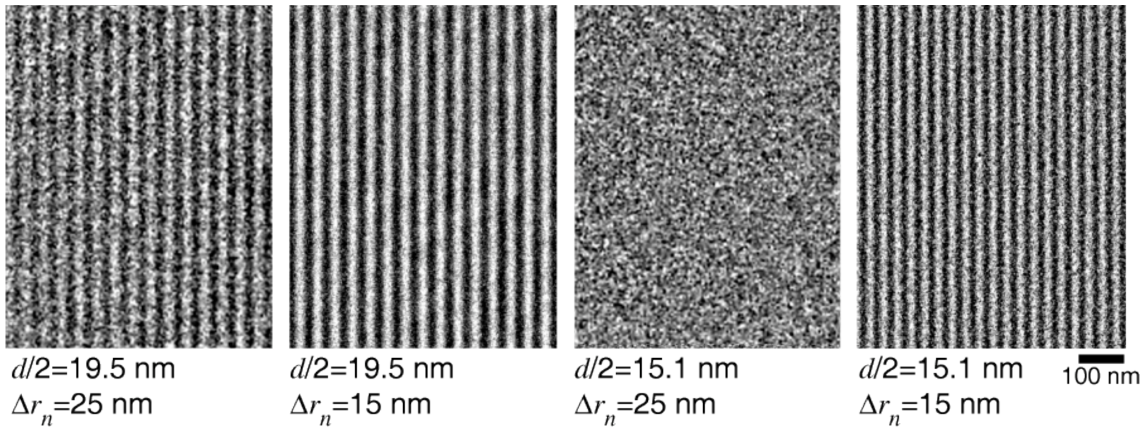
1. Overcrowding
2. Focal length too small - bad for sample rotation
3. Depth of focus too small - bad for tomography reconstructions
4. Solution to all: harder x-rays
5. But it's not easy

TXM (Göttingen)



THE BERKELEY CENTER FOR X-RAY OPTICS RESOLUTION TEST

Chao et al 2005



Test objects are thin transverse slices of multilayer coatings of the given layer thickness

- Microscope: XM-1, ALS BM beam line 6.1
- X-ray energy: 815 eV
- Objective zone plate:
  - outer zone width: 15 nm
  - diameter: 30 µm
  - thickness: 80 nm of gold
  - number of zones: 500
  - focal length: 0.3 mm
  - depth of focus: ≈ 0.1 µm
- Condenser zone plate:
  - outer zone width: 55 nm
  - diameter: 9 mm
  - writing time (in 2003): 48 hours

- Ratio $\sigma = NA_{cond}/NA_{obj}$: 0.38
- Theoretical resolution: 12 nm (matched NAs)
- Measured resolution: ≤ 15 nm (actual NAs)

Comments:

- Definition of resolution: the half-period of the finest square wave that can be imaged with 26.5% modulation - similar to the Rayleigh criterion for a two-point object
- This zone plate is technically impressive but it will be hard to use especially for tomography on account of the short focal length and DOF
- They would be shorter still in the water window but longer in linear proportion to the energy at higher energies

ZERNIKE PHASE CONTRAST

ESRF

Principle:

