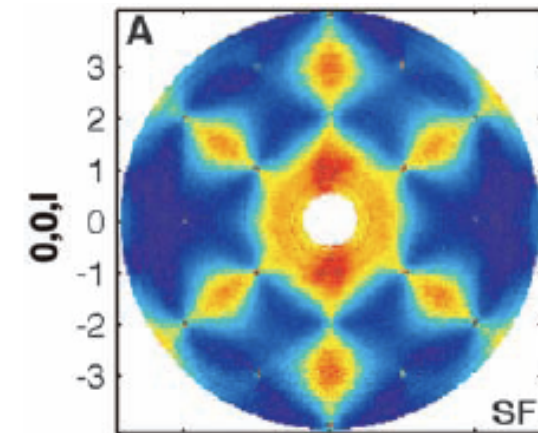
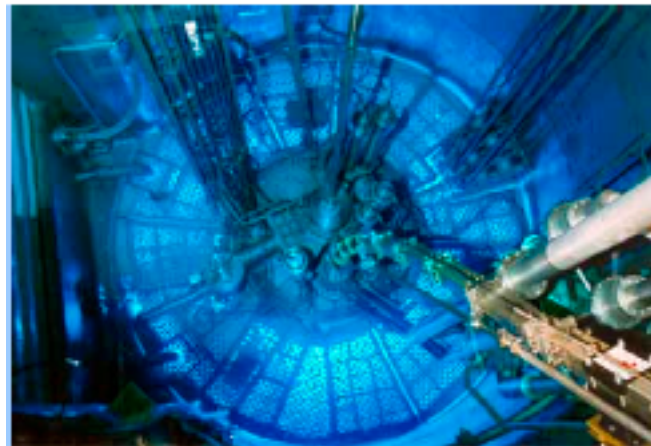
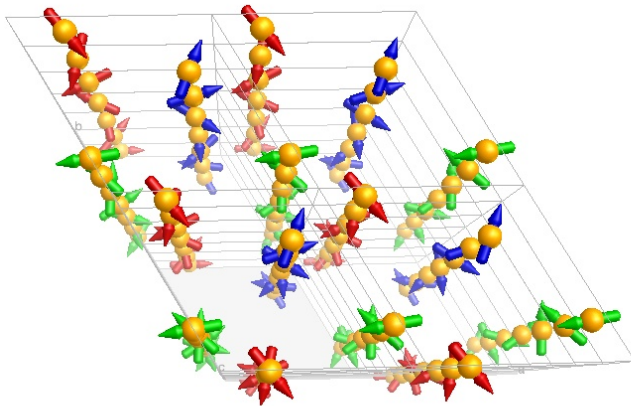


Magnetic frustration and multiferroics

Virginie Simonet

Institut Néel, CNRS/Université Grenoble Alpes, Grenoble, France



Outline of the lecture

Reminder: Conventional Magnetism

Magnetic frustration

- Introduction
- Competition of interactions
- Geometrical frustration
- Quantum case
- Role of the anisotropy

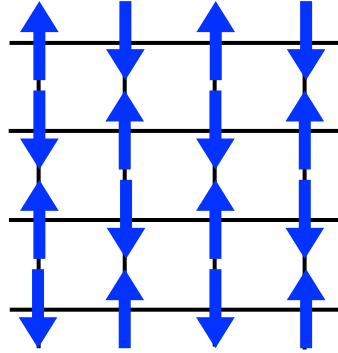
Multiferroism

- Introduction
- About symmetries and chirality
- Examples

Sorry, I will only speak about neutron scattering!

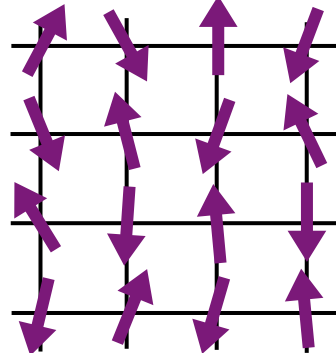
Reminder: conventional magnetism

Temperature \ll interactions



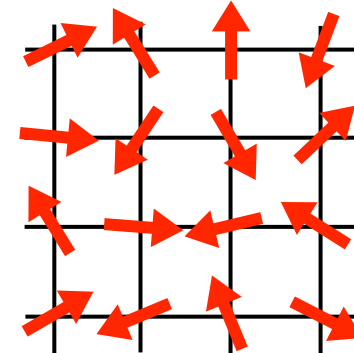
Ordered state
small fluctuations

Temperature \approx interactions

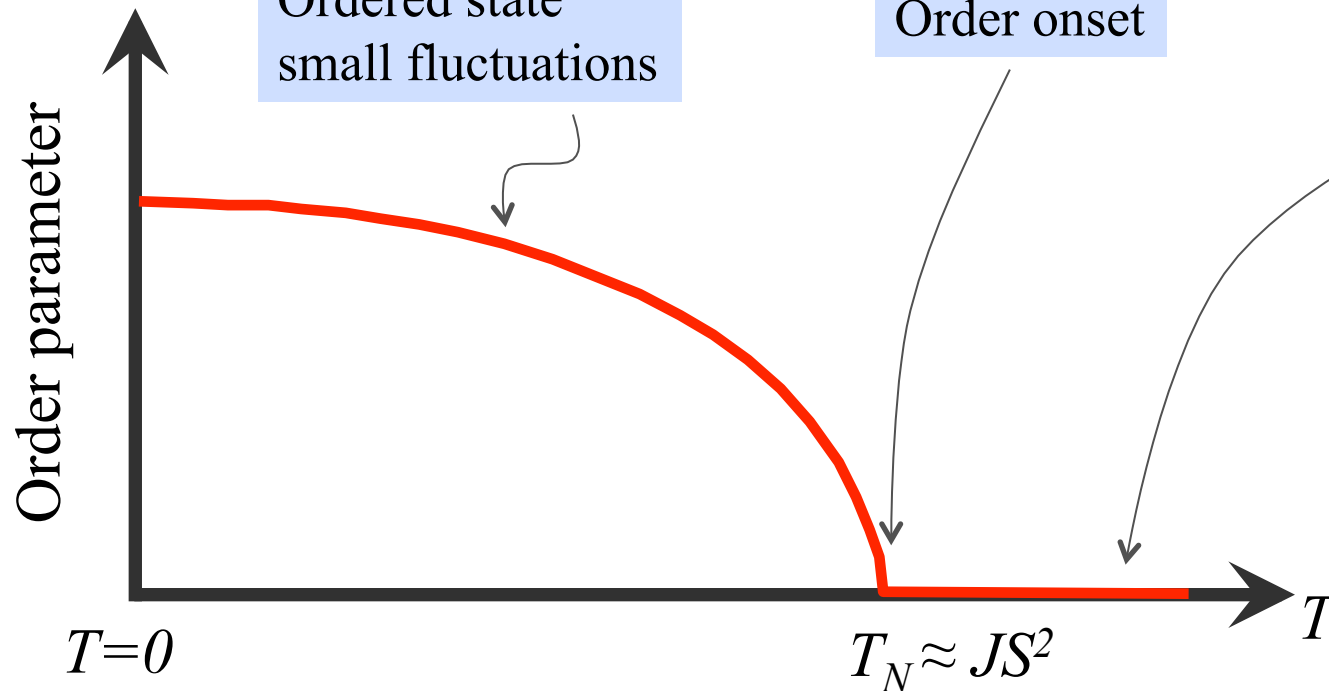


Order onset

Temperature \gg interactions



Fluctuating disordered



Heisenberg Hamiltonian

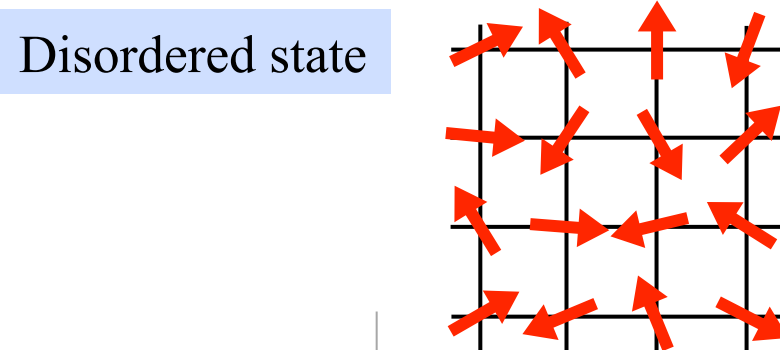
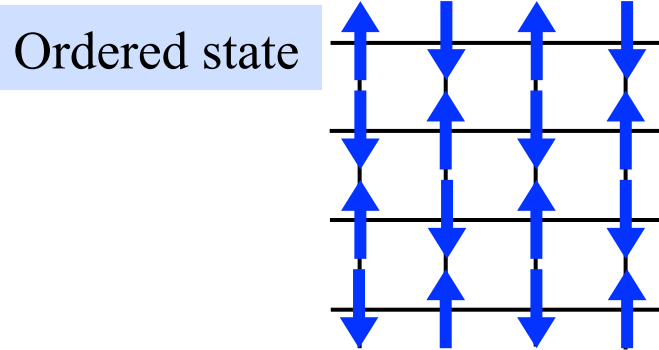
$$\mathcal{H} = - \sum_{i,j} \vec{S}_i \tilde{J}_{ij} \vec{S}_j$$

$J < 0$ AFM

Reminder: conventional magnetism

Temperature \ll interactions

Temperature \gg interactions

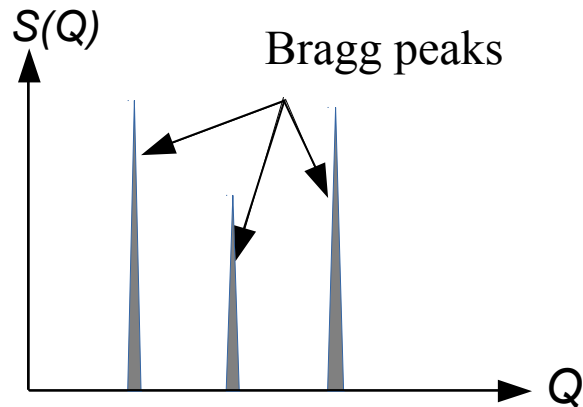


- **Spin pair correlation function**

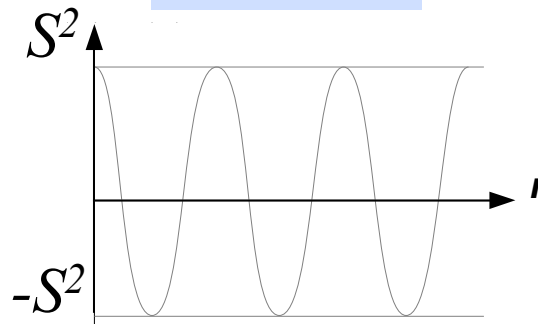
$$\langle S_i(0)S_j(r) \rangle$$

$$\langle S_i(0)S_j(r) \rangle = S^2 \delta_{ij}$$

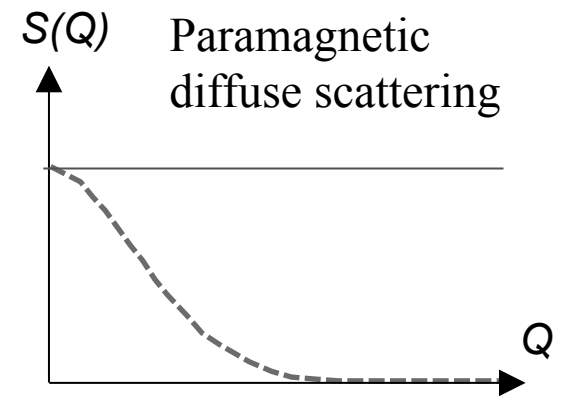
Reciprocal space



Real space



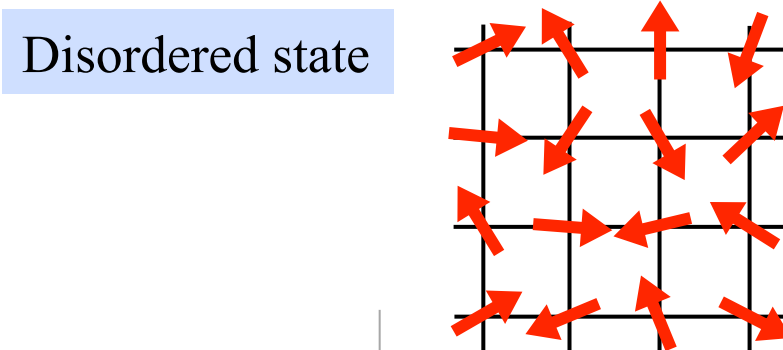
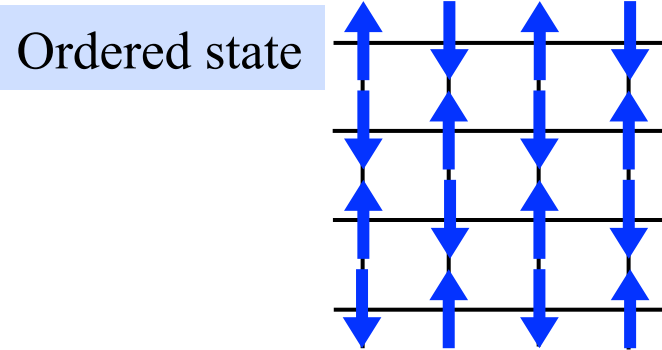
Reciprocal space



