Quantitative coherence and application to x-ray beam lines

by

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Lecture 2 of the series

COHERENT X-RAYS AND THEIR APPLICATIONS

A series of tutorial–level lectures edited by Malcolm Howells*

*ESRF Experiments Division

CONTENTS

- Information on remaining lectures in the series (missed last session due to lack of time)
- Review of material presented last session
- Quantitative description of spatial coherence
- How to determine the spatial coherence of the illumination from a given source - (van-Cittert-Zernike theorem)
- Examples: circular pinhole source, how much coherence does a zone plate need?
- How to predict the diffraction pattern of an object with partially coherent illumination (Generalised Schell's theorem)
- Example: why do many beam lines have horizontal stripes?

A pdf file of this presentation and others in the series will be at **http://www.esrf.fr/events/announcements/Tutorials**

Practical aspects of temporal and spatial coherence for hard X-rays,
how can we measure spatial coherence?

Definition of the coherence requirements on mirrors, crystals and windows,
how close are we to meeting them?

Consequences of failing to meet requirements, strategies for improvement,
what remains to be done?

What new challenges do the “purple-book” experiments pose for optics?

Coherence matching for nanofocusing optics,
single-bounce single-capillary reflectors versus compound refractive lenses
and Fresnel zone plates

COHERENCE AND X-RAY MICROSCOPES

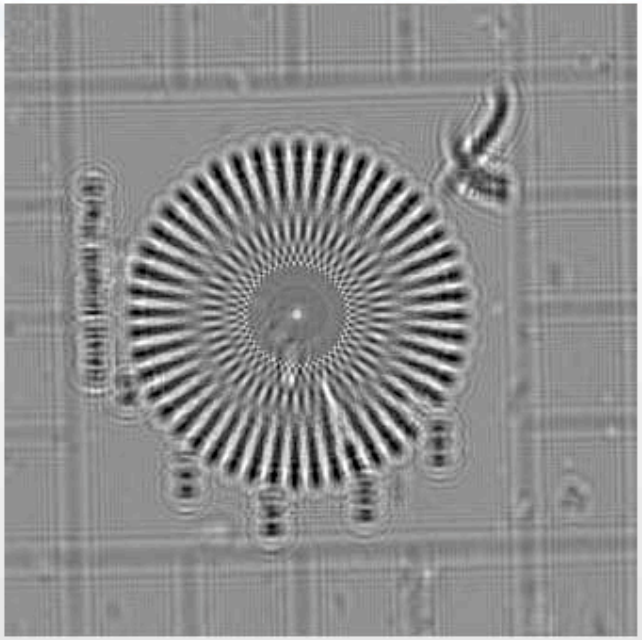
By Malcolm Howells, ESRF Experiments Division, May 26, (CTRL room)

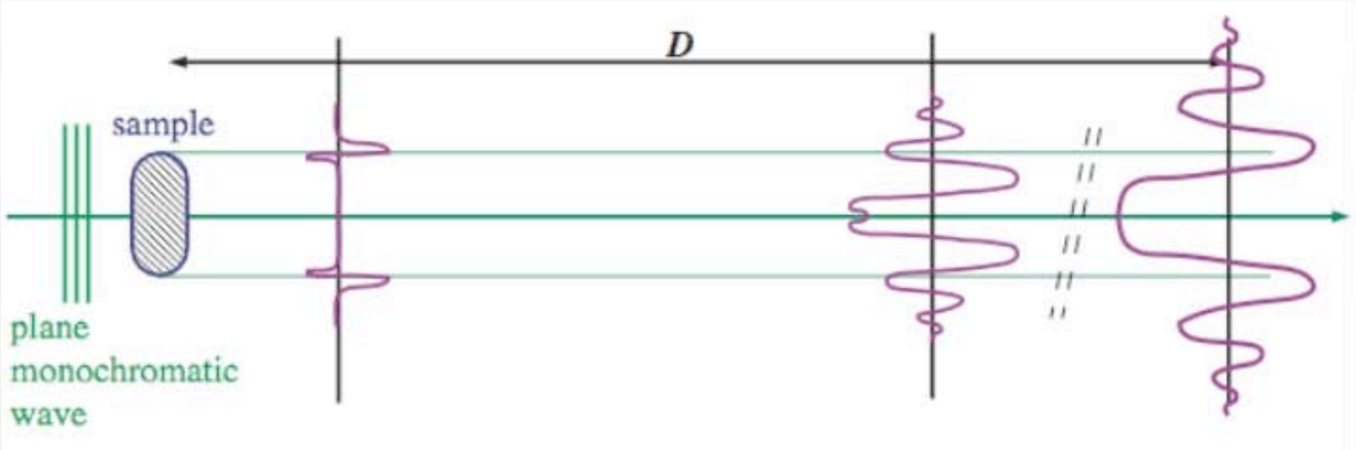
- Introduction to x-ray microscopes at synchrotrons
- Zone plates
- Transmission x-ray microscopes (TXMs) and scanning transmission x-ray microscopes (STXMs)
- Sample illumination (condenser) systems, should the illumination be coherent, is the image coherent?
- Role of beam angle in determining resolution, Fourier optics treatment
- Contrast transfer, reciprocity, influence of coherence on resolution
- Coherence and Zernike phase contrast, Wigner phase contrast
- Are microscopes flux or a brightness experiments?
- Some example results.

Phase-contrast x-ray imaging in two and three dimensions

Peter Cloetens, June 2, room 500

- Propagation-based phase-contrast imaging
  - coherence conditions
  - the forward problem; Fresnel diffraction
  - in-line holography

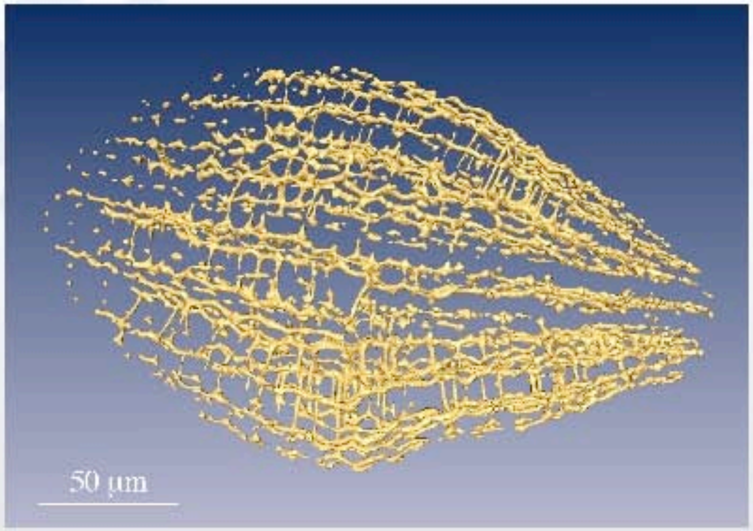


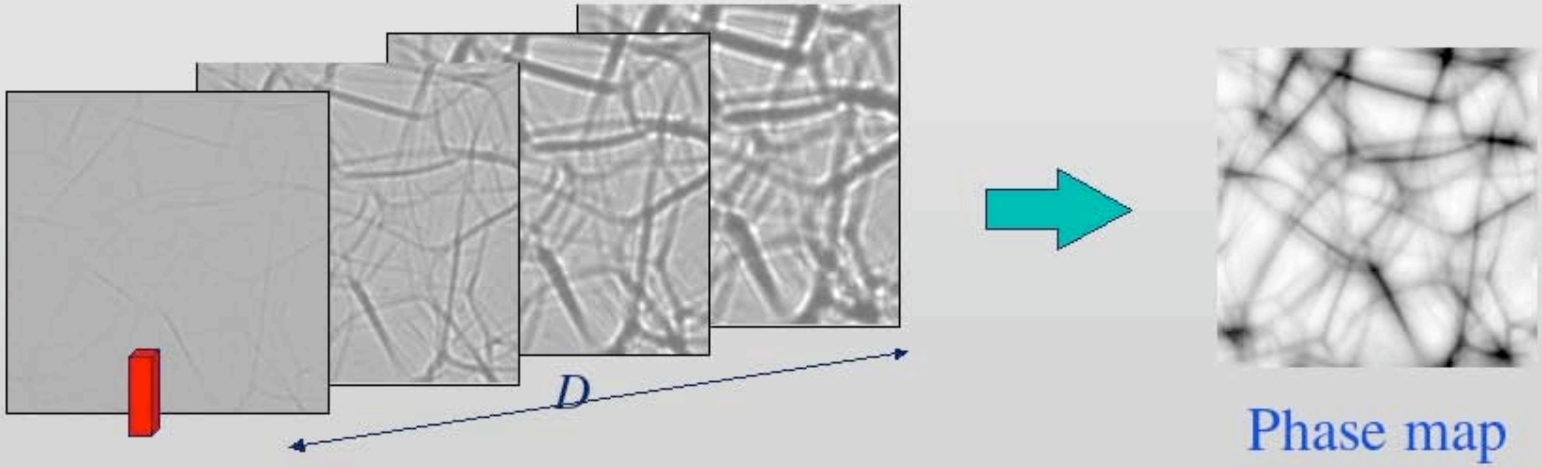


Phase-contrast x-ray imaging in two and three dimensions

Peter Cloetens, June 2, room 500

- The inverse problem
  - phase retrieval methods
- Three-dimensional imaging
  - holo-tomography
- Projection microscopy
  - KB-based imaging





Coherence activities at the ESRF: Scanning transmission x-ray microscopy
By Jean Susini, ESRF Experiments Division

- **Basic principles**
- **Optical design**
  - **Zone plates vs to Kirkpatrick-Baez systems?**
  - **High-beta vs low-beta sources?**
  - **Source demagnification vs use of long beamlines?**
- **Spectromicroscopy and energy tuning**
  - **Use of multi-keV x-rays**
  - **Chemical mapping and X-ray fluorescence**
  - **Radiation damage**
- **Some applications**

Synchrotron based microprobe techniques

X-Ray Fluorescence

- Composition
- Quantification
- Trace element mapping

X-ray Diffraction & scattering

- Long range structure
- Crystal orientation mapping
- Stress/strain/texture mapping

