Resonant Elastic and Inelastic X-ray Scattering

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- Introduction
- Scattering cross-sections, refraction and absoprtion
- Resonant Elastic X-ray Scattering
 - Magnetic order: Novel MITs in 5d transition metal oxides
 - Mulitipolar order: Orbital ordering
- Resonant Inelastic X-ray Scattering
 - Magnons and crystal-field excitations: 5d transition metal oxides
- Summary

Elastic Scattering: groundstates and order



Inelastic Scattering: excitations

Dispersion Relations



Excitation spectrum, direct measure of interactions: strength, range, symmetry Quasi-particle spectroscopy



1D: Fractional spinon excitations in the quantum Heisenberg antiferromagnetic chain Mourigal et al. Nature Physics (2013)



3D: Quantum and classical criticality in a dimerized quantum antiferromagnet Merchant et al. Nature Physics (2014)

What do we measure in a scattering experiment?

Cross-section and scattering length



The Total Cross - section is obtained by integrating over all solid angle

$$\sigma = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega = 4\pi b^2$$

X-rays and Neutrons **Basic properties and Scattering lengths** Photon **Neutron** Charge: 0 0 1.675 x 10⁻²⁷ Kg Mass: 0 Spin: 1 $\frac{1}{2}$ -1.913 μ_N **Magnetic Moment:** 0 **Scattering lengths:** r₀=2.82 x 10⁻⁵ Å Sensitivity to b~r₀ Structure: (Short range nuclear forces) (E field photon with e) Sensitivity to $r_0(\hbar\omega/mc^2)$ $\mathbf{b}_{mag} \sim \mathbf{r}_0$ **Magnetism:** (E, H field photon with e and μ_B) $(\mu_n.B_{dipp})$ Resonant $r_0 = \frac{1}{4\pi\epsilon} \frac{e^2}{mc^2} = 2.82 \times 10^{-15} m$ 100 r₀! Scattering:

Source Brilliance (or spectral brightness)





Quite generally we expect

$$I_{SC} = \Phi_0 \times \Delta\Omega \times \text{Scattering efficiency factor} = \Phi_0 \times \Delta\Omega \times \left(\frac{d\sigma}{d\Omega}\right)$$

This defines the **Differential Cross - section**

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of particles scattered per second into detector}}{\text{Incident Flux } \times \text{Detector solid Angle}} = \frac{I_{sc}}{\Phi_0 \Delta \Omega}$$
Elastic

The Total Cross - section is obtained by integrating over all solid angle

$$\sigma = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

This Partial Differential Cross - section

$$\left(\frac{d\sigma}{d\Omega dE_f}\right) = \frac{\text{Particles scattered per second into detector in energy window } dE_f}{\text{Incident Flux } \times \text{Detector solid Angle} \times dE_f}$$

Scattering kinematics

• Momentum transfer

$$\hbar \mathbf{Q} = \hbar \left(\mathbf{k}_i - \mathbf{k}_f \right)$$

• Energy transfer

$$\hbar\omega = E_i - E_f$$



- Scattering event independent variables: (Q,ω)
 - Elastic Scattering $\hbar\omega = 0$ Crystallography
 Inelastic Scattering $\hbar\omega \neq 0$ Spectroscopy
- FT conjugate variables
 - \circ Q ~ 2 π /(length)
 - \circ ħω ~ 2πħ/(time)

Non-resonant charge scattering from unbound electrons Thomson cross-section

Classical calculation of the electric field reradiated from a free electron



$$\mathbf{E}_{rad} \propto \frac{-e}{R} \mathbf{a}_X(t') \sin \Psi \equiv \frac{e}{R} \mathbf{a}_X(t') \left(\hat{\boldsymbol{\epsilon}} \cdot \hat{\boldsymbol{\epsilon}}'\right)$$
$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}} = r_0^2 \left|\hat{\boldsymbol{\epsilon}} \cdot \hat{\boldsymbol{\epsilon}}'\right|^2$$

Single atom



$$\begin{pmatrix} \frac{d\sigma}{d\Omega} \end{pmatrix}_{\text{Th.}}^{atom} = r_0^2 \left[f^0(\mathbf{Q}) \right]^2 \left| \hat{\boldsymbol{\epsilon}} \cdot \hat{\boldsymbol{\epsilon}}' \right|^2$$

$$f^0(\mathbf{Q}) = \int \rho(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \qquad \begin{array}{c} \text{Atomic} \\ \text{form} \\ \text{form} \\ \text{factor} \end{array}$$

$$\mathbf{Q} = \mathbf{k} - \mathbf{k}'$$

Wave vector transfer

X-ray Resonant Scattering from bound electrons

Dispersion corrections

From electrons bound in atoms expect:

$$f(\mathbf{Q},\omega) = f^{0}(\mathbf{Q}) + f'(\omega) + i f''(\omega)$$

Forced, damped oscillator model









Relationship between scattering and refraction

Electric field $\mathbf{E}(t) \Rightarrow \mathbf{P}(t)$ (electric dipole/V)

$$\mathbf{P}(t) = \varepsilon_0 \chi \mathbf{E}(t) = (\varepsilon - \varepsilon_0) \mathbf{E}(t)$$

where

$$P(t) = \frac{-Nex(t)}{V} = -\rho ex(t) = -\rho e \left(-\frac{e}{m}\right) \frac{E_0 e^{-i\omega t}}{\left(\omega_0^2 - \omega^2 - i\omega\Gamma\right)}$$
$$\Rightarrow \frac{P(t)}{E(t)} = \varepsilon - \varepsilon_0 = \left(\frac{e^2 \rho}{m}\right) \frac{1}{\left(\omega_0^2 - \omega^2 - i\omega\Gamma\right)}$$