Reminder: conventional magnetism

Temperature \ll interactions

Temperature \gg interactions

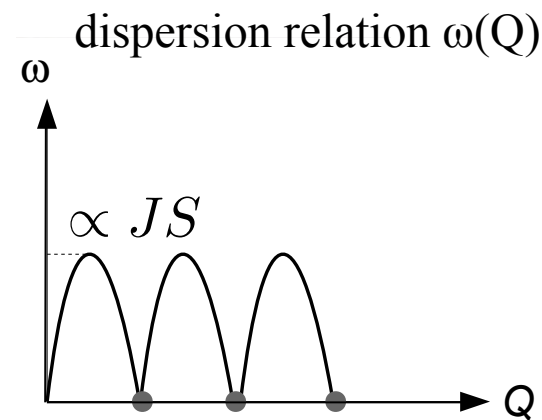
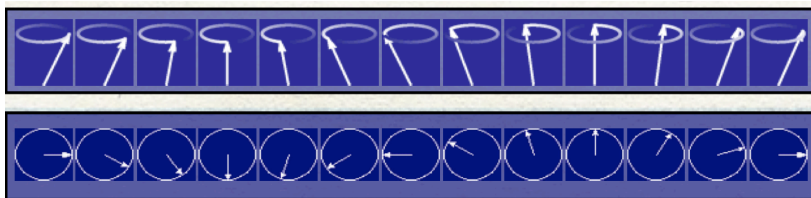


- **Spin pair correlation function**

$$\langle S_i(0)S_j(r) \rangle$$

$$\langle S_i(0)S_j(r) \rangle = S^2 \delta_{ij}$$

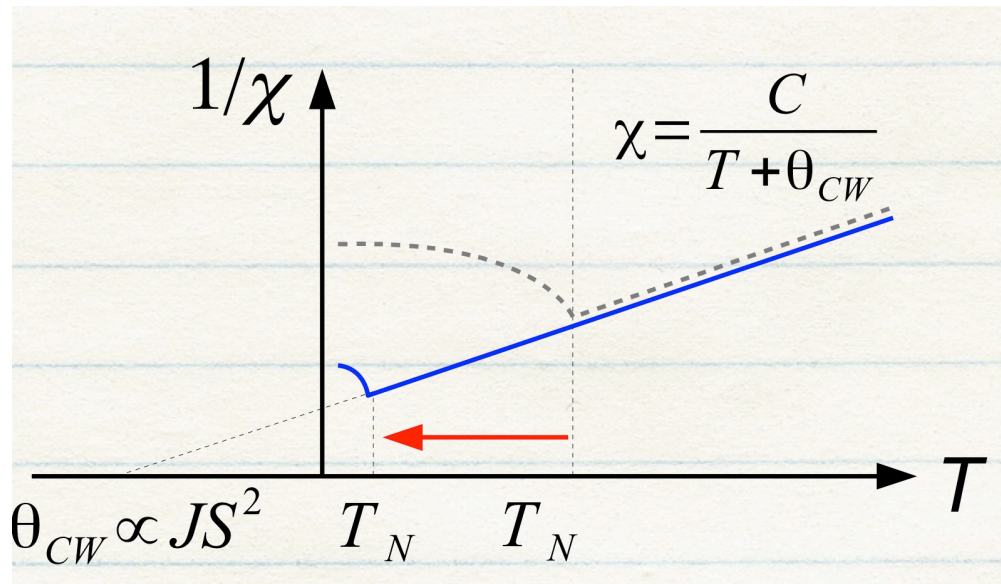
- **Excitations:**
spin waves (magnons)



Magnetic frustration: introduction

Is it possible to impede the onset of magnetic ordering?

→ Yes, through dimensional effects, introduction of disorder and **frustration**
 Consequences on spin pair correlations, excitations, neutron signatures?



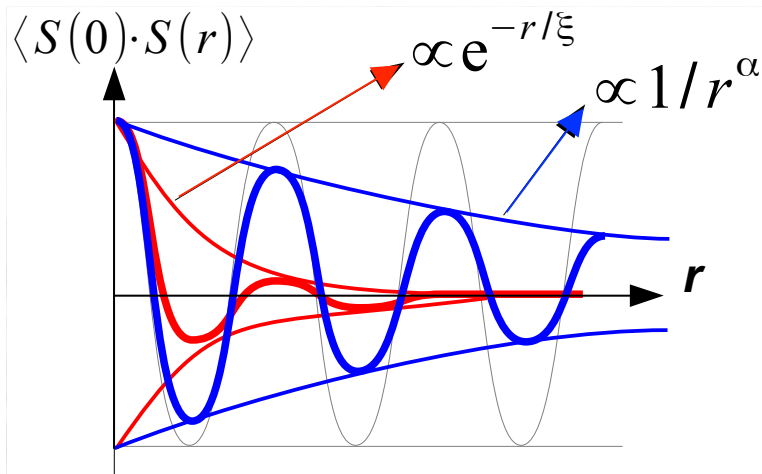
Frustration index $f = \frac{\theta_{CW}}{T_N}$

Magnetic frustration: introduction

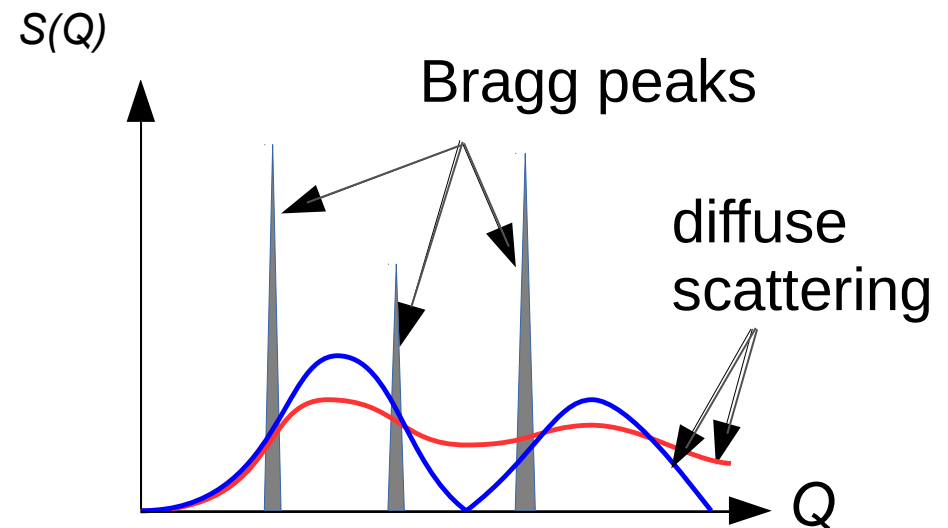
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Real space

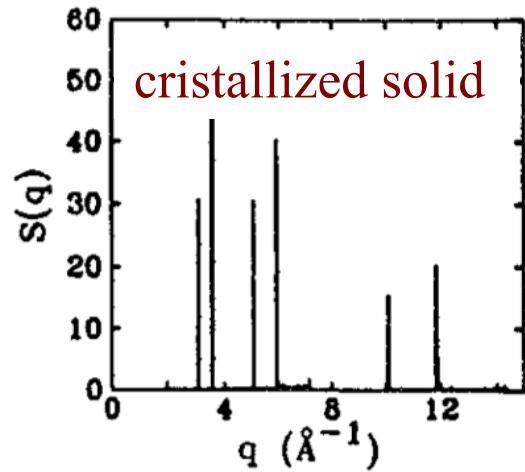


Reciprocal space

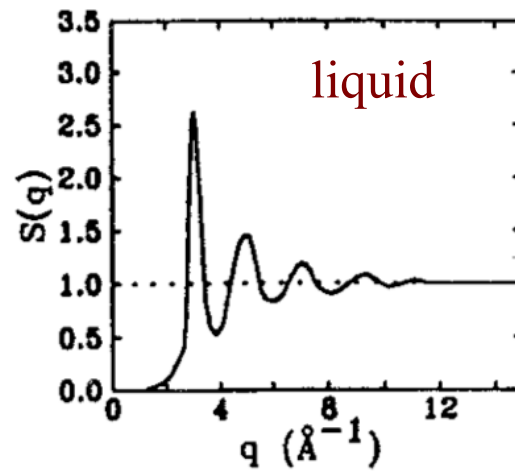


Magnetic frustration: introduction

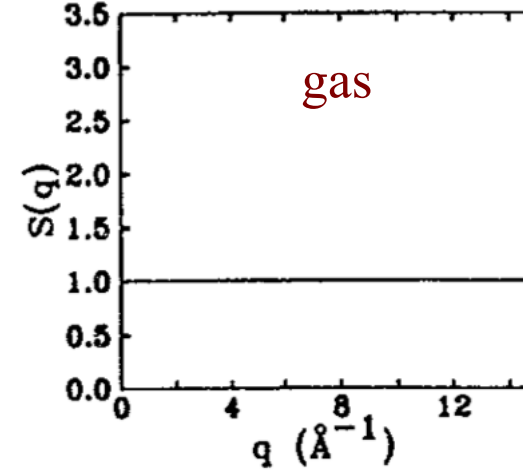
atomic states



Magnetic order

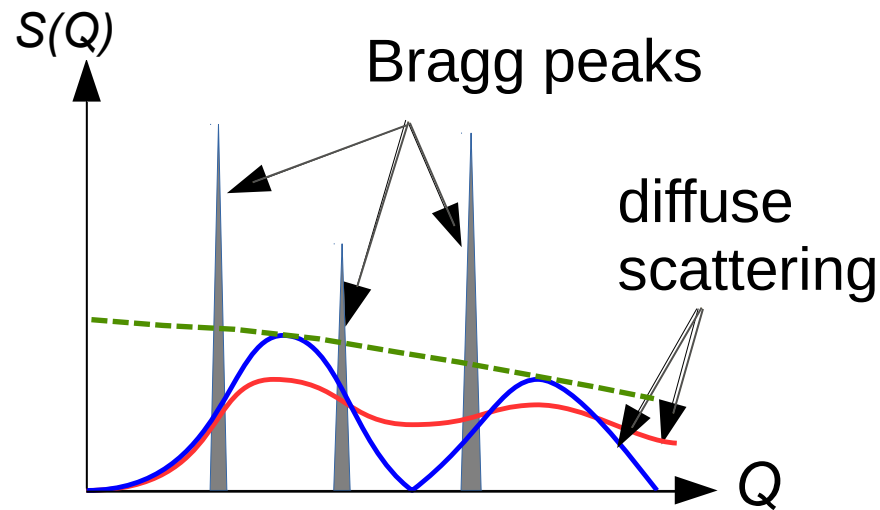


Spin liquid



paramagnet

Magnetic states

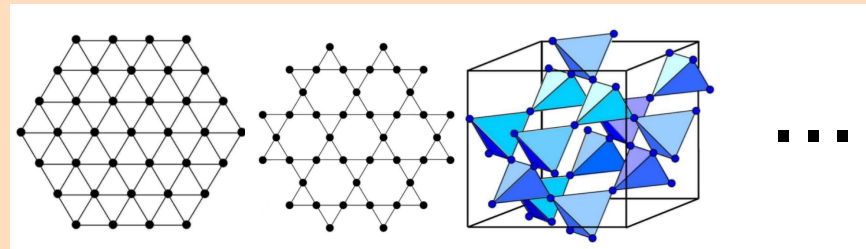


Magnetic frustration: introduction

One or several competing constraints can not be satisfied simultaneously

Important ingredients:

- Topology of the lattice

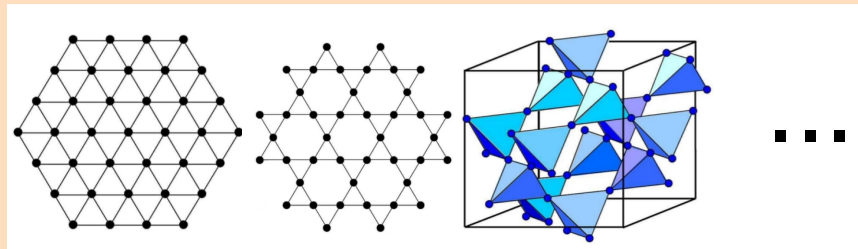


Magnetic frustration: introduction

One or several competing constraints can not be satisfied simultaneously

Important ingredients:

- Topology of the lattice



- Competing interactions: superexchange (nearest/further neighbors) dipolar, DM

$$\mathcal{H} = - \sum_{i,j} \vec{S}_i \tilde{J}_{ij} \vec{S}_j$$

$$\vec{D}_{ij} \cdot (\vec{S}_i \times \vec{S}_j)$$

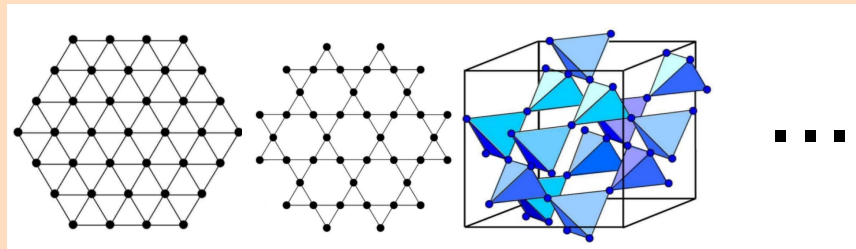
$$Dr_{nn}^3 \sum_{i>j} \left[\frac{\mathbf{S}_i \cdot \mathbf{S}_j}{|\mathbf{r}_{ij}|^3} - \frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{|\mathbf{r}_{ij}|^5} \right]$$

Magnetic frustration: introduction

One or several competing constraints can not be satisfied simultaneously

Important ingredients:

- Topology of the lattice



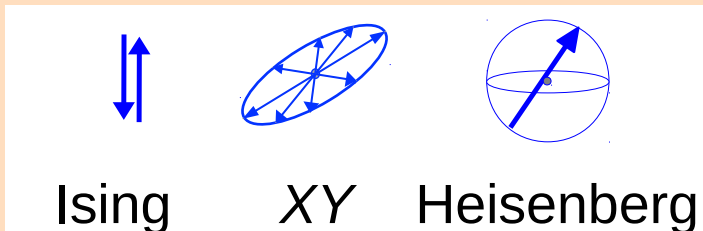
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- Anisotropy, spin dimension and norm (classical versus quantum)



$$S=1/2, 1, 3/2, \dots, \infty$$

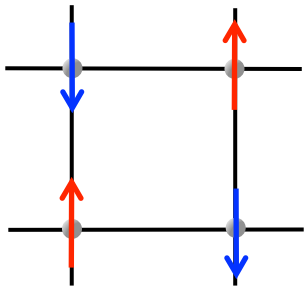
- Disorder, thermal and quantum fluctuations

Magnetic frustration: competition of interactions

One or several competing constraints can not be satisfied simultaneously

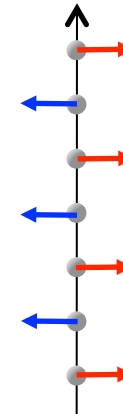
Through competing interactions:

Square lattice with Ising spins



AFM J_1

Spin chain with Ising spins

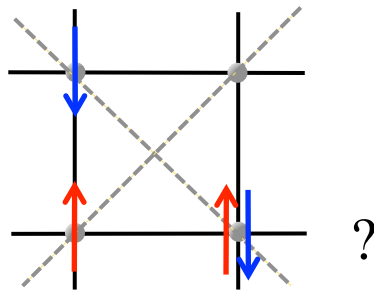


Magnetic frustration: competition of interactions

One or several competing constraints can not be satisfied simultaneously

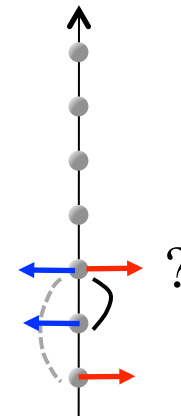
Through competing interactions:

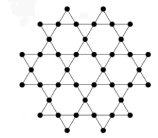
Square lattice with Ising spins



$$\text{AFM } J_1 \approx \text{AFM } J_2$$

Spin chain with Ising spins

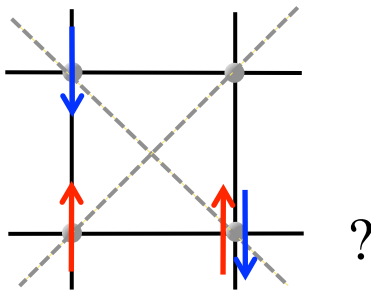




One or several competing constraints can not be satisfied simultaneously

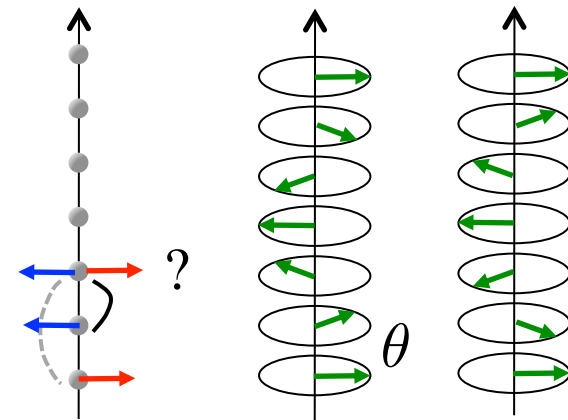
Through competing interactions:

Square lattice with Ising spins



Spin chain from Ising to Heisenberg (XY) spins

$$\text{AFM } J_1 \approx \text{AFM } J_2$$



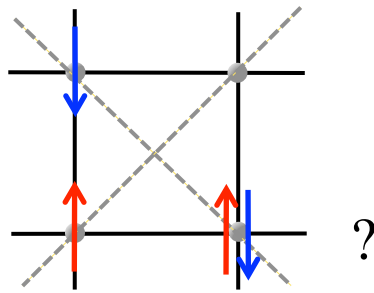
Helical order for $J_1 \leq 4J_2$
 with $\theta = \arccos\left(\frac{-J_1}{4J_2}\right)$

Magnetic frustration: competition of interactions

One or several competing constraints can not be satisfied simultaneously

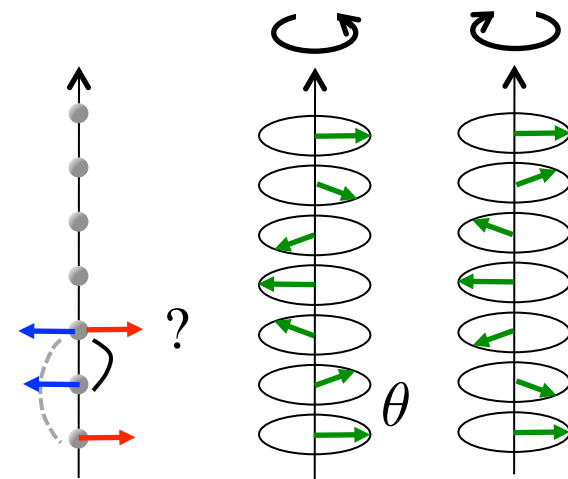
Through competing interactions:

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Spin chain from Ising to Heisenberg (XY) spins

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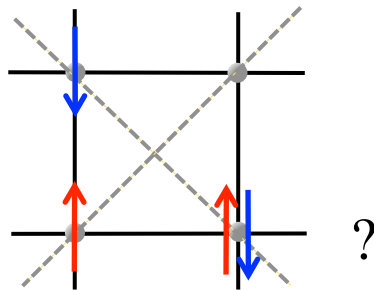


Magnetic frustration: competition of interactions

One or several competing constraints can not be satisfied simultaneously

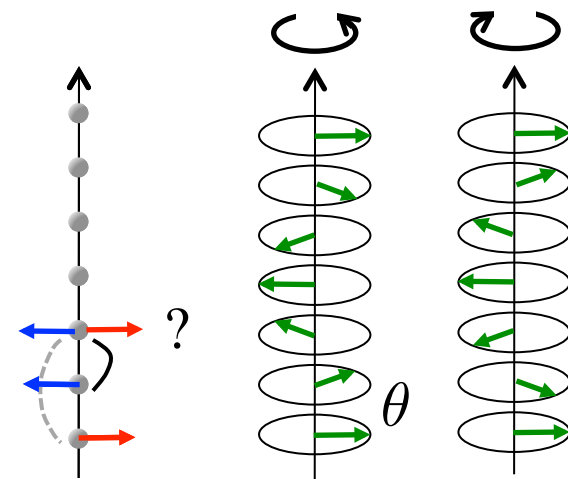
Through competing interactions:

Square lattice with Ising spins



Spin chain from Ising to Heisenberg (XY) spins

$$AFM J_1 \approx AFM J_2$$



→ Prevents magnetic order or find a compromise with exotic magnetic orders (non collinear)

Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

Through the geometry of the lattice:

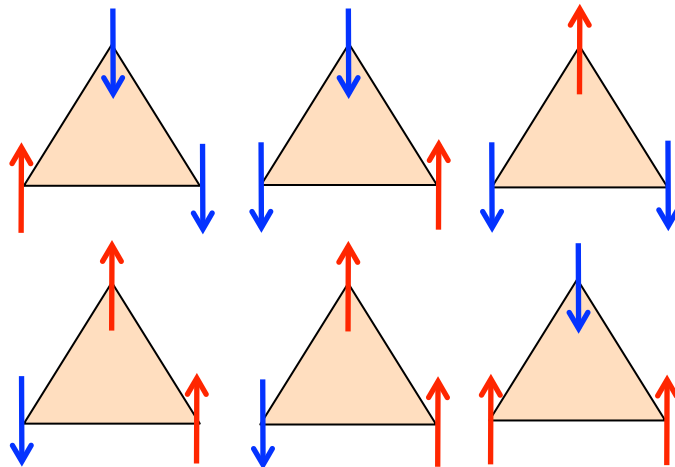
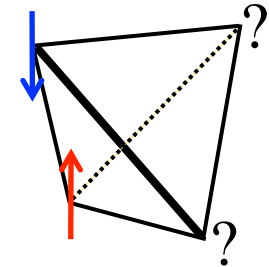
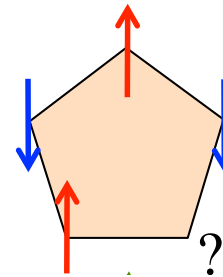
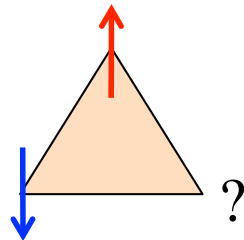
Polygons with odd number of edges

Ex. triangles,

Pentagons, tetrahedron

Antiferromagnetic interactions

Ising



Degeneracy
(measured by finite entropy)

Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

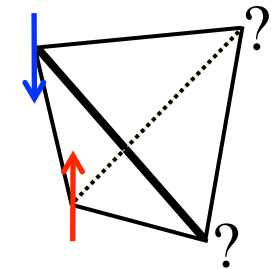
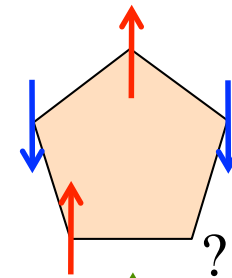
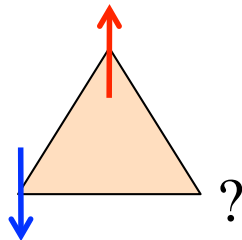
Through the geometry of the lattice:

Polygons with odd number of edges

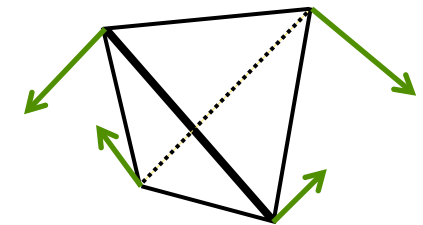
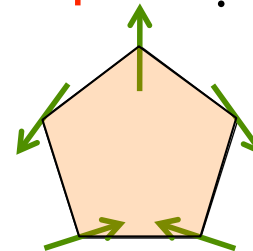
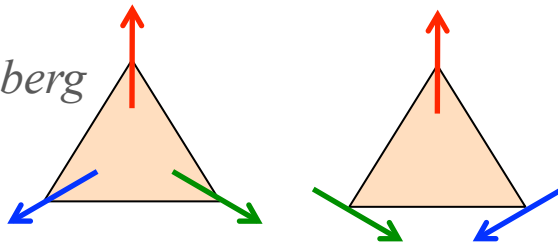
Ex. triangles,