Phase contrast X-ray imaging

- 2D/3D Morphology
- High resolution
- Density mapping

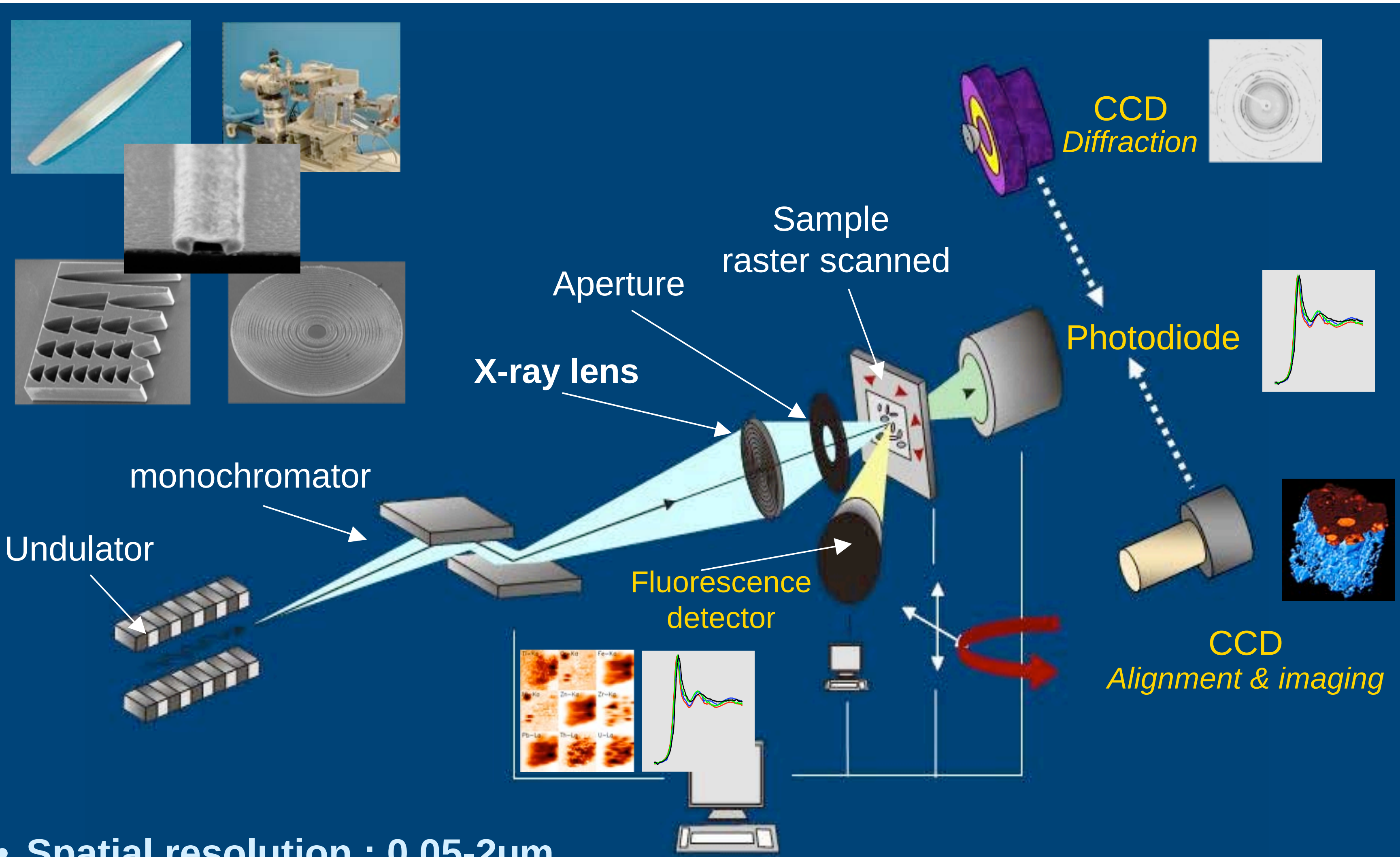
Infrared FTIR-spectroscopy

- Molecular groups & structure
- High S/N for spectroscopy
- Functional group mapping

X-ray spectroscopy

- Short range structure
- Electronic structure
- Oxidation/speciation mapping

Synchrotron based hard X-ray microprobe



- **Spatial resolution : 0.05-2µm**
- **Spectral resolution : ΔE/E ~ 10^{-2} - 10^{-4}**
- **Averaged flux : 10^{10} – 10^{13}photons/s/µm^2**

- The basic idea: measure the intensities, compute the phases
- Why the need for coherence?
- Schematic experiments and algorithms
- Oversampling: is it necessary?
- History, active groups and their achievements
- Experimental realities and limitations, computational challenges
- Radiation damage limitations
- Resolution-flux scaling, ways around the damage limit?
- Alternative schemes – ptychography

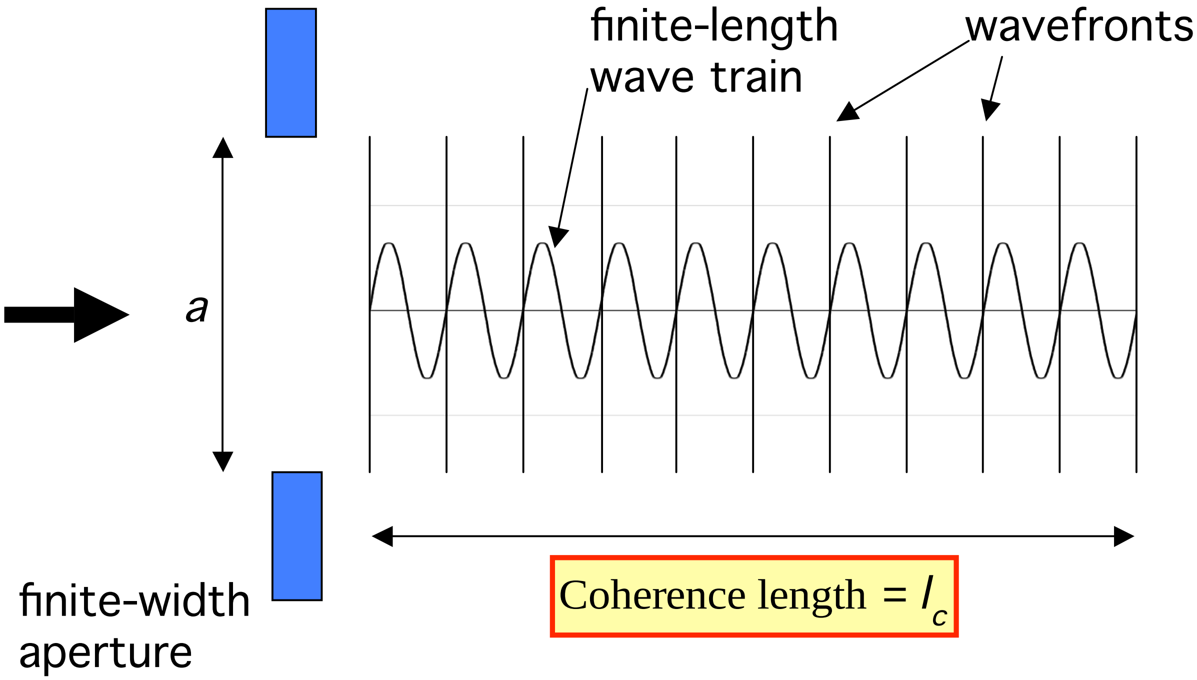
X-ray Photon Correlation Spectroscopy

Anders Madsen, June 30

- XPCS overview
- Speckle
- Correlation functions
- Setup and detectors for XPCS
- Scientific highlights
- Complementary methods
- XPCS at 4th generation sources

- Summary of present achievements and future projections in CXDI and other coherence techniques
- Details of ALS results and their implications
- How do we know the resolution?
- Detectors, multiple exposures
- Choice of wavelength
- Resolution-exposure-time tradeoffs with a purpose-built beam line
- Beam-line design, the Berkeley COSMIC project
- What are reasonable performance expectations for the future?
- Benefits of the ESRF upgrade, Comparison with other techniques
- New opportunities for time-resolved and damage-avoiding experiments with x-ray free-electron lasers
- Conclusion

THE DEGREE OF TEMPORAL COHERENCE IS DETERMINED BY THE LENGTH OF THE WAVE TRAIN (MONOCHROMATICITY)



For a wave train of N periods

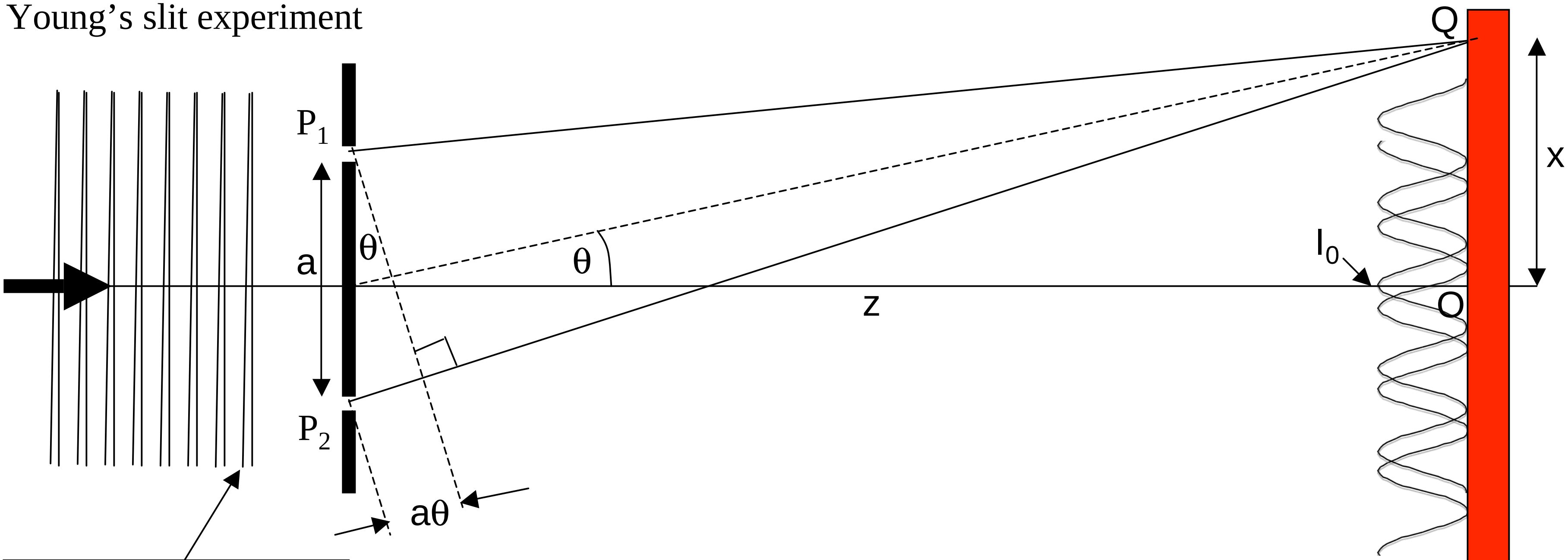
$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu} = \frac{\Delta E}{E} \approx \frac{1}{N}$$