Sample pixel $\Rightarrow$ small phase change ϕ

Emerging wave field = $e^{i\phi} \approx 1 + i\phi$

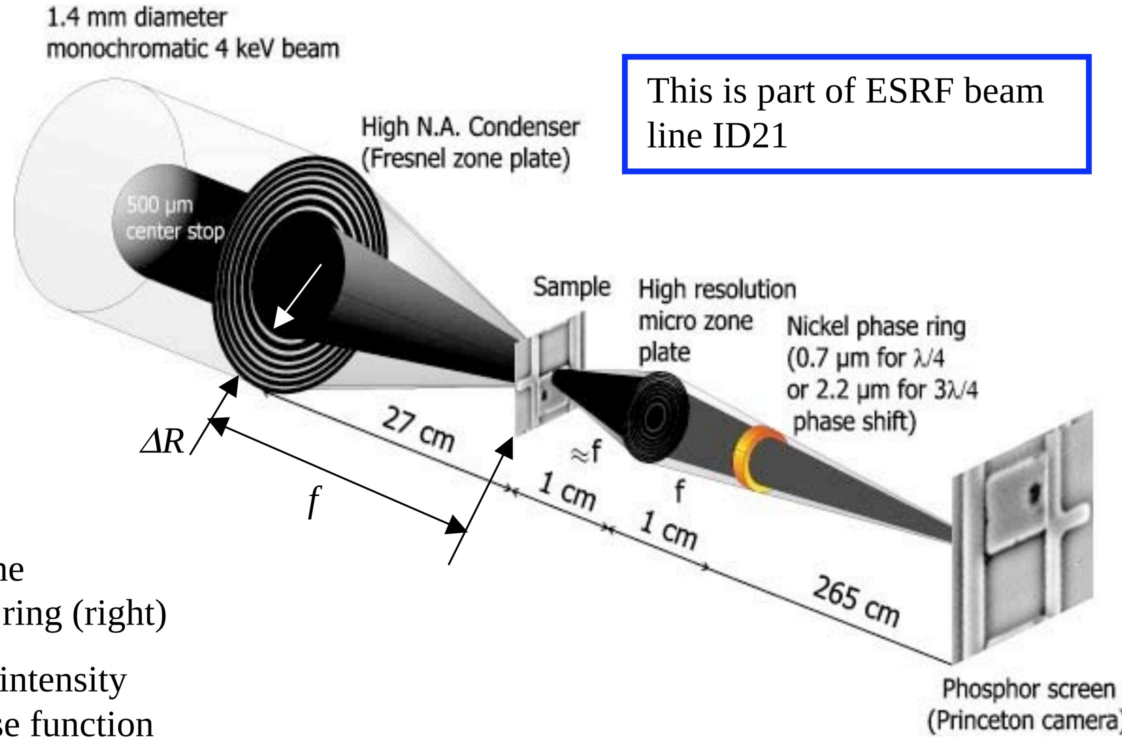
Intensity = $|1 + i\phi|^2 = 1 + \phi^2$