Pentagons, tetrahedron

Ising



XY or Heisenberg



$$\begin{aligned} \mathcal{H} &= -J(S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1) \\ &= -\frac{J}{2}(S_1 + S_2 + S_3)^2 + cst \end{aligned}$$

Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

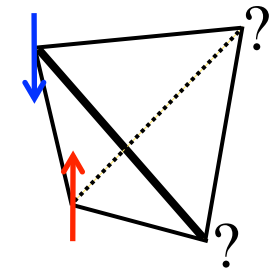
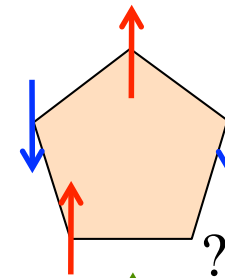
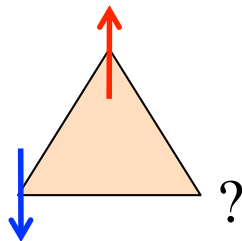
Through the geometry of the lattice:

Polygons with odd number of edges

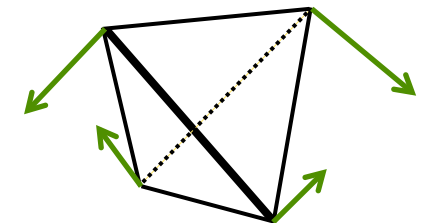
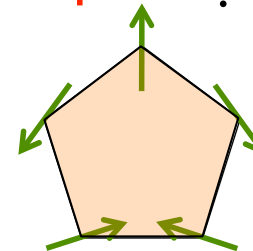
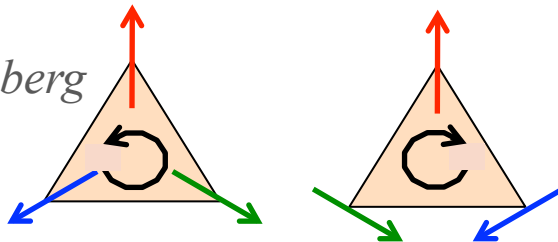
Ex. triangles,

Pentagons, tetrahedron

Ising



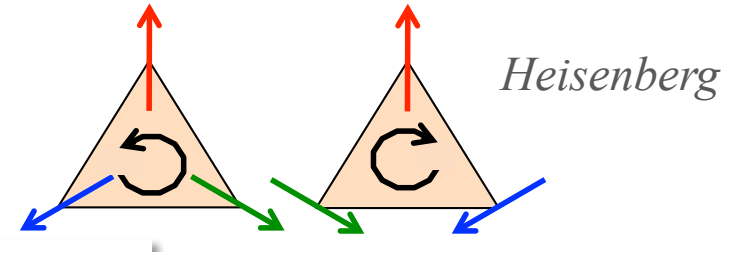
XY or Heisenberg



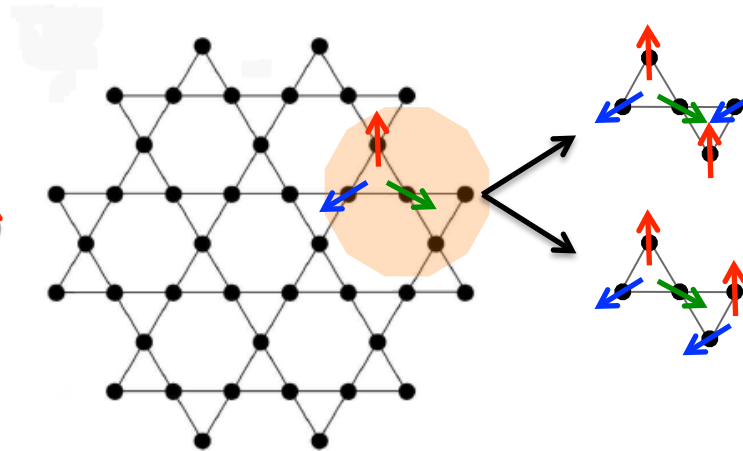
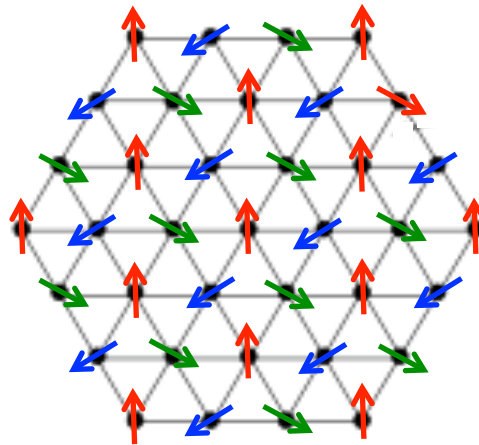
Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

Through the geometry of the lattice:
 Polygons with odd number of edges
 Ex. triangles,
 Pentagons, tetrahedron



role of connectivity: **triangular** (Néel order) vs **kagomé** (spin liquid)



Degeneracy

Geometrical frustration

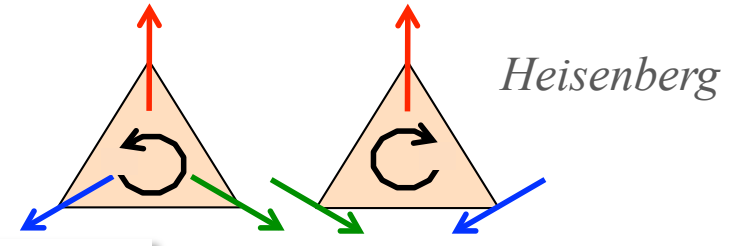
One or several competing constraints can not be satisfied simultaneously

Through the geometry of the lattice:

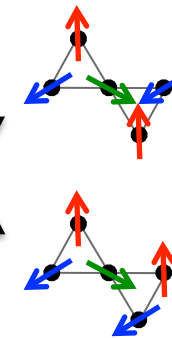
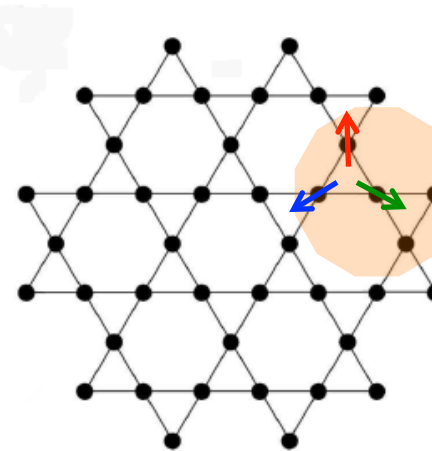
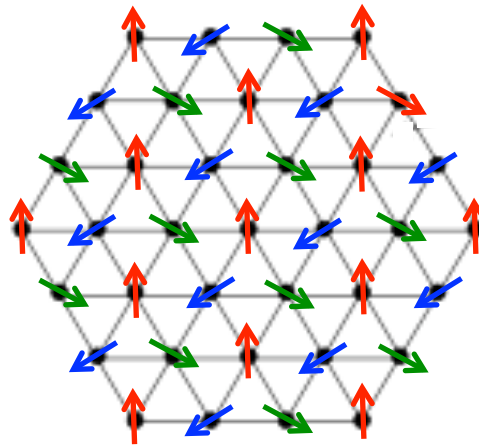
Polygons with odd number of edges

Ex. triangles,

Pentagons, tetrahedron



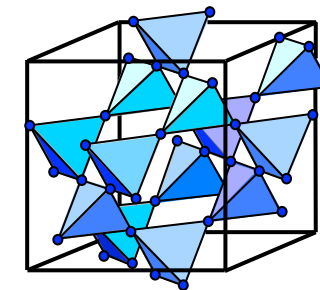
role of connectivity: **triangular** (Néel order) vs **kagomé** (spin liquid)

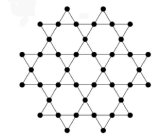


Degeneracy

Macroscopic degeneracy

also for the 3D **pyrochlore** (corner-sharing tetrahedra) lattice

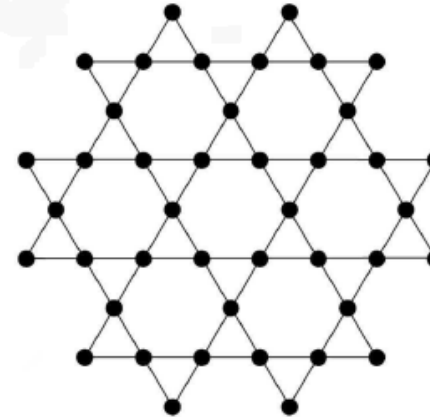




Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

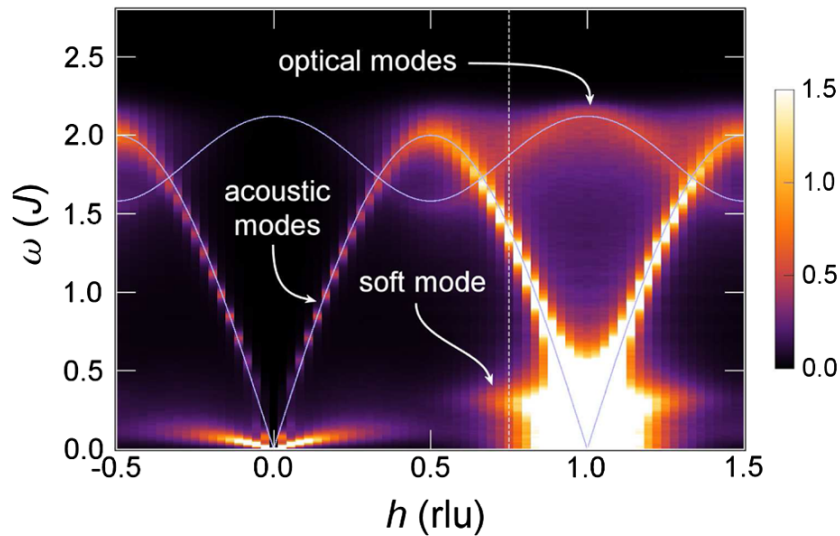
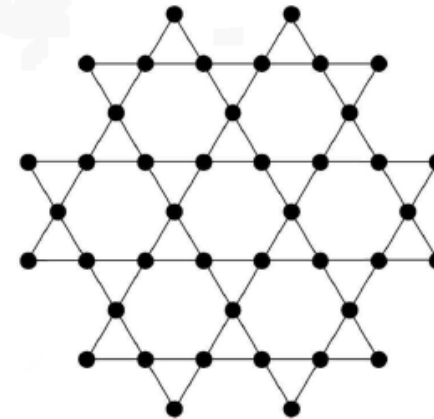
Spin liquids: exotic excitations



Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

Spin liquids: exotic excitations



Well defined excitations in the spin liquid state

Robert et al. PRL 2008

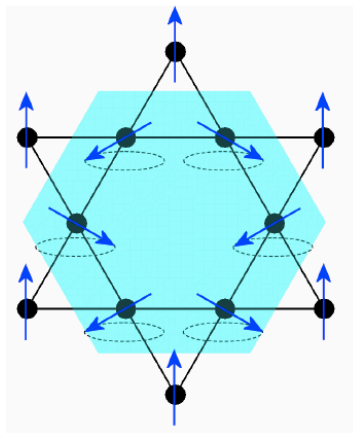
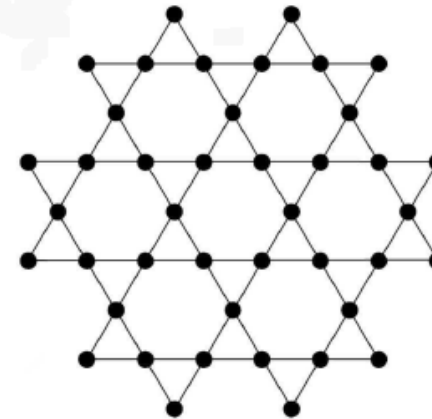
Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