$$\boxed{l_c = N\lambda = \frac{\lambda^2}{\Delta\lambda}}$$

- The main point is to make sure that the coherence length is long compared to all path differences between interfering rays in the experiment
- If this is done then the illumination is called ***quasimonochromatic*** and temporal coherence effects are removed from consideration

THE DEGREE OF SPATIAL COHERENCE IS DETERMINED BY THE DEGREE OF COLLIMATION

Young's slit experiment



Second wave tilted by ε=λ/(4a) giving an additional path lag of λ/4 of the signal from P_2 relative to that from P_1

- The fringe blurring caused by $\pm\lambda/4$ path change is considered tolerable so we say that P_1 and P_2 are "coherently" illuminated
- If the beam spread FULL angle is *A* (equals $\pm\varepsilon$) then the coherence width *a* is given by the $aA \approx \lambda/2$
- The equation $aA \approx \lambda/2$ is important and *defines* a spatially coherent beam

WHAT IS COHERENT OPTICS?

EARLIER WE SAID:

- *Optical coherence exists in a given region of a wave field if the phase difference between every pair of points in that region has a definite value which is constant with time*
- *The sign of good coherence is the ability to form interference fringes of good contrast*
- Now suppose that a set of points P_i in some region radiates signals coherently in the above sense.
- Thus the intensity at Q is given by the COHERENT SUM $I_Q = \left|\sum_i u_i\right|^2$ *where u_i are the appropriately delayed arriving complex amplitudes*
- **Note that the complex amplitudes of the signals are summed first. After that the square modulus is taken. This is the essence of coherent optics**
- If the points P_i radiated signals with a random phase relationship then the intensity would be given by the INCOHERENT SUM $I_Q = \sum_i |u_i|^2$ (adding intensities)
- The coherent sum can give coherence effects (fringes) the incoherent sum cannot

RULES FOR CALCULATING A COHERENT DIFFRACTION PATTERN

RULE 1: WAVE PASSING THROUGH A TRANSPARENCY

$$u_{\mathrm{EXIT}}(x,y) = u_{\mathrm{P}}(x,y)\, t(x,y)$$

RULE 2: WAVE PROPAGATING TO THE DETECTOR

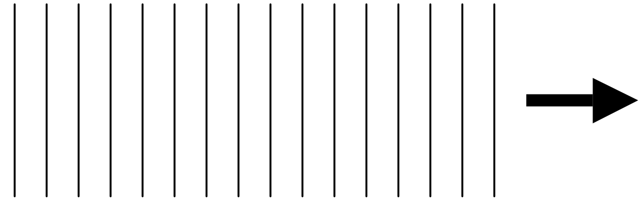
First $u_{\mathrm{Q}} = \dfrac{1}{i\lambda z}\displaystyle\int_{-\infty}^{+\infty} u_{\mathrm{P}}(x,y)\, t(x,y)\, \mathrm{e}^{\frac{i\pi}{\lambda z}\left[(x_1-x)^2+(y_1-y)^2\right]} dx\, dy$ then $I_Q(x_1,y_1) = |u_{\mathrm{Q}}|^2$

This is the diffraction integral in the Fresnel approximation - we see that the output wave is a convolution of an input wave field with the point spread function

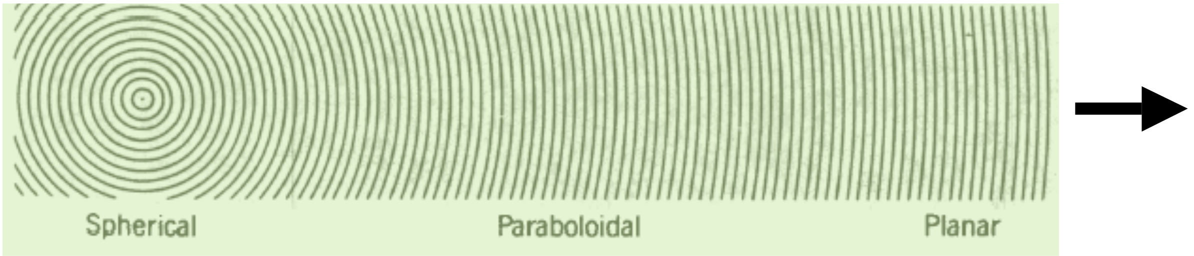
First $\underbrace{u_Q(x_1,y_1)}_{\text{output}} = \underbrace{u_{\mathrm{P}}(x,y)\, t(x,y)}_{\text{input}} * \underbrace{\left.\dfrac{1}{i\lambda z}\mathrm{e}^{\frac{i\pi}{\lambda z}\left[x^2+y^2\right]}\right|_{x_1,y_1}}_{\text{point spread function}}$ then $I_Q(x_1,y_1) = |u_Q(x_1,y_1)|^2$

EXAMPLES OF HIGHLY COHERENT BEAMS

1. A monochromatic plane wave



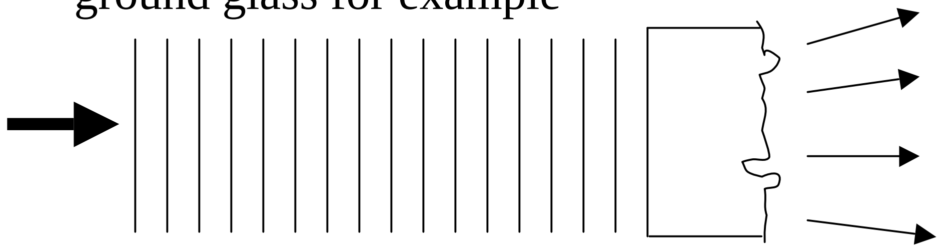
2. A monochromatic spherical wave



3. Beam from a single mode laser well above threshold

4. Beam from an incoherent monochromatic source (such as an undulator plus monochromator) that is "slitted down" so that only its coherence width is utilised

5. An initially coherent beam that has passed through a random phase disturber - ground glass for example



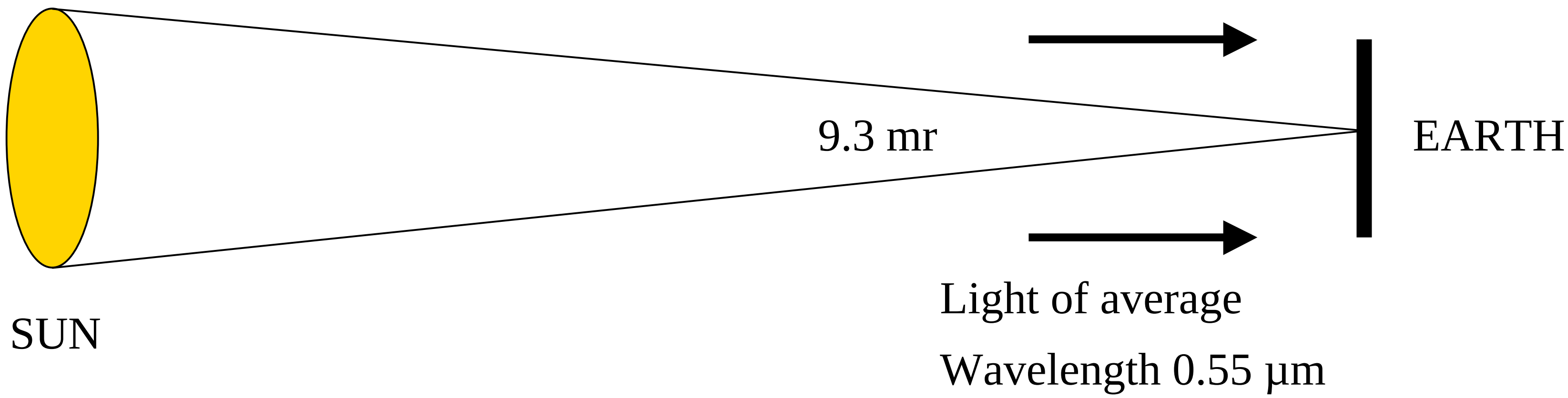
No longer a plane wave but still coherent - phase differences in the exit plane are scrambled but still constant in time

EXAMPLES OF HIGHLY INCOHERENT BEAMS

- A coherent beam that has been passed through a rotating ground-glass screen or equivalent - wood is sometimes used for x-rays - the time scale with which phase shifts change must be short compared to the integration time of the detector
- A coherent beam whose phase space as been processed by a "coherence buster" - designed to enlarge its phase-space area (meaning its width-angular-width product) in both horizontal and vertical
  - in microscopes it increases the range of angles (frequencies) of the illumination
  - in lithography it reduces unwanted coherence effects such as "fringing"
- Sources that are large and close - strip lights
- Sources with a broad spectrum, not monochromatised - bending magnet
- The sun - see next slide

HOW COHERENT IS VISIBLE SUNLIGHT?