The refractive index is defined by

$$n^{2} = \frac{c^{2}}{v^{2}} = \frac{\varepsilon}{\varepsilon_{0}}$$
$$\Rightarrow n^{2} = 1 + \left(\frac{e^{2}\rho}{\varepsilon_{0}m}\right) \frac{1}{\left(\omega_{0}^{2} - \omega^{2} - i\omega\Gamma\right)}$$

For X-rays, $\omega \gg \omega_0 \gg \Gamma$

$$n \approx 1 - \frac{1}{2} \left(\frac{e^2 \rho}{\varepsilon_0 m \omega^2} \right) = 1 - \frac{2\pi \rho r_0}{k^2}$$

$$\delta = \frac{2\pi\rho_a r_0 \left(f^0(0) + f'(\hbar\omega) \right)}{k^2} \qquad \beta = -\frac{2\pi\rho_a r_0 f''(\hbar\omega)}{k^2}$$





Relationship between scattering and refraction



Scattering and refraction: different ways of understanding the same phenomena

X-ray absorption edges



Relationship scattering, refraction and absorption



$$\int_{a}^{b} = -\left(\frac{1}{2\pi\rho_{a}r_{0}}\right) + \frac{1}{2k} = -\left(\frac{1}{4\pi r_{0}}\right) + \frac{1}{2k}$$
Absorption is proportional to the imaginary part of the forward scattering amplitude (Optical Theorem)

Theoretical Framework

Cross-sections and the interaction Hamiltonian

Task is to determine the differential cross-section:

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\text{Number of particles scattered per second into detector}}{\text{Incident Flux } \times \text{Detector solid Angle}}$$
$$= \frac{W}{\Phi_0(\Delta\Omega)}$$

The transition rate probability W to 2nd order

$$W = \frac{2\pi}{\hbar} \left| \left\langle f \left| \boldsymbol{H}_{I} \right| i \right\rangle + \sum_{n} \left| \frac{\langle f \left| \boldsymbol{H}_{I} \right| n \rangle \langle n \left| \boldsymbol{H}_{I} \right| i \rangle}{E_{i} - E_{n}} \right|^{2} \rho \left(E_{f} \right) \right|$$

Interaction Hamiltonian H_I : describes interaction between radiation and target

Density of final states

$$\rho\left(E_{f}\right)dE_{f} = \rho\left(\mathbf{k}_{f}\right)d\mathbf{k}_{f}$$

Box normalisation implies

$$\rho\left(E_{f}\right)dE_{f} = \rho\left(k_{f}\right)k_{f}^{2}\Delta\Omega dk_{f}$$

$$\therefore \quad \rho\left(E_{f}\right) = \frac{V}{(2\pi)^{3}}k_{f}^{2}\Delta\Omega \frac{dk_{f}}{dE_{f}}$$

To first order

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{1}{\Phi_0} \frac{2\pi}{\hbar} \left| \left\langle f \left| \boldsymbol{H}_{\boldsymbol{I}} \right| i \right\rangle \right|^2 \frac{V}{(2\pi)^3} \mathbf{k}_f^2 \frac{d\mathbf{k}_f}{dE_f}$$



Valid for neutrons and X-rays



Non-resonant charge scattering from unbound electrons Quantum mechanical calculation

Hamiltonian of single electron in an electromagnetic field:

 $\mathcal{H}_0 = \frac{p^2}{2m} + V \qquad \mathbf{p} \to \mathbf{p} + e\mathbf{A} \qquad \mathbf{B} = \nabla \times \mathbf{A} \quad \text{Canonical momentum}$ $\mathcal{H}_0 \to \mathcal{H}_0 + \mathcal{H}_I \qquad \mathcal{H}_I = \frac{e\mathbf{A} \cdot \mathbf{p}}{m} + \frac{e^2 \mathbf{A}^2}{2m}$

Quantization of the electromagnetic field vector potential:

$$\mathbf{A}(\mathbf{r},t) = \sum_{u} \sum_{\mathbf{k}} \sqrt{\frac{\hbar}{2\epsilon_0 \omega V}} \hat{\boldsymbol{\epsilon}}_u \left[a_{u,\mathbf{k}} e^{i(\mathbf{k}\cdot\boldsymbol{r}-\omega t)} + a_{u,\mathbf{k}}^{\dagger} e^{-i(\mathbf{k}\cdot\boldsymbol{r}-\omega t)} \right]$$
$$a_{u,\mathbf{k}}|n\rangle = \sqrt{n}|n-1\rangle \qquad a_{u,\mathbf{k}}^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle \quad \rightarrow \mathcal{E}_{\mathrm{rad}} = \hbar\omega (a_{u,\mathbf{k}}^{\dagger}a_{u,\mathbf{k}}+1/2)$$