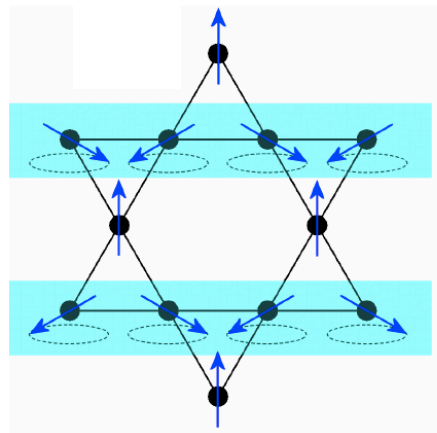
Spin liquids: exotic excitations

$$\begin{aligned} \mathcal{H} &= -J(S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1) \\ &= -\frac{J}{2}(S_1 + S_2 + S_3)^2 + cst \end{aligned}$$

Heisenberg spins : weathervane modes
 → flat modes in ordered states



$\sqrt{3} \times \sqrt{3}$



$q = 0$

Geometrical frustration

One or several competing constraints can not be satisfied simultaneously

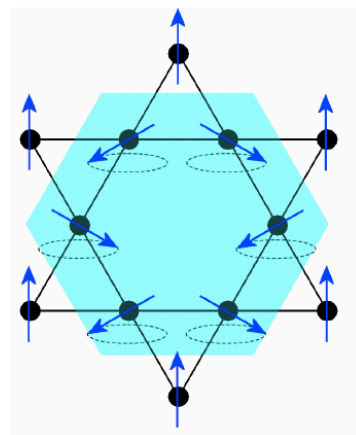
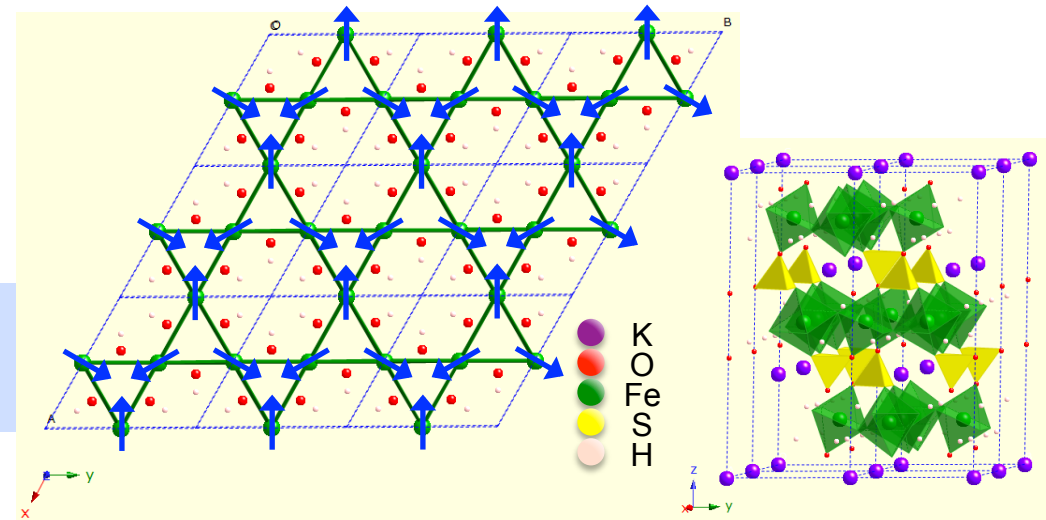
Spin liquids: exotic excitations

$$\mathcal{H} = -J(S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1)$$

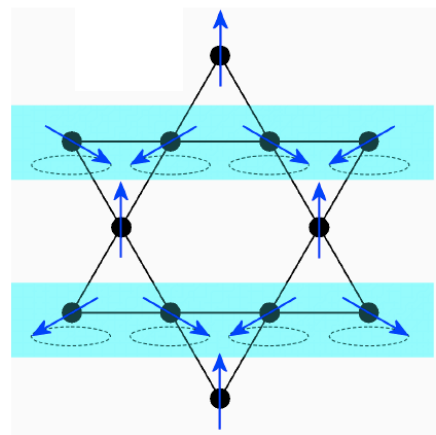
$$= -\frac{J}{2}(S_1 + S_2 + S_3)^2 + cst$$

Heisenberg spins : weathervane modes
 → flat modes in ordered states

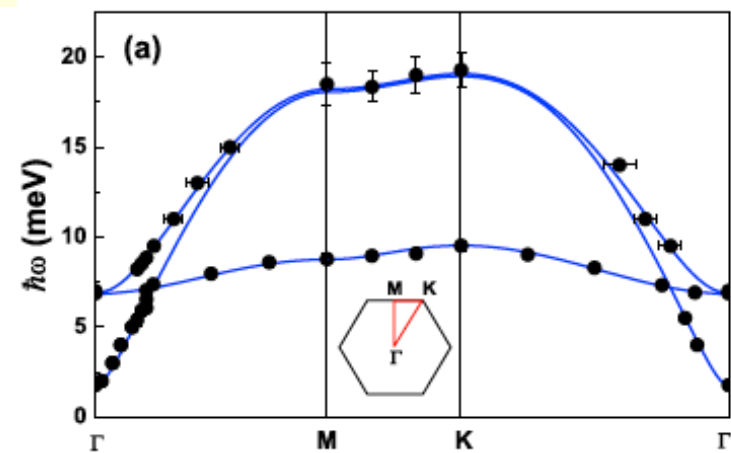
Example Fe jarosite



$$\sqrt{3} \times \sqrt{3}$$



$$q = 0$$



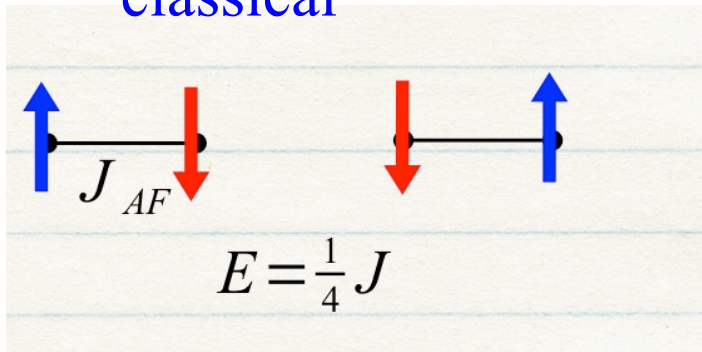
Matan et al. PRL 2006

Magnetic frustration : quantum case

valence bond, entanglement of 2 spins 1/2

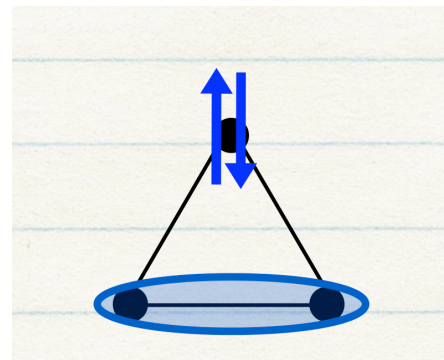
$$\mathcal{H} = -JS_1 \cdot S_2$$

classical



quantum

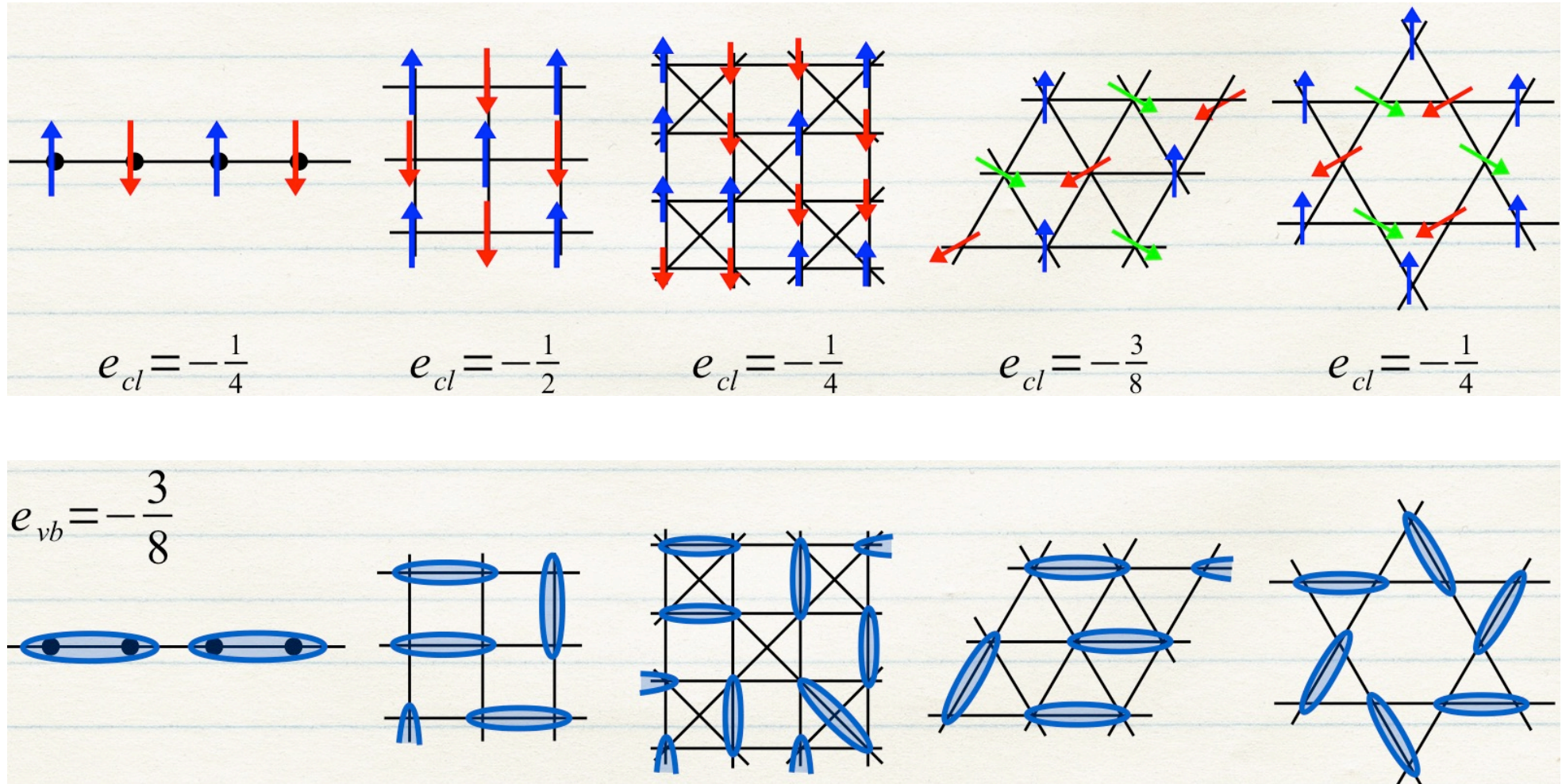
$$\begin{aligned}
 |GS\rangle &= \begin{array}{c} \uparrow \text{---} \downarrow \\ \downarrow \text{---} \uparrow \end{array} + \begin{array}{c} \downarrow \text{---} \uparrow \\ \uparrow \text{---} \downarrow \end{array} \\
 &= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \equiv \text{oval}
 \end{aligned}$$