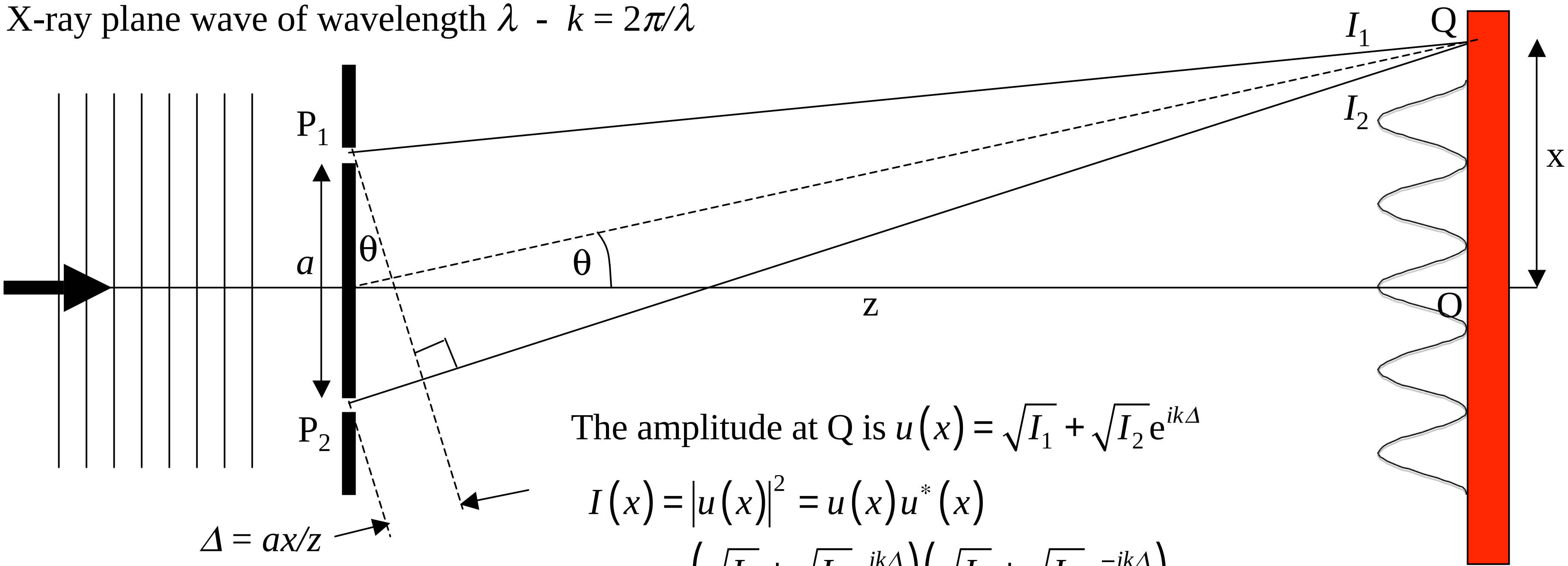
Born and Wolf section 10.4.2



- All sources, however large and close have *some* degree of coherence however small
- In this case using the criterion for what we will call "nearly perfect coherence" the width of the coherently illuminated area at the earth is 19 µm - more on how to calculate this later today

YOUNG'S SLITS EXPERIMENT IN COHERENT ILLUMINATION

X-ray plane wave of wavelength λ - $k = 2\pi/\lambda$



In this case P_1 and P_2 are coherently iluminated

The amplitude at Q is $u(x) = \sqrt{I_1} + \sqrt{I_2}\,e^{ik\Delta}$

$$I(x) = |u(x)|^2 = u(x)u^*(x)$$

$$= \left(\sqrt{I_1} + \sqrt{I_2}\,e^{ik\Delta}\right)\left(\sqrt{I_1} + \sqrt{I_2}\,e^{-ik\Delta}\right)$$

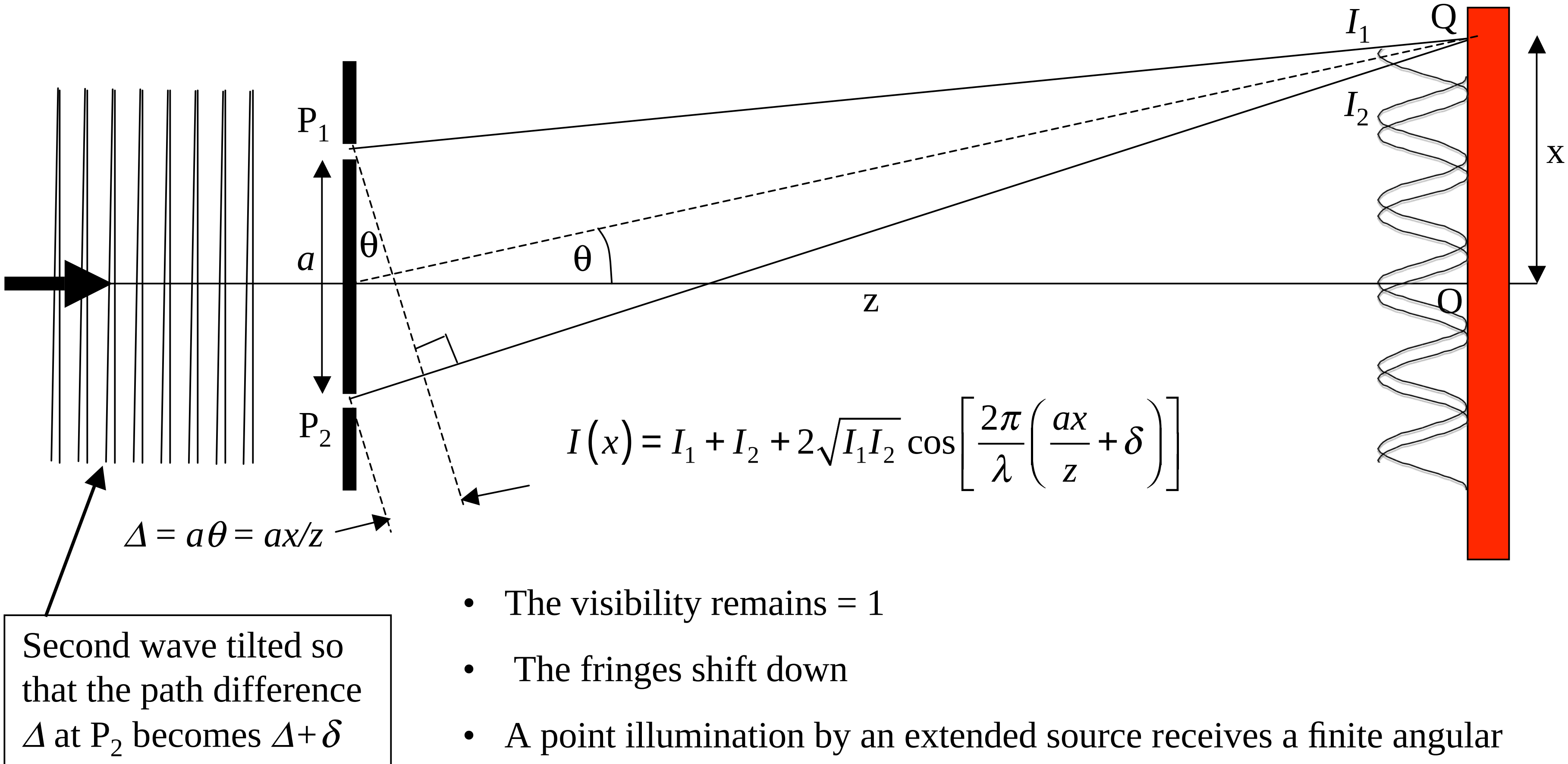
$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(k\Delta)$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi a x}{\lambda z}\right)$$

The fringe visibility **V** is given by

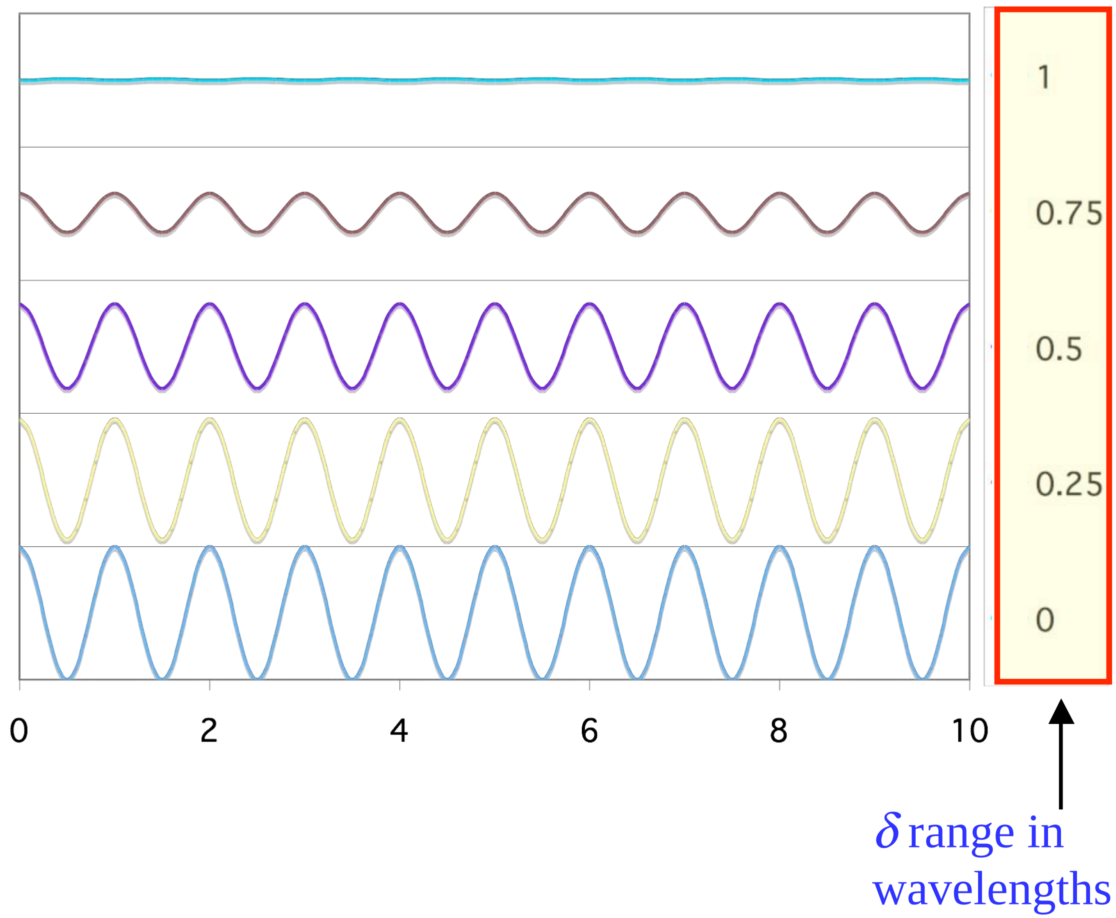
$$\mathbf{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{4\sqrt{I_1 I_2}}{2(I_1 + I_2)} \boxed{= \ 1} \text{ when } I_1 = I_2$$