Apply a $\pi/2$ phase change to the 1

Intensity = $|i1 + i\phi|^2 = 1 + 2\phi + \phi^2$

ϕ makes a linear change to the intensity

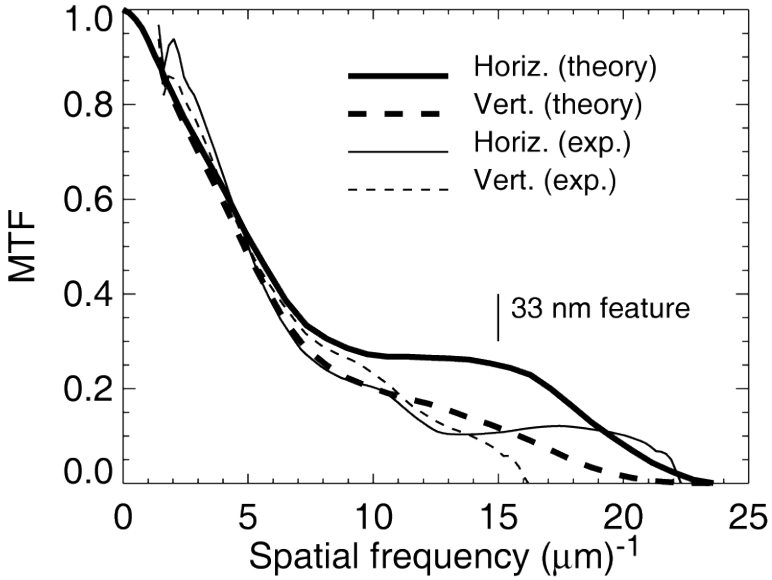
This is part of ESRF beam line ID21



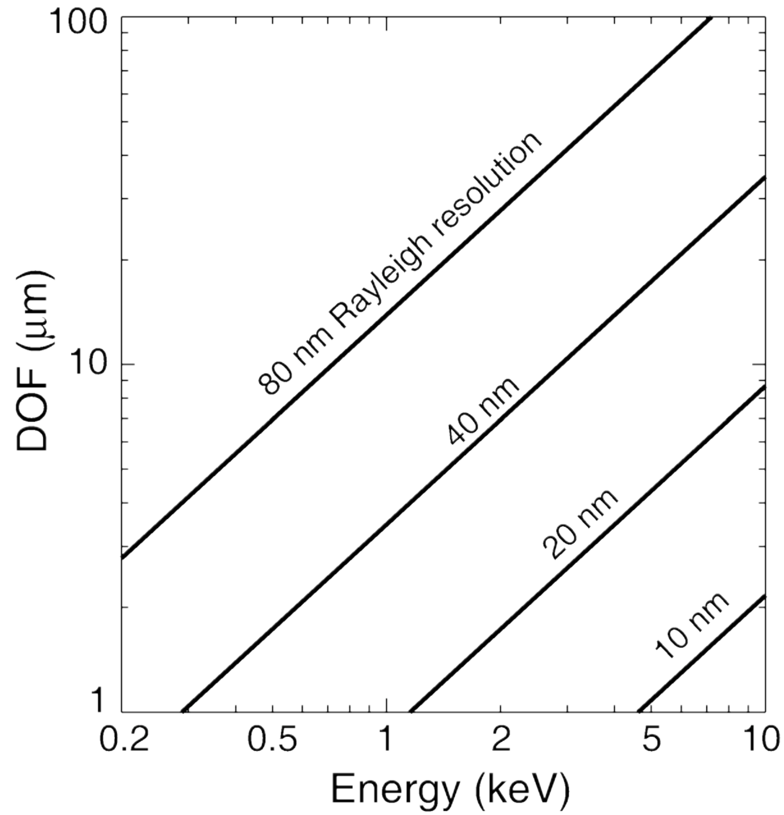
- In practice we apply a $\pi/2$ phase change to the *undiffracted* beam by means of the Ni phase ring (right)
- With good coherence this delivers an image intensity pattern which is a linear mapping of the phase function

- However with bad coherence it renders phase jumps as a zero-crossing function and a phase rectangle function as two zero crossings with a flat zero area in between - in other words the rectangle looks nothing like a rectangle and we have a serious fidelity problem
- The solution is better coherence - in fact to make the rectangle look like a rectangle we need to choose ΔR so that the coherence width $w_c = \lambda f/\Delta R$ of the illumination is at least equal to the size of the rectangle (or other feature)
- There is price for this - one has to throw away modes and therefore flux

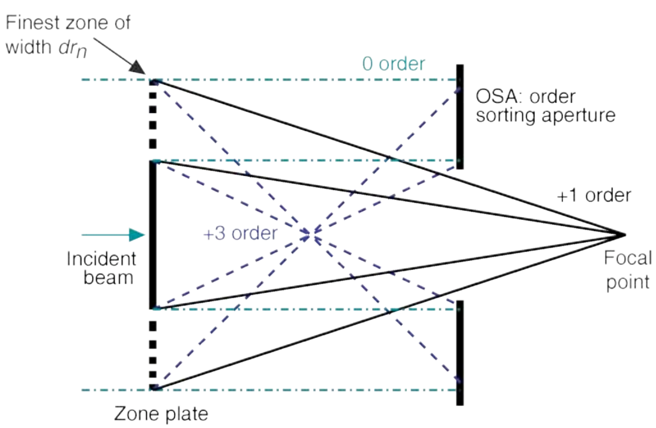
Title



1.0
0.8
0.6
0.4
0.2
0.0
MTF
0
5
10
15
20
25
Spatial frequency (µm)-1
Horiz. (theory)
Vert. (theory)
Horiz. (exp.)
Vert. (exp.)
33 nm feature