4 ground states

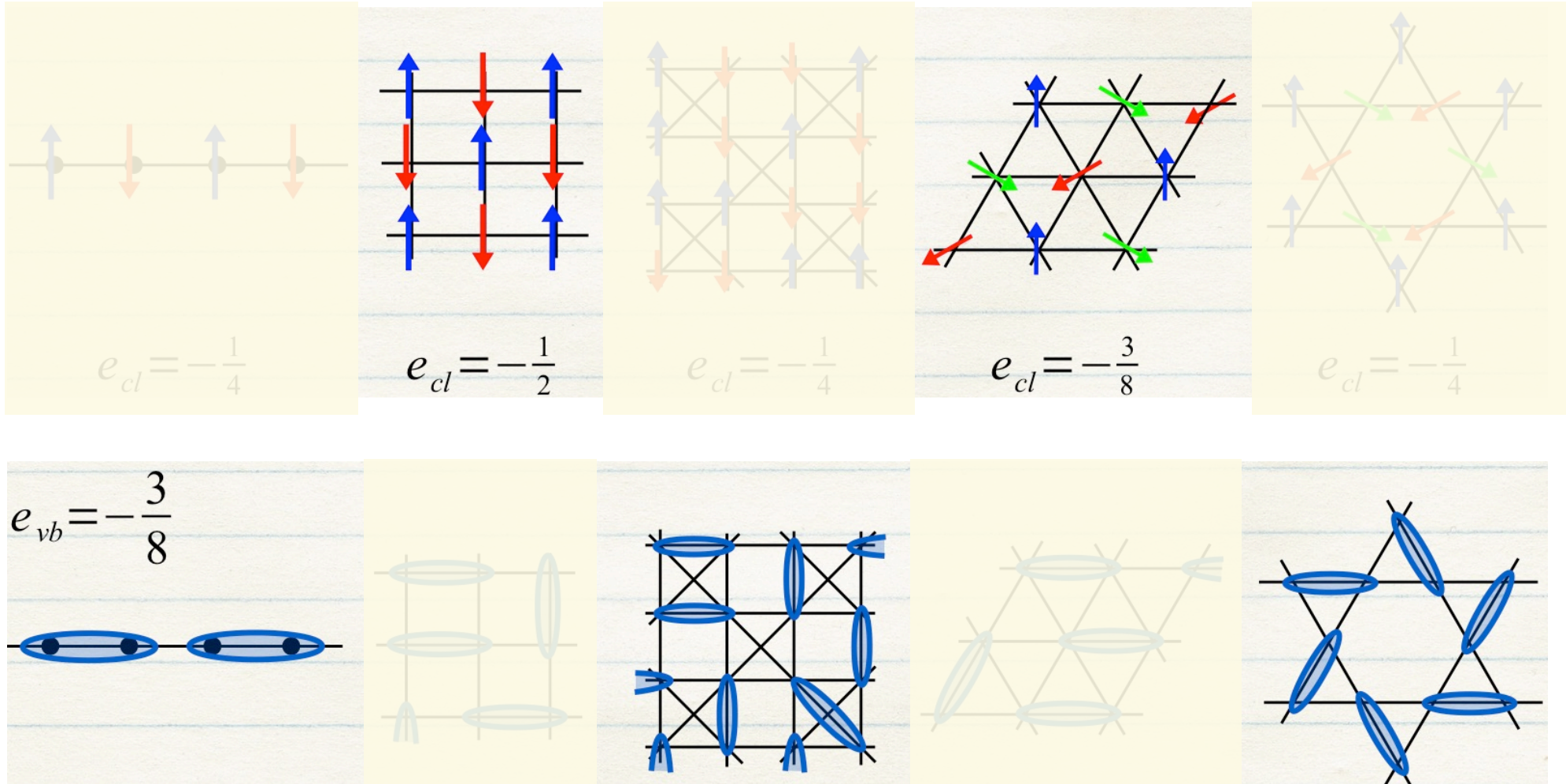
Magnetic frustration : quantum case

Rough comparison of the energies : classical vs quantum ground states



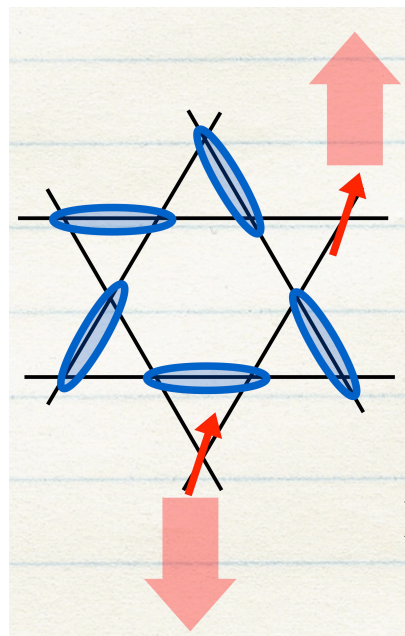
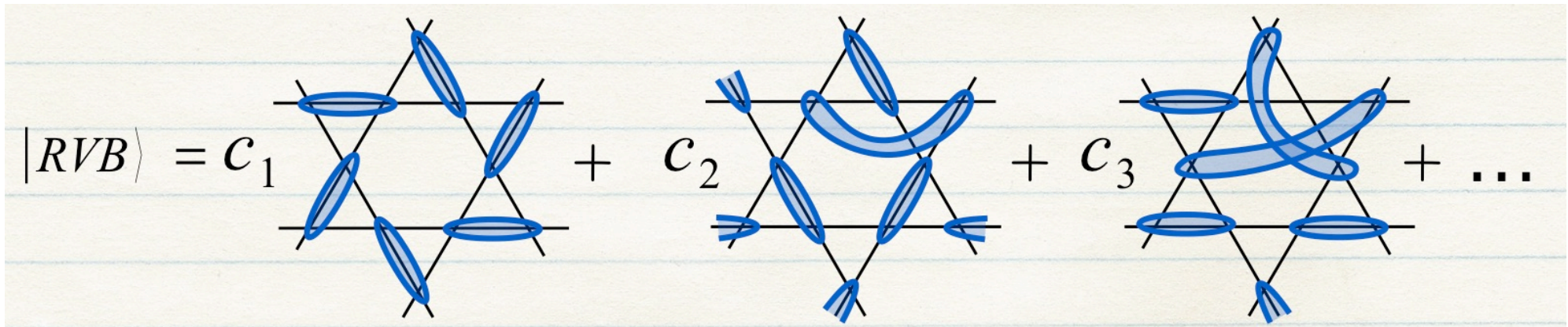
Magnetic frustration : quantum case

Rough comparison of the energies : classical vs quantum ground states



Magnetic frustration : quantum case

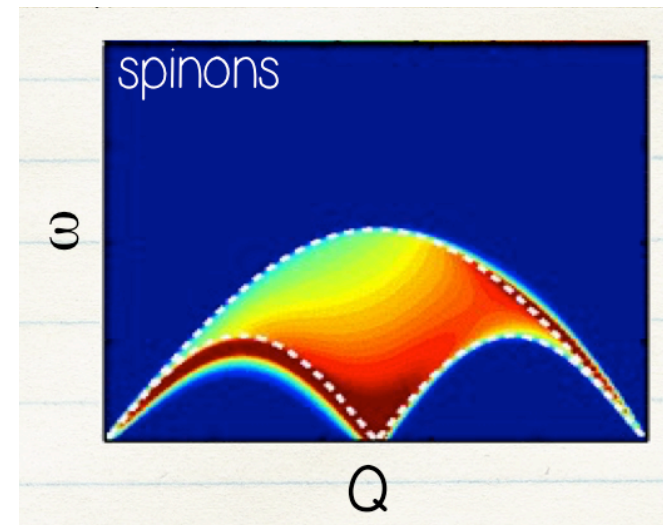
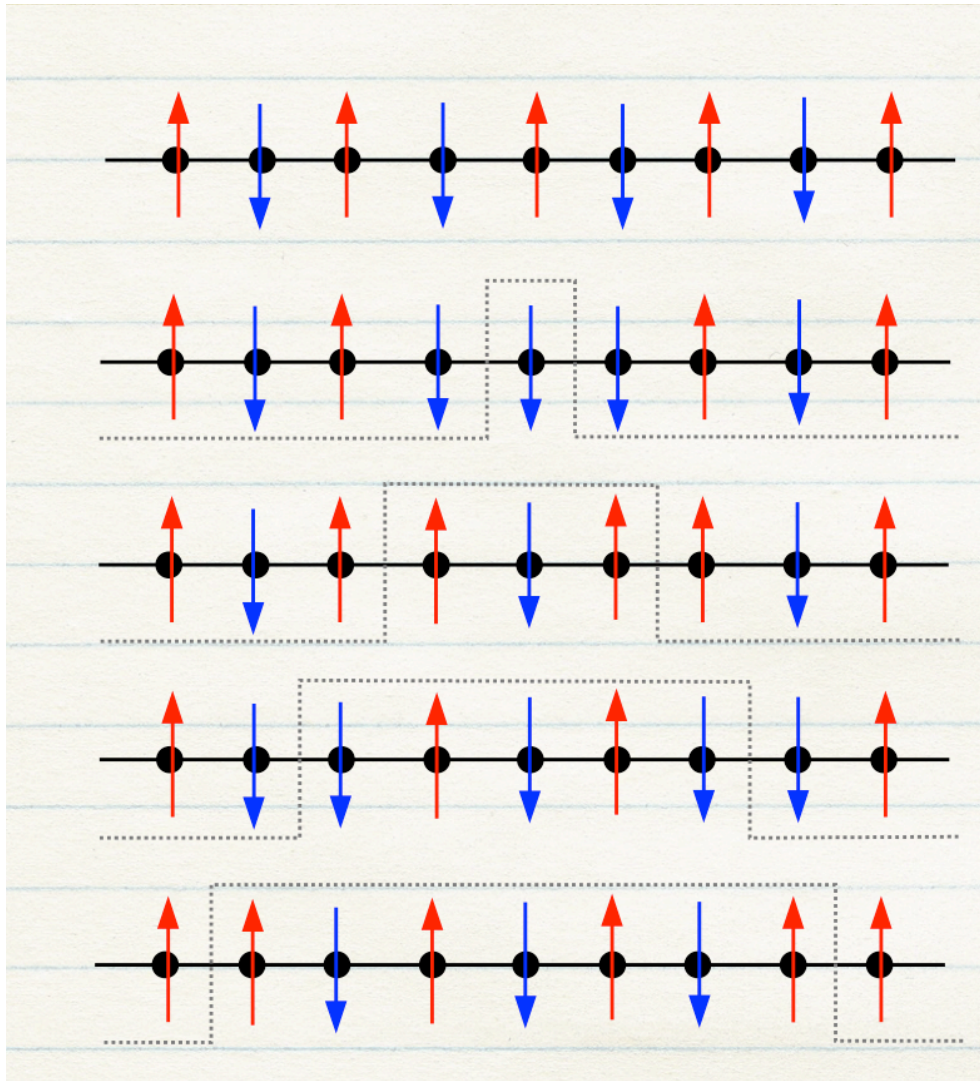
Quantum spin liquid on a kagome lattice: resonating valence bond model



Excitations : deconfined spinons (continuum)

Magnetic frustration : quantum case

Fractionalized excitations: spinons continuum

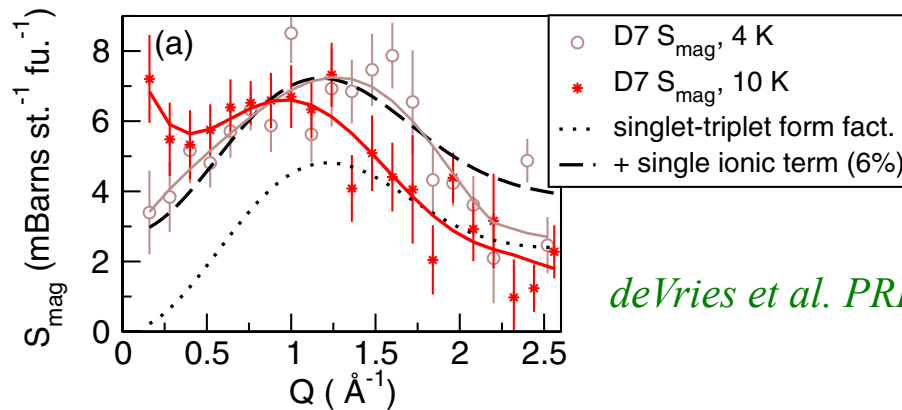


Magnetic frustration : quantum case

Quantum spin liquid on a kagome lattice: resonating valence bond model

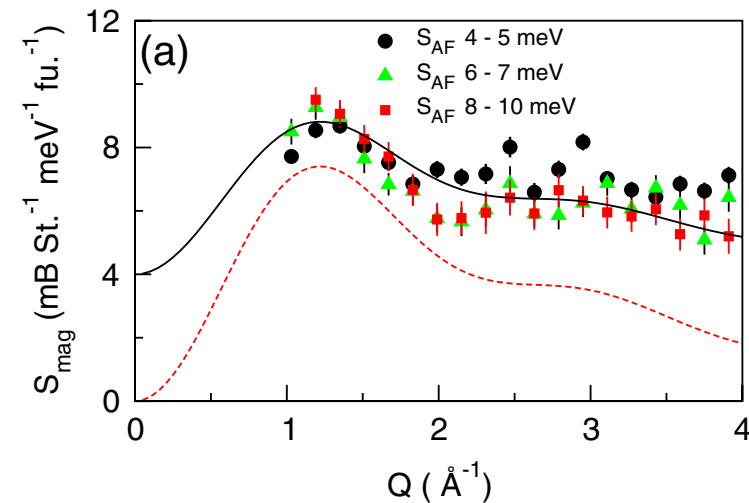
Experimental search for kagome quantum spin liquid : one candidate **Herbertsmithite**,

- **powder measurements** : diffuse spin correlations, dynamics weakly dependent of energy and temperature : signature of spin liquid with spinon excitations?