INTRODUCE A TILTED WAVE TO REPRESENT IMPERFECT COLLIMATION



- The visibility remains = 1
- The fringes shift down
- A point illumination by an extended source receives a finite angular spread and a **range** of values of δ
- The fringe systems due to all the values of δ are then averaged together
- Resulting in a blurring of the fringes and reduced visibility (contrast)

FRINGE CONTRAST WHEN THE ILLUMINATING BEAM HAS ANGULAR SPREAD



- The graphs show the loss of fringe contrast when the fringe patterns with all δ values in the given δ range were averaged from $-\delta/2$ to $+\delta/2$
- δ range equals zero is the coherent case
- Note that there is no change in the phase of the fringes because the angular spread was symmetrical
- The zero and maximum of the intensity for each plotted fringe pattern are the axes immediately above and below the plot
- When δ equals one wavelength for example the total beam angular spread is $1\lambda/P_1P_2$

MATHEMATICAL DESCRIPTION OF THE EFFECT OF SPATIAL COHERENCE

- To capture the essence of spatial coherence we make the following restriction
- Illumination is **quasimonochromatic**

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi a x}{\lambda z}\right)$$ Fringes with coherent illumination

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mu_{12}\cos\left(\frac{2\pi a x}{\lambda z}\right)$$ Fringes with partially coherent illumination

The quantity $\boldsymbol{\mu}_{12}$ satisfies $\boxed{0 \le |\boldsymbol{\mu}_{12}| \le 1}$ and is known as the complex coherence factor

It is the normalised form of $\mathbf{J}_{12}$ the ***mutual intensity*** of the light at P_1 and P_2 where

$\mathbf{J}_{12} = \langle \mathbf{u}(\mathrm{P}_1,t)\mathbf{u}^*(\mathrm{P}_2,t)\rangle$ and $\mathbf{u}(\mathrm{P}_1,t)$, $\mathbf{u}(\mathrm{P}_2,t)$ are the complex signals at P_1 and P_2

$$\mu_{12} = \frac{\mathbf{J}_{12}}{\sqrt{I(\mathrm{P}_1)\,I(\mathrm{P}_2)}}$$

Thus μ_{12} is a correlation function of the signals at the typical points P_1 and P_2 and represents their coherence. The definitions of $\mathbf{u}$, $\mathbf{J}_{12}$ and μ_{12} take into account that the signals due to real light (including undulator x-rays) are random processes which we do not consider further

MEASUREMENT OF SPATIAL COHERENCE

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \mu_{12} \cos\left(\frac{2\pi a x}{\lambda z}\right)$$

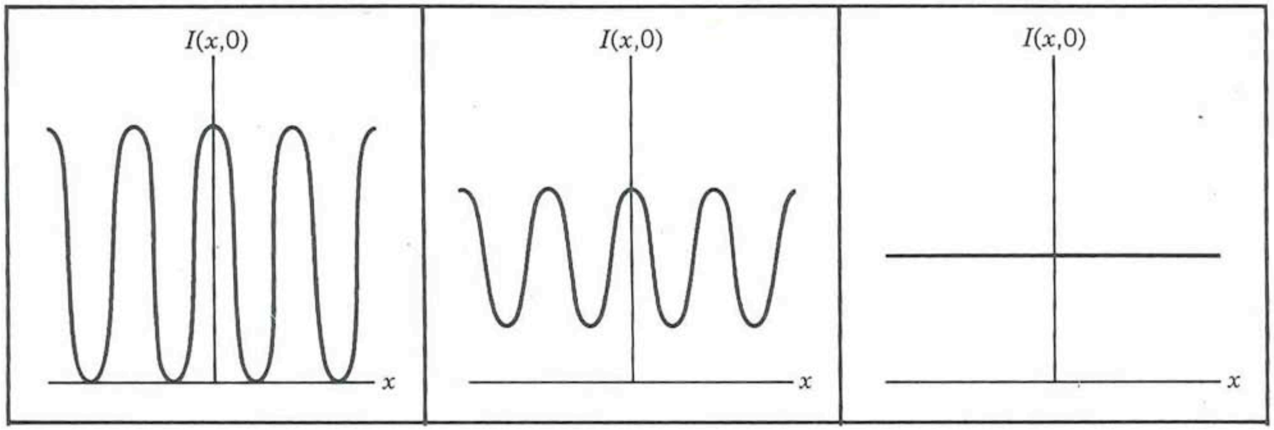
The fringe visibility **V** is now given by

$$\mathbf{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{4\sqrt{I_1 I_2}\ |\mu_{12}|}{2(I_1 + I_2)} \boxed{= \ |\mu_{12}|} \text{ when } I_1 = I_2$$

The fringe visibility (contrast) (and therefore also the **complex coherence factor**) is an experimentally measurable quantity

EFFECT OF μ_{12} ON FRINGE CONTRAST: SUMMARY

$\mu_{12} = 1$ $\quad$ $0 < \mu_{12} < 1$ $\quad$ $\mu_{12} = 0$



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\left(\frac{2\pi a x}{\lambda z}\right)$$

coherent illumination

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mu_{12}\cos\left(\frac{2\pi a x}{\lambda z}\right)$$

partially coherent illumination

$$I(x) = I_1 + I_2$$

incoherent illumination

THE VAN CITTERT-ZERNIKE THEOREM

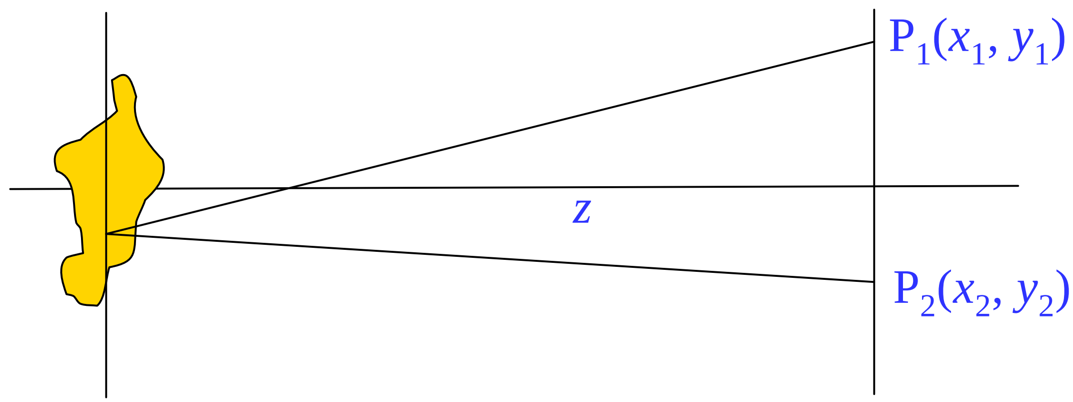
So we know that μ_{12} is an index showing the goodness of spatial coherence between two points but how do we know what the value of μ_{12} is? The van-Cittert-Zernike theorem tells us

[van-Cittert 1934]

[Zernike 1938]

Planar source $I(x_s,y_s)$

Receiving plane



x_s, y_s plane

x, y plane

Assumptions:

- x_s, x_1 etc << z
- ray angles are small
- incoherent source
- quasimonochromatic light
- spatially stationary light

THE VAN CITTERT-ZERNIKE THEOREM: STATEMENT AND MEANING

$$\mu_{12}(\Delta x,\ \Delta y) = \frac{e^{-i\psi} \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} I(x_s, y_s)\, e^{\frac{2\pi i}{\lambda z}[x_s\,\Delta x + y_s\,\Delta y]}\, dx_s dy_s}{\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} I(x_s, y_s)\, dx_s dy_s}$$

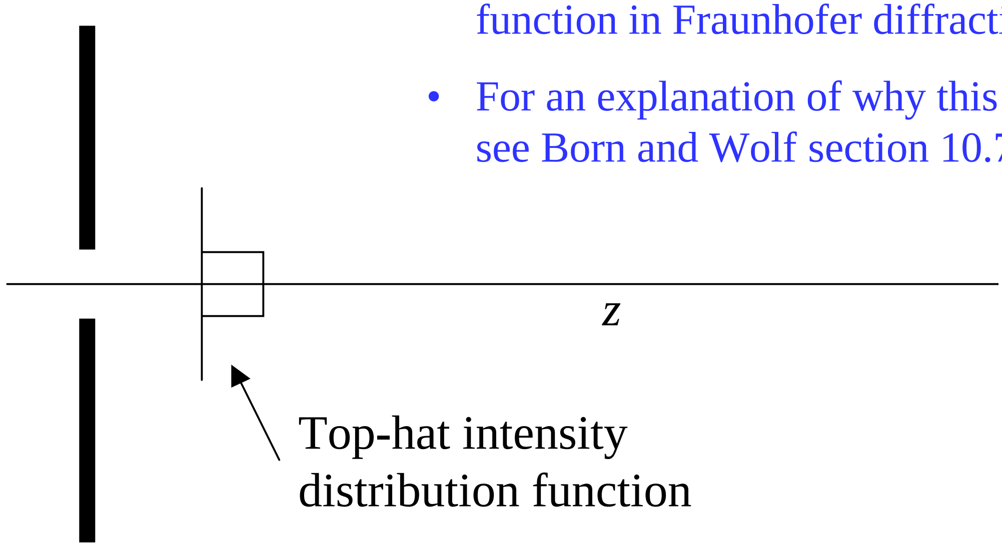
where $\psi = \frac{\pi}{\lambda z}\left[\left(x_1^2 + x_2^2\right) - \left(y_1^2 + y_2^2\right)\right]$