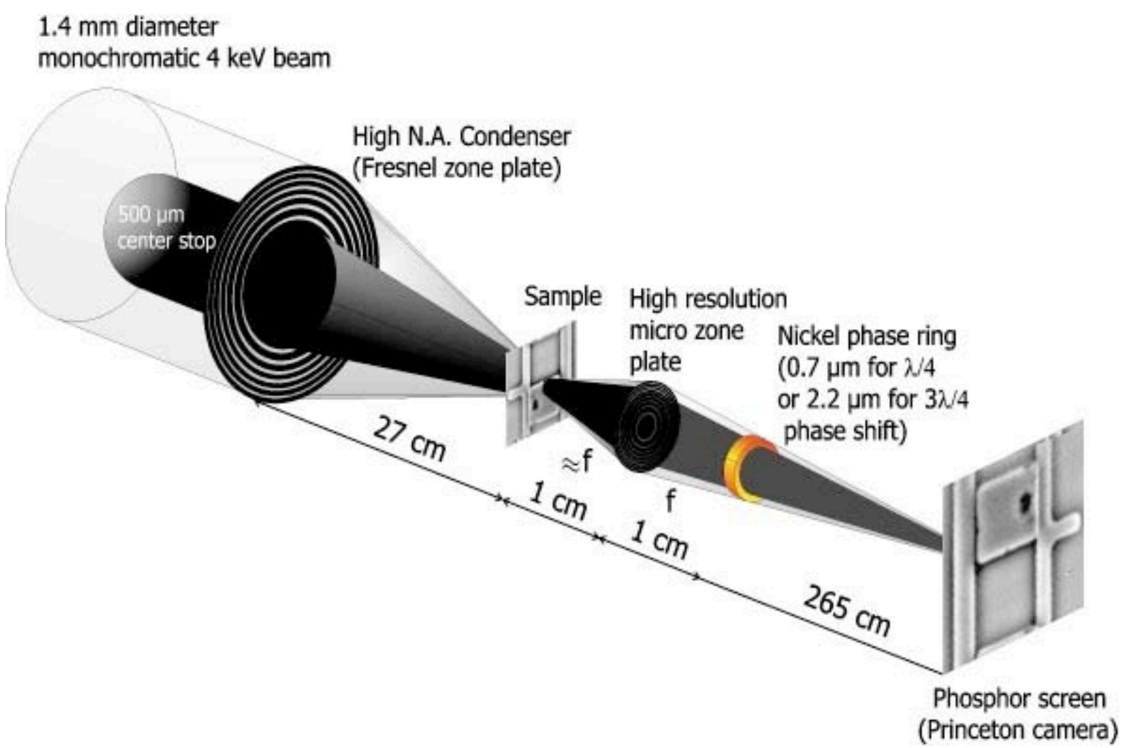


100
10
1
DOF (µm)
0.2
0.5
1
2
5
10
Energy (keV)
80 nm Rayleigh resolution
40 nm
20 nm
10 nm



Finest zone of
width dr_n
0 order
OSA: order
sorting aperture
+1 order
+3 order
Incident
beam
Focal
point
Zone plate

Title



1.4 mm diameter
monochromatic 4 keV beam
500 µm
center stop
High N.A. Condenser
(Fresnel zone plate)
Sample
High resolution
micro zone
plate
Nickel phase ring
(0.7 µm for λ/4
or 2.2 µm for 3λ/4
phase shift)
27 cm
≈f
1 cm
f
1 cm
265 cm
Phosphor screen
(Princeton camera)

Title

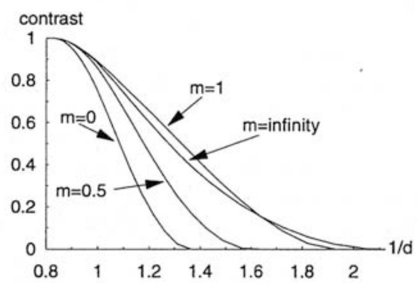


Fig. 2. Image contrast of a two-point object imaged with $\epsilon_j = \epsilon_k = 0$ and coherence parameters m of 0, 0.5, 1, and infinity; d is in units of λ/NA_o.