deVries et al. PRL 2009

Polarized neutron spectrometer D7@ILL
 XYZ polarization analysis



Time-of-flight spectrometer IN4@ILL

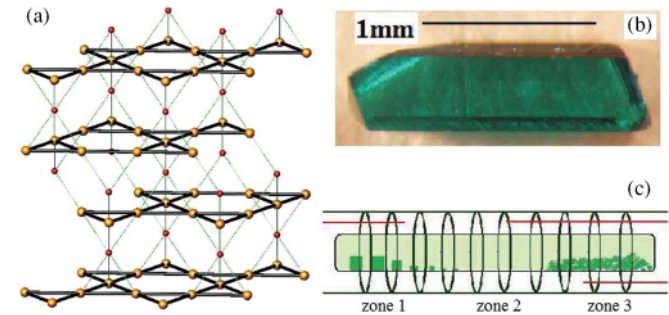
Magnetic frustration : quantum case

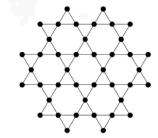
Quantum spin liquid on a kagome lattice: resonating valence bond model

Experimental search for kagome quantum spin liquid : one candidate **Herbertsmithite**,

- 14 years to succeed in growing a **single crystal!**

Han et al. PRB 2011





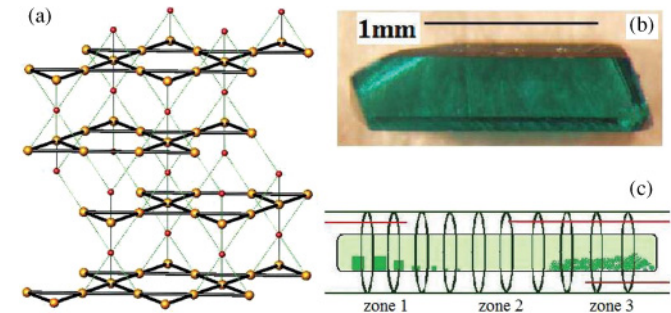
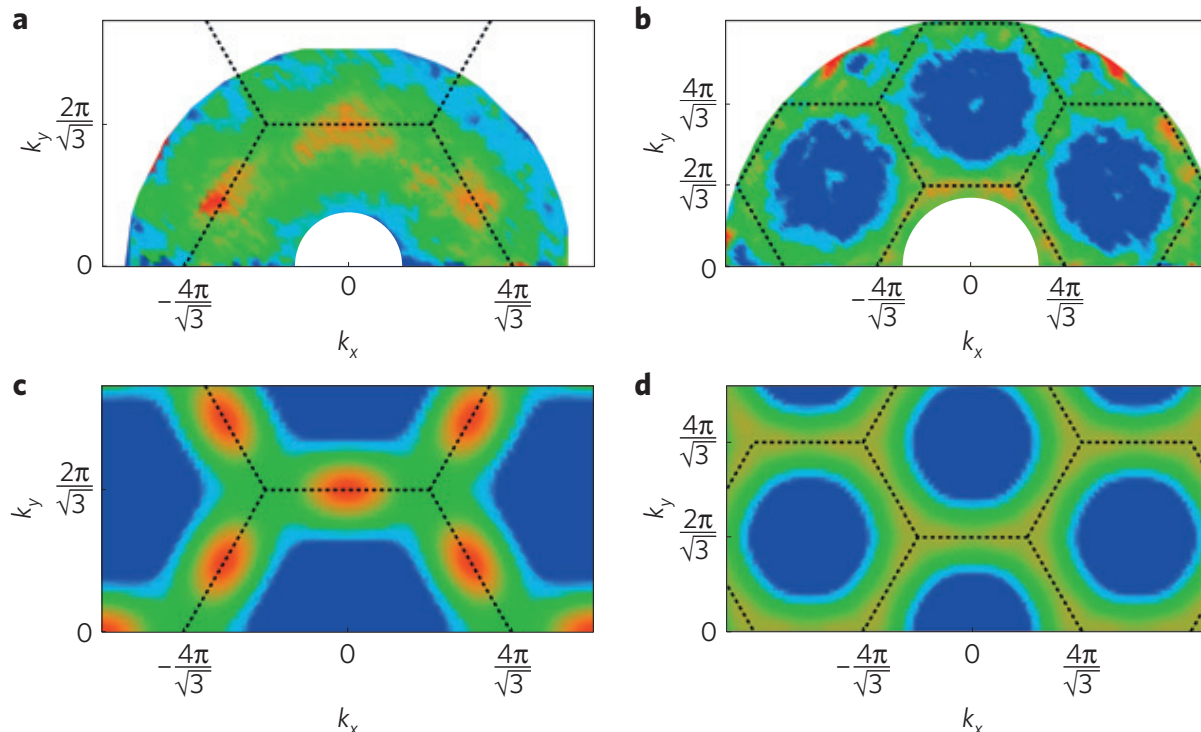
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Han et al. PRB 2011

Dynamical structure factor



Experiment: Han et al. Nature 2012

Calculations: Punk et al. Nat. Phys. 2014

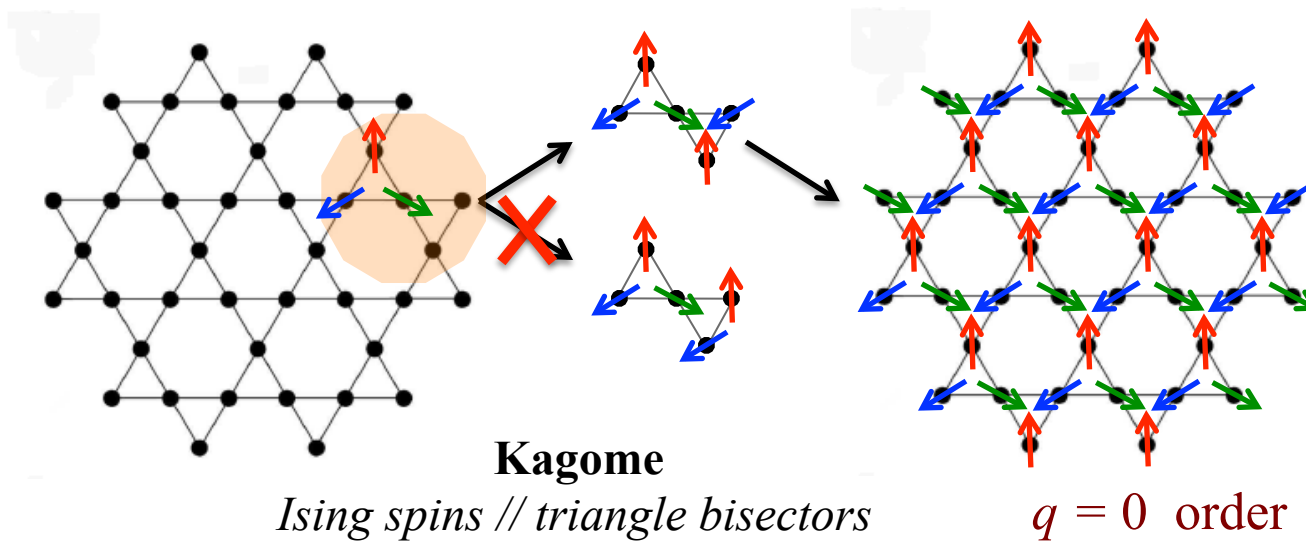
Cl: Quantum spin liquid with complex fractionalized excitations

Magnetic frustration: role of anisotropy

Spin liquids : fragile state with strong degeneracies, spin-spin liquid-like pair correlation, flat modes
 → easily destabilized by small perturbations (DM, dipolar, second neighbor interactions, disorder...)

Antiferromagnetic interactions

Example: Release of degeneracy by the multiaxial anisotropy

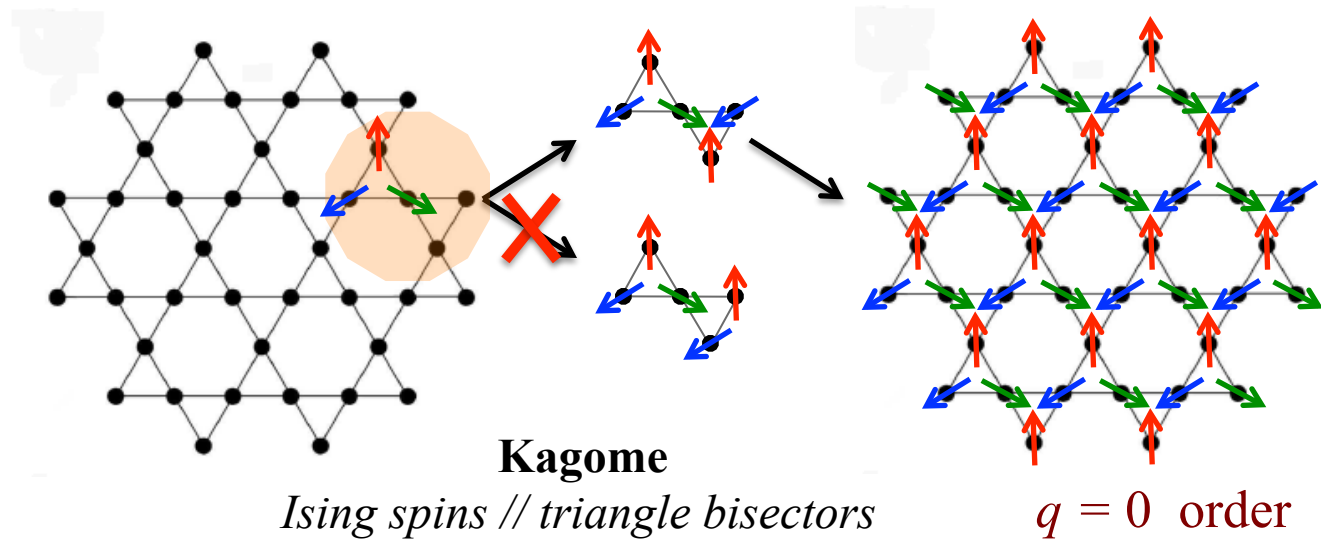


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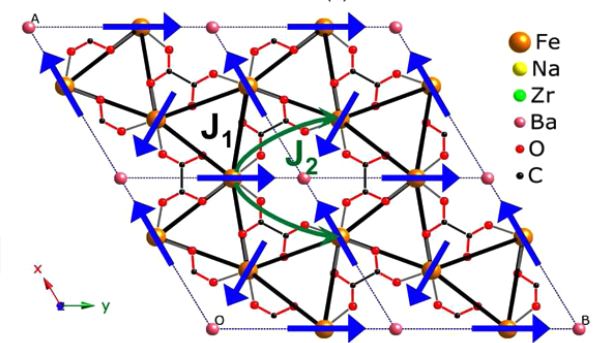
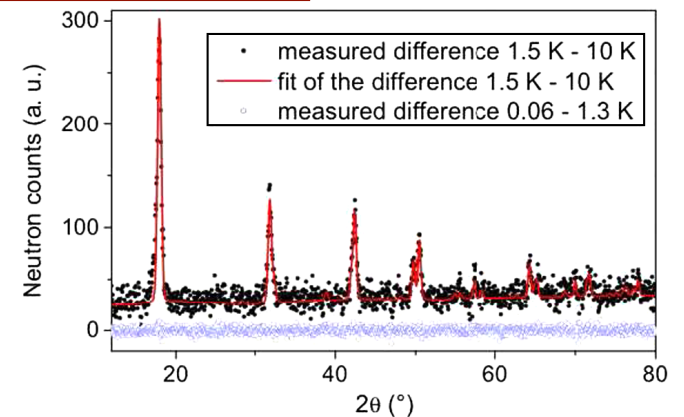
Antiferromagnetic interactions

Example: Release of degeneracy by the multiaxial anisotropy



Ex: $\text{Na}_2\text{Ba}_3[\text{Fe}^{\text{II}}_3(\text{C}_2\text{O}_4)_6][\text{A}^{\text{IV}}(\text{C}_2\text{O}_4)_3]$