In words:
The complex coherence factor μ_{12} is equal to the normalized Fourier transform of the source intensity distribution [Goodman 1985] eq 2.6-10

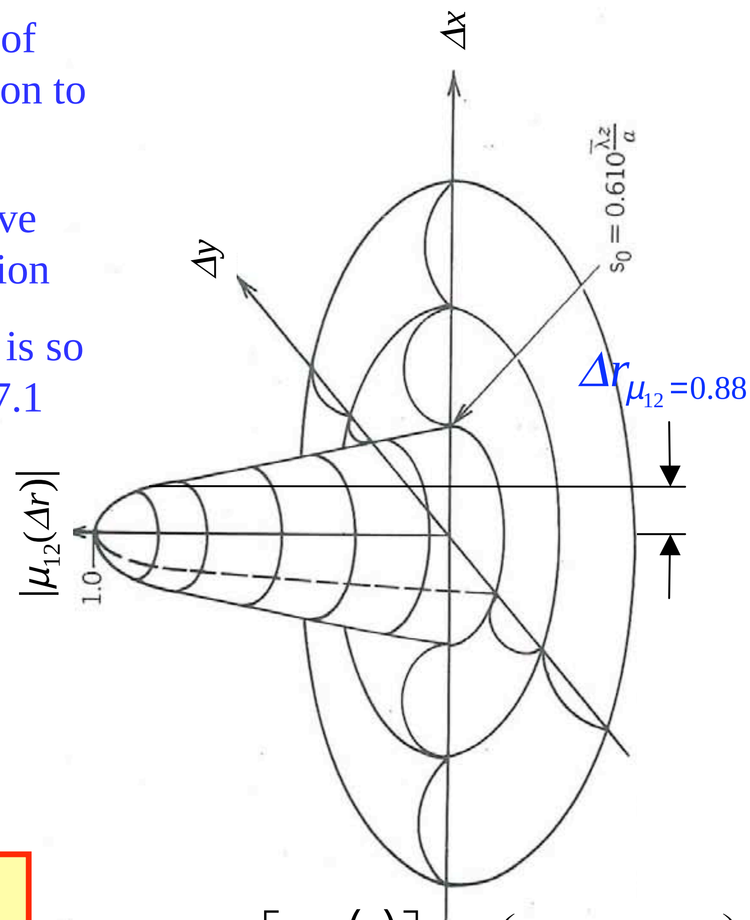
- The distribution of values of the source points (x_s, y_s) is essentially a mapping of the angles of the rays from the source points to any given receiving point
- Thus the theorem is nothing more than a precise statement of our earlier demonstrations that angular spread determines spatial coherence
- Obviously if the detector is moved further away from the source the angular spread of the arriving rays will diminish
- This causes the coherence width to increase (improve)
- This general behavior is called "coherence by propagation"

THE VAN CITTERT-ZERNIKE THEOREM: example

Circular pinhole with radius r_0 uniformly and incoherently illuminated



- We take the Fourier transform of the intensity distribution function to obtain μ_{12}
- Similarly to calculating the wave function in Fraunhofer diffraction
- For an explanation of why this is so see Born and Wolf section 10.7.1

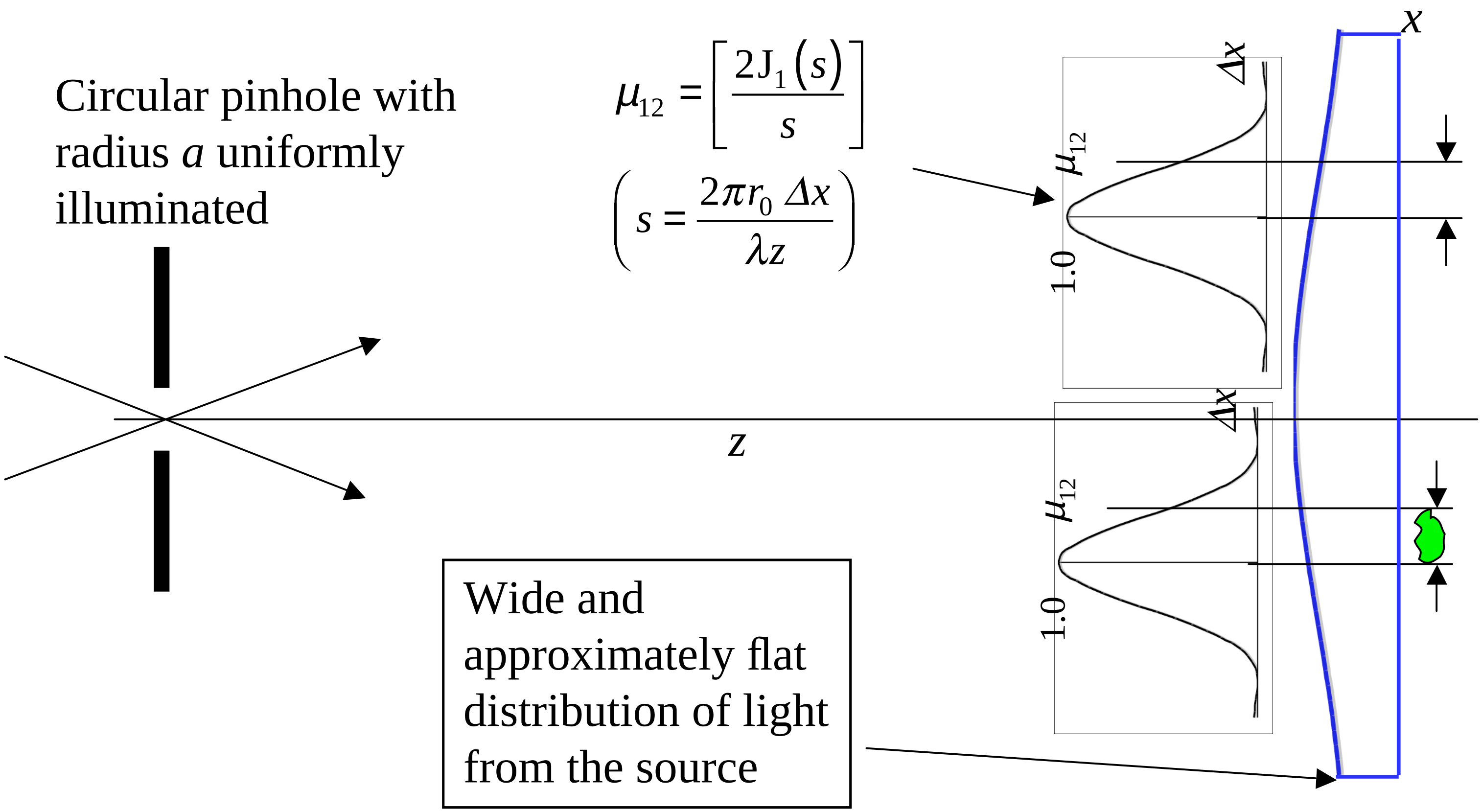


To illuminate a sample with "very good" coherence $\mu_{12} > 0.88$ say (Born and Wolf 10.4-31) sample size must be less than $\Delta r_{\mu_{12}=0.88}$ which is one definition of the "**coherence width**"

$$\mu_{12}(\Delta r) = \left[\frac{2J_1(s)}{s}\right] \qquad \left(s = \frac{2\pi r_0\, \Delta r}{\lambda z}\right)$$

(Fourier transform of a top hat)

THE VAN CITTERT-ZERNIKE THEOREM: discussion



ANY pair of points with this spacing Δy has μ_{12}=0.5

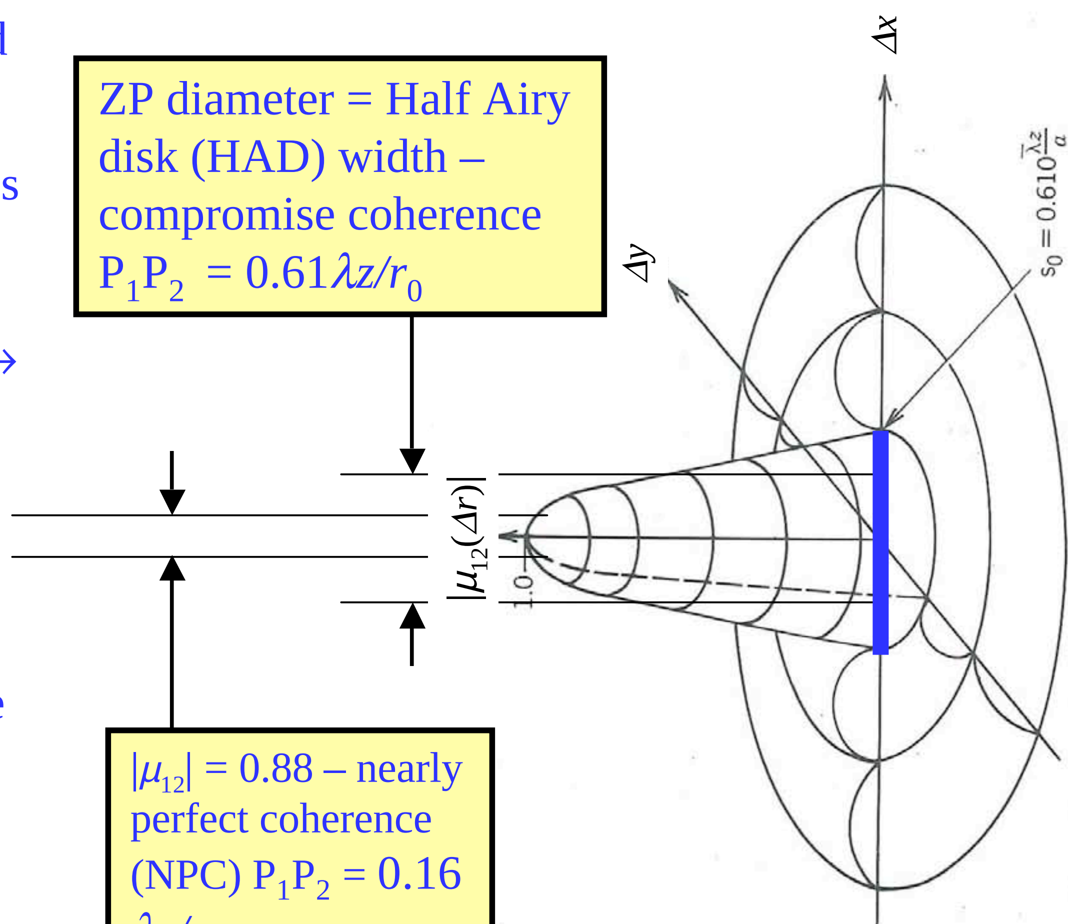
- Note that the coherence width is often much smaller than the illuminated width
- Only a small part of the light can be used for coherence experiments - the rest is wasted!
- However it can be anywhere in the illuminated area - **due to spatial stationarity!**