Lhotel et al. PRL 2011

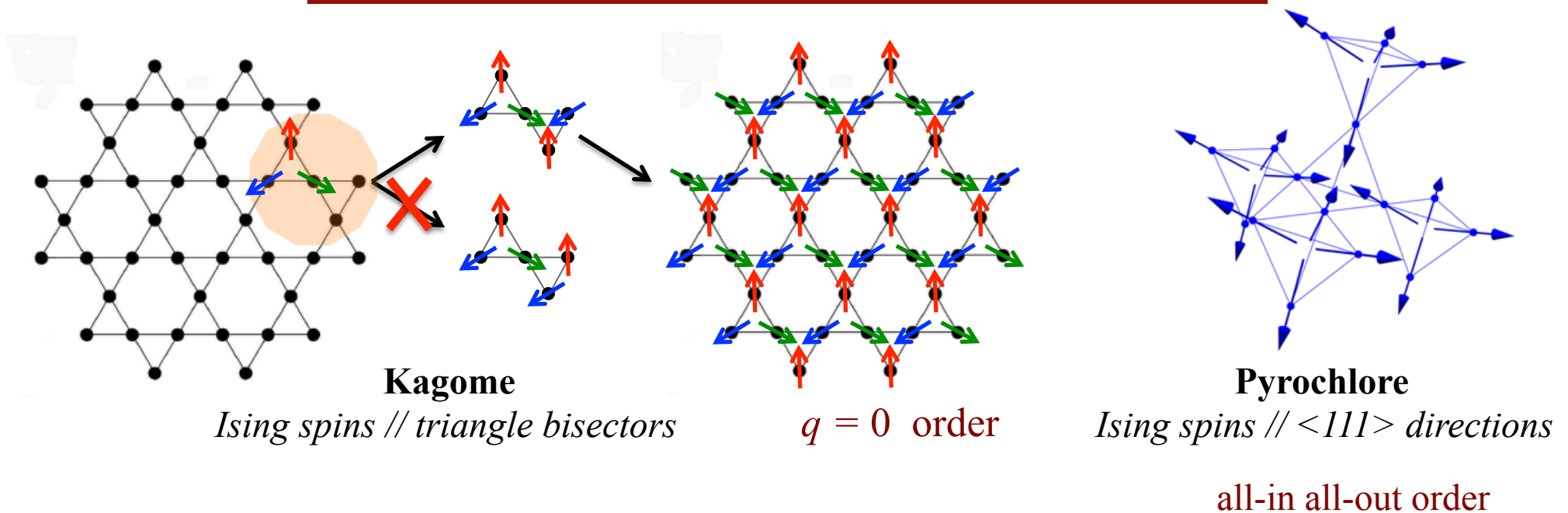


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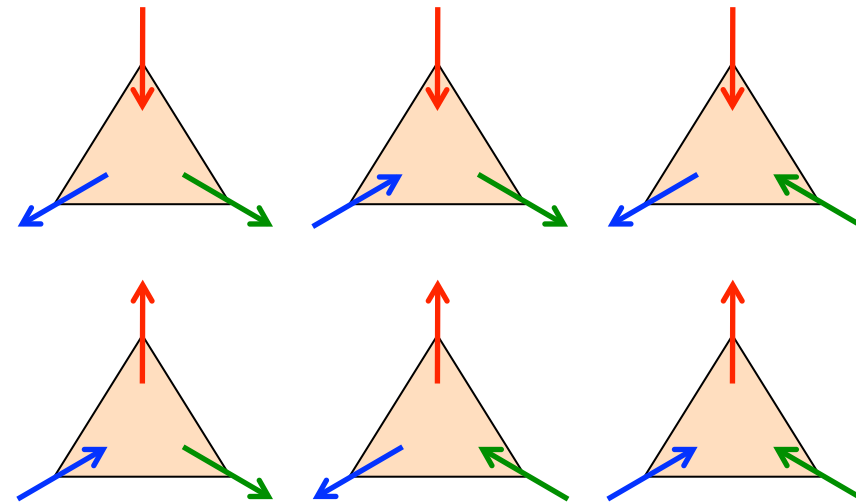
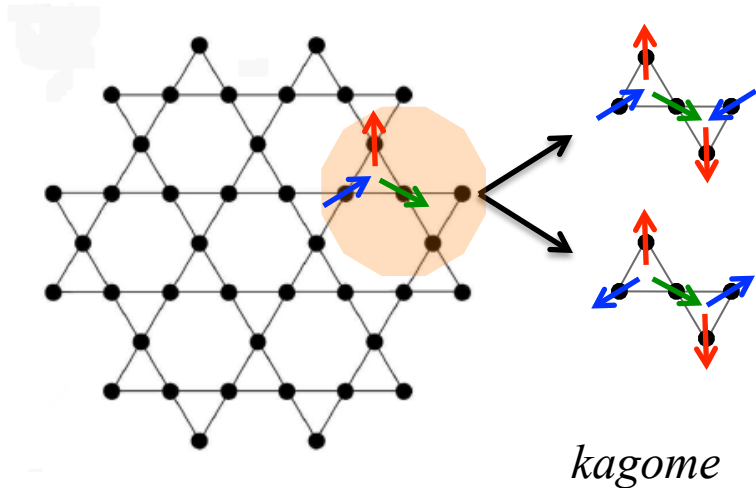
Magnetic frustration: role of anisotropy

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What about ferromagnetic interactions + multiaxial anisotropy?



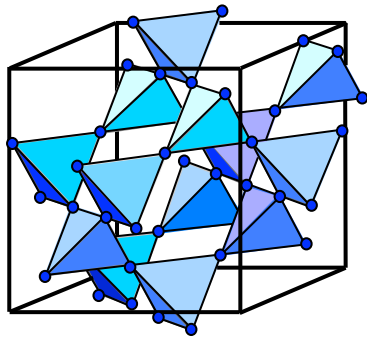
Degenerate ground state



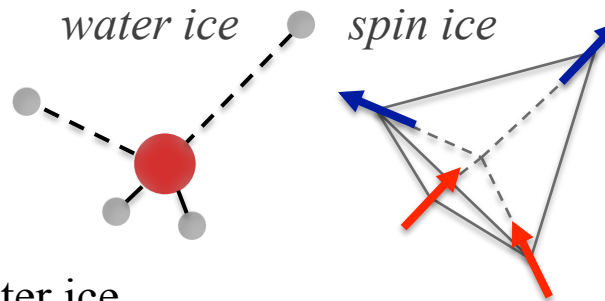
Two-in one-out or two-out on-in ice rule

Spin ice = special case of spin liquids

Magnetic frustration: role of anisotropy



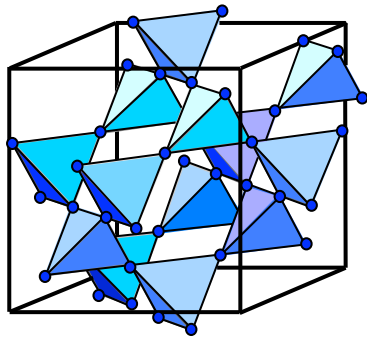
Spin ices in pyrochlores
Multiaxial anisotropy + ferromagnetic interactions



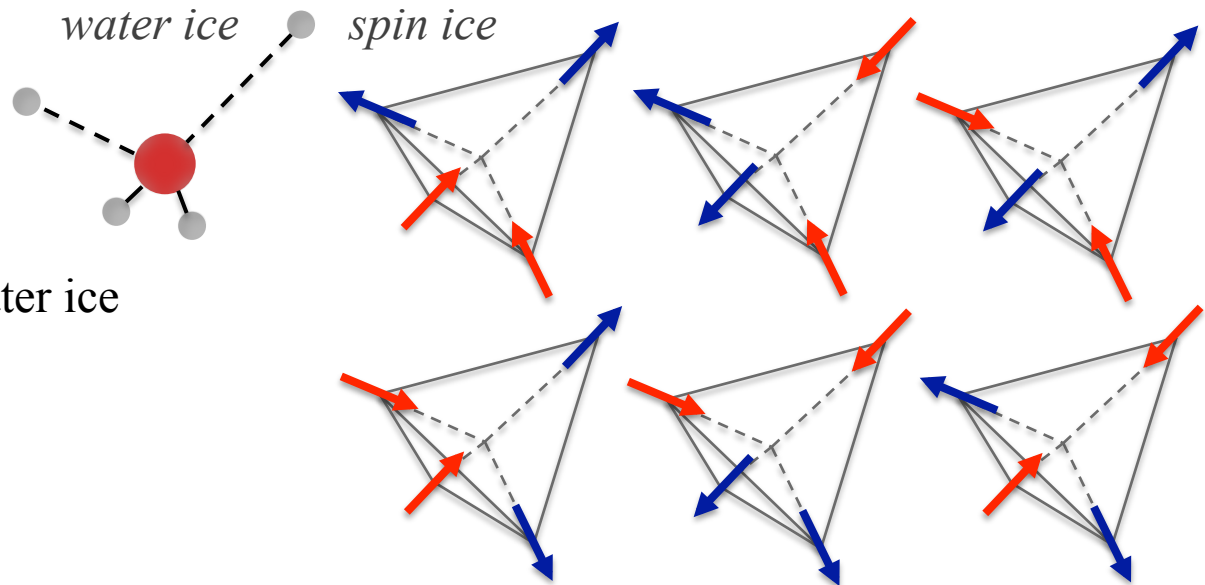
Analogy with proton position in water ice
(2 close, 2 far)

Ice-rules: 2 spins in, 2 spins out

Magnetic frustration: role of anisotropy



Spin ices in pyrochlores
 Multiaxial anisotropy + ferromagnetic interactions



Analogy with proton position in water ice
 (2 close, 2 far)

Ice-rules: 2 spins in, 2 spins out
 6 possible states / tetrahedron
 => **Macroscopic degeneracy**

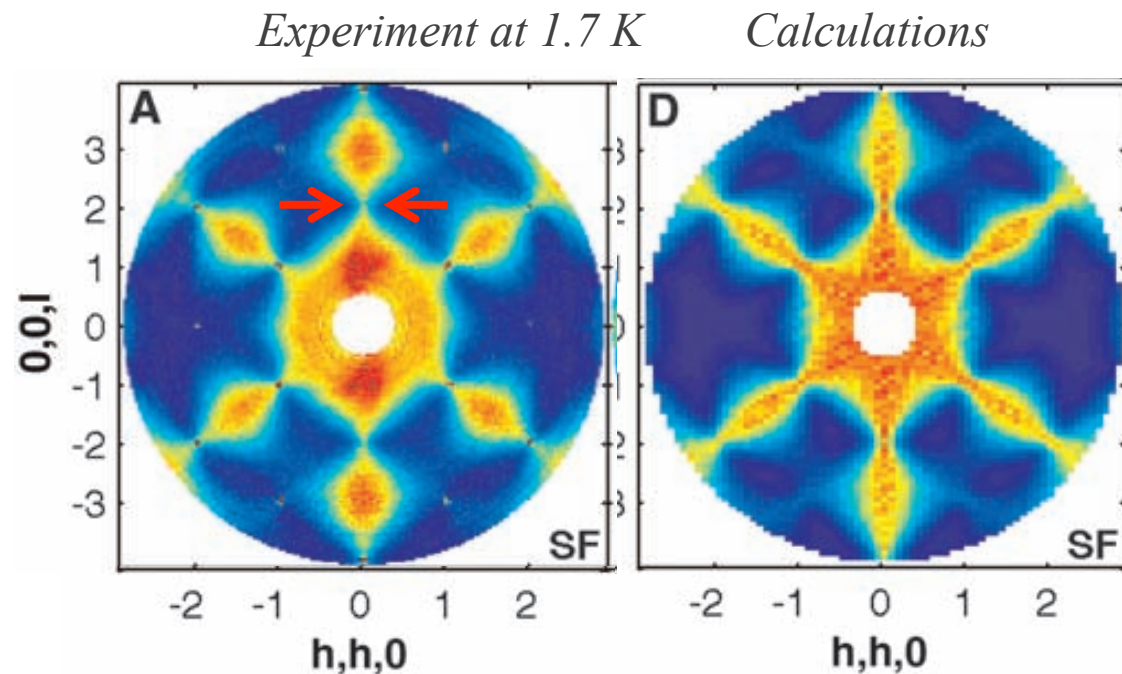
Harris et al., PRL 1997

Magnetic frustration: role of anisotropy

Spin ices in pyrochlores
Multiaxial anisotropy + ferromagnetic interactions

Ice-rules

→ power-law correlations and **pinch-points** in reciprocal space



Fennell et al., Science 2009

Single-crystal spin-flip diffuse scattering map on D7@ILL

Magnetic frustration: summary

Magnetic frustration = wonderful playground to observe new magnetic behaviors:
Macroscopic degeneracy, spin liquid, spin ice, fractionalized excitations ...

Easily destabilized towards complex magnetic orders:
order by disorder, additional interactions, anisotropies, classical and quantum
fluctuations ...

→ non collinear, chiral, partially fluctuating magnetic order with complex (H,T) phase
diagram...

Why neutron scattering is a suitable tool?

Small magnetic signal at very low temperature,
access to spin correlation functions and diffuse scattering,
use of polarized neutrons,
inelastic probe,
complex magnetic order,
complex (H, P, T) phase diagram