COMPROMISE COHERENCE CHOICES

- Example: how coherently should you illuminate a zone plate
- The **diameter** of the zone plate is the max spacing between points P_1P_2 from which signals must interfere – consider two cases →
- HAD gains a factor $(0.61/0.16)^2$ = 14.5 in flux compared to NPC
- Similarly it loses about 10% in zone plate spatial resolution – ultimately a flux-resolution trade
- See Born and Wolf eqs 10.4-30 and 31 for the P_1P_2 numbers

ZP diameter = Half Airy disk (HAD) width – compromise coherence $P_1P_2 = 0.61\lambda z/r_0$

$|\mu_{12}| = 0.88$ – nearly perfect coherence (NPC) $P_1P_2 = 0.16\,\lambda z/r_0$



= the diameter of the Airy disk

$$|\mu_{12}(\Delta r)| = \left[\frac{2J_1(s)}{s}\right] \quad \left(s = \frac{2\pi r_0\,\Delta r}{\lambda z}\right)$$

(Fourier transform of a top hat)

DIFFRACTION BY A TRANSPARENCY FUNCTION WITH PARTIALLY COHERENT ILLUMINATION

- If we can do this we can solve many practical forward diffraction problems in beam-line optics
- We start by simplifying to a source that is symmetrical about the axis
- We can solve this problem by putting together two ideas we have already seen (1) the van-Cittert-Zernike theorm and (2) the propagation rule of coherent optics in the Fresnel approximation - let's remind ourselves of these

$$\mu_{12}(\Delta x,\ \Delta y) = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} I(x_s, y_s)\, e^{\frac{2\pi i}{\lambda z}\left[x_s\, \Delta x + y_s\, \Delta y\right]} dx_s dy_s}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} I(x_s, y_s)\, dx_s dy_s}$$

In words:
The complex coherence factor μ_{12} is equal to the normalized Fourier transform of the source intensity distribution [Goodman 1985] 2.6-10

$$I_Q^{(PW)}(x_1, y_1) = \left| \frac{1}{i\lambda z} \int_{-\infty}^{+\infty} t(x, y)\, e^{\frac{i\pi}{\lambda z}\left[(x_1 - x)^2 + (y_1 - y)^2\right]} dx\, dy \right|^2 = \left| \left. t(x, y) * \frac{1}{i\lambda z} e^{\frac{i\pi}{\lambda z}\left[x^2 + y^2\right]} \right|_{x_1, y_1} \right|^2$$

- We have changed the notation in the last equation slightly to correspond to the case when the incoming complex amplitude $u_P(x, y)$ is a plane wave

A GENERALIZED SCHELL'S THEOREM

Schell 1961, Nugent (1990), Cloetens (2001), Goodman (1985)

- The form of Schell's theorem that we wish to use and can use with the results given so far is the following

$$\boxed{I(x_1,y_1)=\frac{1}{\lambda^2 z^2}\int\int_{-\infty}^{+\infty}\mu_{12}(\Delta x,\Delta y)\,\overline{I_Q^{(PW)}(x_1,y_1)}\Big|_{\Delta x,\Delta y}\exp\left[-2\pi i\left(x_1\Delta x+y_1\Delta y\right)\right]d(\Delta x)\,d(\Delta y)}$$

- where the bar over $I_Q^{(PW)}(x_1,y_1)$ indicates that its Fourier transform is to be used
- $\mu_{12}(\Delta x, \Delta y)$ is found by the van-Cittert-Zernike theorem and $I_Q^{(PW)}(x_1,y_1)$ by the coherent propagation rule (previous slide)
- The calculation of the diffraction patterns formed with an extended source (partially coherent illumination) using this method should therefore be feasible for anyone running IDL or MATLAB

GENERALIZED SCHELL'S THEOREM: DISCUSSION IN TERMS OF THE AUTOCORRELATION THEOREM

ESRF

- From the definition two slides back, we can see that $I_Q^{(PW)}(x_1, y_1)$ is the modulus squared of the Fourier transform of the quantity

$$t_{QFF}(x,y) = t(x,y)\exp\left[\frac{i\pi}{\lambda z}\left(x^2+y^2\right)\right]$$

- In other words it is the power spectrum of t_{QFF} – therefore $\overline{I_Q^{(PW)}(x_1, y_1)}$, which is what we need for Schells's theorem, can be seen (by the autocorrelation theorem) to be equal to the autocorrelation function $\mathbf{T}(\Delta x, \Delta y)$ of $t_{QFF.}$ – Schell's theorem can thus be written

$$I(x_1,y_1) = \frac{1}{\lambda^2 z^2}\int\int_{-\infty}^{+\infty} \mu_{12}(\Delta x, \Delta y)\ \mathbf{T}(\Delta x, \Delta y)\ \exp\left[-2\pi i\left(x_1\Delta x + y_1\Delta y\right)\right] d(\Delta x)\, d(\Delta y)$$

where the autocorrelation function of t_{QFF} is defined as

$$\mathbf{T}(\Delta x, \Delta y) \equiv \int\int_{-\infty}^{+\infty} t_{QFF}\left(\overline{x}-\frac{\Delta x}{2}, \overline{y}-\frac{\Delta y}{2}\right) t_{QFF}^{*}\left(\overline{x}+\frac{\Delta x}{2}, \overline{y}+\frac{\Delta y}{2}\right) d(\overline{x})\, d(\overline{y})$$

- This agrees with Goodman 1985 equation 5.7-10 - *except for the use of* $t_{QFF}(x,y)$ *instead of* $t(x,y)$
- The version given by Goodman using $t(x,y)$ (*i. e.* without the quadratic phase factor) , is the original version which applies in the Fraunhofer approximation
- The version given here (with the quadratic phase factor) is the generalization by [Nugent 1990] and is also valid in the Fresnel approximation - this improvement is very important in x-ray applications

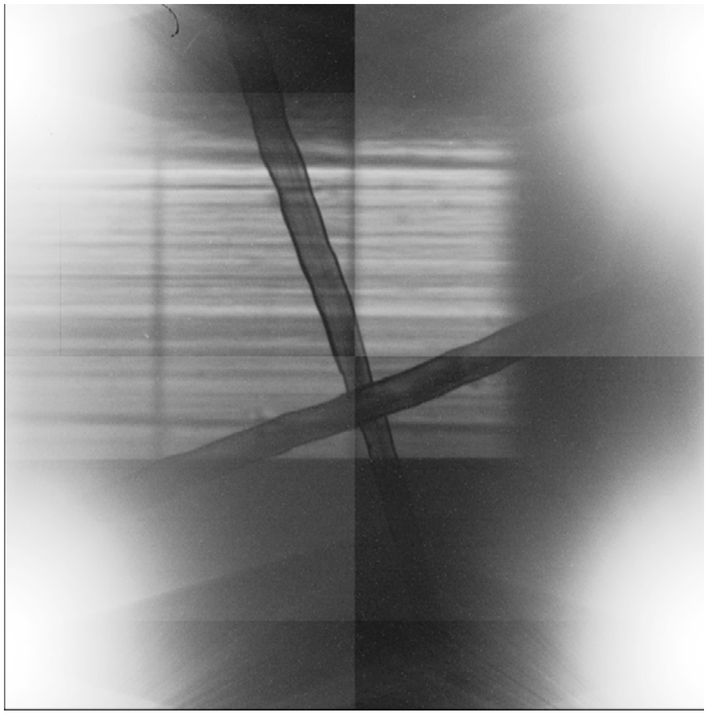
GENERALIZED SCHELL'S THEOREM: APPLICATIONS

ESRF

- The theorem can be applied in any problem where the diffracting object can be represented as a complex transparency function
- Examples are:
  - planar diffractive optics such as zone plates and computer-generated holograms
  - 3D diffracting objects that satisfy the eikonal approximation - the participating transverse features are much larger than $\sqrt{\lambda d}$ where d is the thickness of the sample or equivalently the exit wave is a projection of the object - this might include CRL's for example
  - "Phase screens" which can often be defined to project the phase effects of features on to an imaginary transmission "screen" perpendicular to the axis - (e.g. phase errors due to mirror figure errors)
- We choose an example from the last category - the effect of mirror figure errors on the beam

WHY DO MANY SYNCHROTRON BEAMS HAVE HORIZONTAL STRIPES?

ESRF

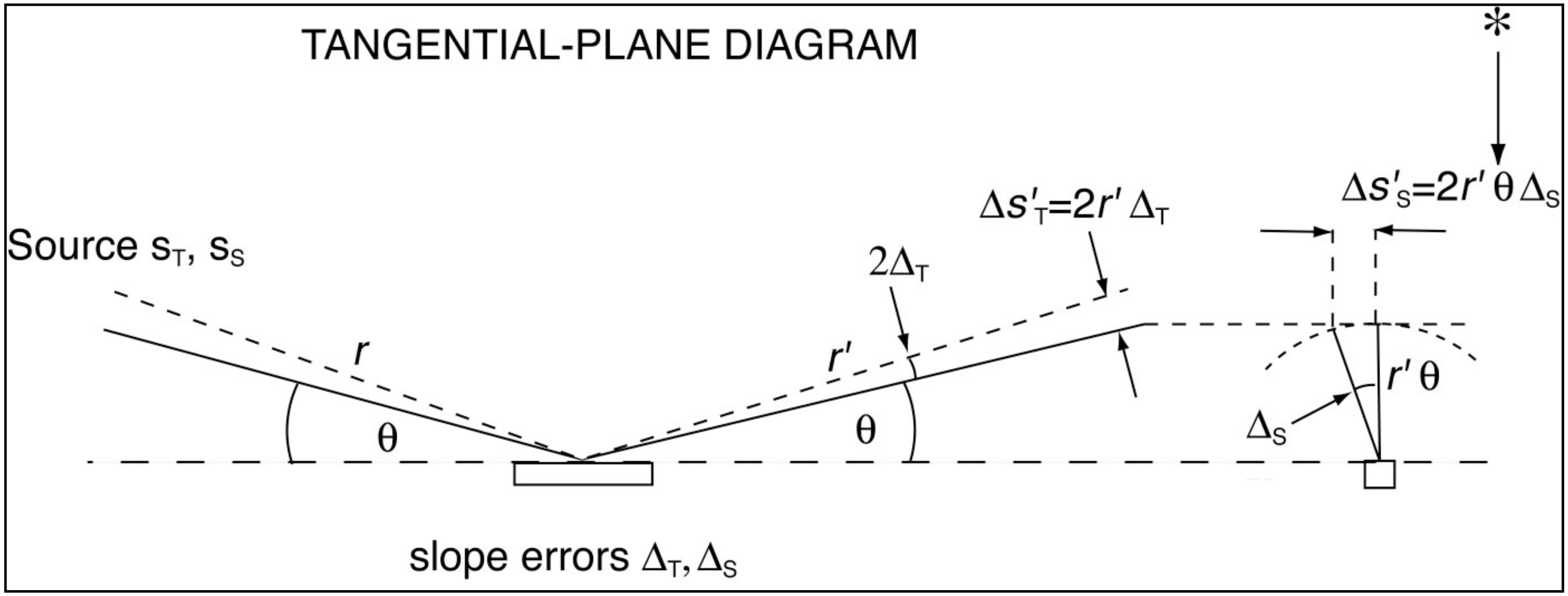


Striped image of the beam from the SLS wiggler (tomography branch line) 25 mA current, X-ray energy 12 keV - Sept 2001 - Marco Stampanoni Thesis

- The stripes are the diffraction pattern of the mirror errors illuminated partially coherently
- The undulator has high coherence in the vertical due to the small height of the source - beam width is only a few times the coherence width
- The mirror plane of incidence is normally the vertical plane so we have tangential scattering (diffraction) in the vertical plane
- We do not see vertical stripes for two reasons
  1. The source size is larger in the horizontal so the coherence is worse
  2. The incidence angle to the mirror is small ($\theta <$ 10 mr) so other things being equal, the angles of sagittal scattering are reduced by a factor of $1/\sin\theta \approx 100$ compared to tangential
- This is the so-called "forgiveness factor" which needs to be explained

DIGRESSION ON THE "FORGIVENESS FACTOR"

ESRF



Requirement: $\Delta s'_T \le \frac{1}{2} s'_T \quad \rightarrow \quad 2r'\Delta_T \le \frac{1}{2}\frac{r'}{r} s_T$

$$\boxed{\Delta_T \le \frac{1}{4}\frac{s_T}{r}}$$

$\Delta s'_S \le \frac{1}{2} s'_S \quad \rightarrow \quad 2r'\theta\Delta_S \le \frac{1}{2}\frac{r'}{r} s_S$

$$\boxed{\Delta_S \le \frac{1}{4}\frac{s_S}{r\theta}}$$

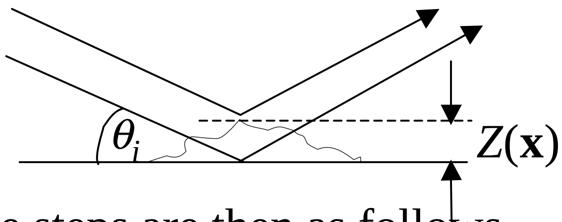
Forgiveness factor

Reasons for the factor 2's:

1. Ray deviation is twice the mirror slope error
2. Quadratic sum of σ and $\sigma/2$ is 1.1σ implying a 10% error which we declare acceptable

HOW DO WE USE SCHELL'S THEOREM TO MODEL A MIRROR PRODUCING A STRIPED BEAM?

- First we project the mirror height errors on to a phase screen perpendicular to the outgoing optical axis
- The phase screen is made by mapping a mirror height error $Z(\mathbf{x})$ to a phase shift $\Delta\phi(\mathbf{x})$ as follows (one could easily use *complex* phases to include absorption)

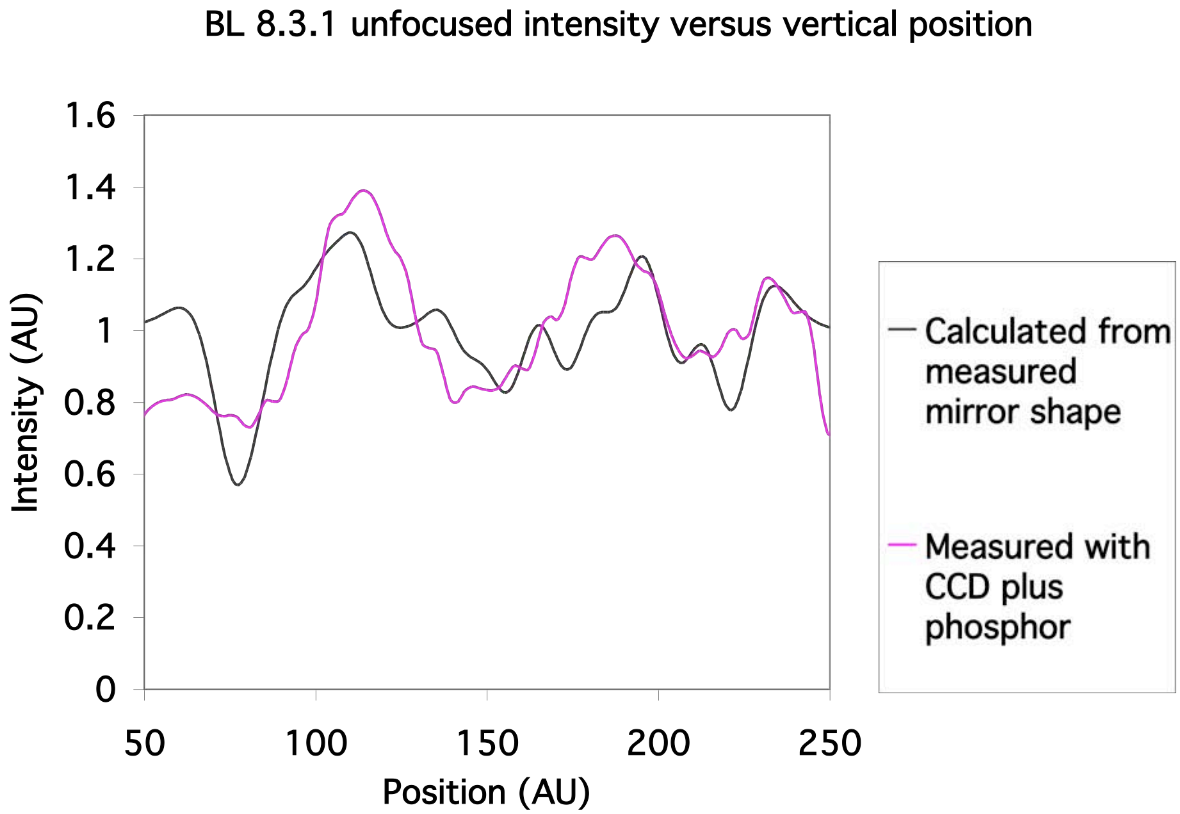


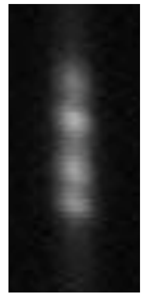
$$\Delta\phi(\mathbf{x}) = \frac{2\pi}{\lambda} 2Z(\mathbf{x})\sin\theta_i = 2\pi f_z Z(\mathbf{x})$$

- The steps are then as follows

1. Determine the effective source size shape and distance - use the van-Cittert-Zernike theorem to get $\mu_{12}(\Delta x, \Delta y)$ at the phase screen
2. Obtain $\left.\overline{I_Q^{(PW)}(x_1, y_1)}\right|_{\Delta x, \Delta y}$ by (a) Applying the propagation rule to the phase screen with a plane wave input, (b) taking the modulus squared and (c) taking the Fourier transform
3. Multiply $\mu_{12}(\Delta x, \Delta y)$ and $\left.\overline{I_Q^{(PW)}(x_1, y_1)}\right|_{\Delta x, \Delta y}$ together and take the Fourier transform

AN ALS EXAMPLE OF CALCULATING THE STRIPE PATTERN





- Photographic measured data (A. Macdowell, 2001)
- Superbend beam line defocused in the vertical but not the horizontal
- Invar mirror - shape from LTP scan
- Calculation done in 1D in IDL
- Qualitative agreement - enough to say that the mirror was responsible

Another intriguing idea is to treat this as an inverse problem - measure the intensity and deduce the mirror errors - solved by Alexi Souvorov et al using CXDI-type methods

A. Souvorov et al., "Deterministic retrieval of surface waviness by means of topography with coherent X-rays" J. Synchrotron Rad. (2002). 9, 223-228