1st order process: destroy photon from incident beam, create one in scattered beam

$$W = \frac{2\pi}{\hbar} |\langle f | \mathcal{H}_{I} | i \rangle|^{2} \rho(E_{f}) = \frac{2\pi}{\hbar} |\langle f | \frac{e^{2} A^{2}}{2m} | i \rangle|^{2} \rho(E_{f})$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Th.}} = \frac{W}{\Phi_{0} \Delta \Omega} = r_{0}^{2} |\hat{\epsilon}_{f} \cdot \hat{\epsilon}_{i}|^{2}$$

$$\underbrace{\frac{\hat{\epsilon}_{\perp} = \sigma}{\Phi_{0} \Delta \Omega} = r_{0}^{2} |\hat{\epsilon}_{f} \cdot \hat{\epsilon}_{i}|^{2}}_{\text{Thomson scattering}}$$

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X-ray Scattering

Non-resonant elastic magnetic

Single spin in electromagnetic field: identify leading order magnetic term



1st order scattering processes:

 $\begin{pmatrix} \frac{d\sigma}{d\Omega} \end{pmatrix}_{\text{Th.}} = r_0^2 \left| \hat{\boldsymbol{\epsilon}}_f \cdot \hat{\boldsymbol{\epsilon}}_i \right|^2 \quad \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mag.}}^{\text{1st order}} = r_0^2 \left(\frac{\hbar\omega}{mc^2} \right)^2 \left| \hat{\boldsymbol{\epsilon}}_f \times \hat{\boldsymbol{\epsilon}}_i \right|^2 \langle \mathbf{s} \rangle^2$

- Non-resonant magnetic scattering is weaker than charge by a factor of 0.0001 at 10 keV
- Non-resonant magnetic scattering (1st order) is proportional to <s>² => Magnetic structures
- Magnetic scattering has a distinctive dependence on photon polarization

X-ray Magnetic Scattering

1st Non-resonant X-ray Magnetic Scattering

NiO, de Bergevin and Brunel (1972)



Tube source: Counts per 4 hours!

1st NREMXS Synchrotron Studies

Holmium, Gibbs et al. (1985)



1st REXMS

Nickel, Namikawa (1985)



Resonant X-ray magnetic scattering (RXMS)

Large enhancement of XRS at L edges of Holmium

Gibbs, Harshman, Isaacs, McWhan, Mills and Vettier (1988)



•100 fold increase when tuned to the L_3 edge

•Two distinct types of transition are observed: one above and one below the edge

• Higher order satellites up to 4th order

•Polarization state changes with order 1^+ : rotated, $\sigma \rightarrow \pi'$ 1^- : unrotated, , $\sigma \rightarrow \sigma'$ •Signal disappears at T

 $\bullet Signal \ disappears \ at \ T_{_N}$

Peaks arise from transitions to bound states
 1⁺: 2p -> 5d Dipole
 1⁻: 2p -> 4f Quadrupole

RXMS is Born: A New Element and Electron Shell Sensitive Probe! Resonant inelastic X-ray scattering studies of elementary excitations Ament et al. Rev. Mod. Phys. 83 705 (2011)

Resonant X-ray scattering

Theoretical Framework

$$w = \frac{2\pi}{\hbar} \sum_{\mathbf{f}} |\langle \mathbf{f} | \mathcal{H}' | g \rangle + \sum_{n} \frac{\langle \mathbf{f} | \mathcal{H}' | n \rangle \langle \mathbf{n} | \mathcal{H}' | g \rangle}{E_g - E_n} |^2 \delta(E_{\mathbf{f}} - E_g)$$
$$E_g = E_g + \hbar \omega_{\mathbf{k}} \qquad E_{\mathbf{f}} = E_f + \hbar \omega_{\mathbf{k}'}$$

When $E_g \sim E_n$ second term dominates. The interaction Hamiltonian to leading order is then $\mathcal{H}' = \frac{e\mathbf{A} \cdot \mathbf{p}}{\mathbf{P}}$

The Kramers-Heisenberg RIXS cross-section is:

$$I(\omega, \mathbf{k}, \mathbf{k}', \boldsymbol{\epsilon}, \boldsymbol{\epsilon}', \omega_{\mathbf{k}}, \omega_{\mathbf{k}'}) = r_0^2 m^2 \omega_{\mathbf{k}}^4 \sum_{\mathbf{f}} |\mathcal{F}_{fg}(\mathbf{k}, \mathbf{k}', \boldsymbol{\epsilon}, \boldsymbol{\epsilon}', \omega_{\mathbf{k}}, \omega_{\mathbf{k}'})|^2 \delta(E_g - E_f + \hbar\omega)$$

RIXS scattering amplitude:

$$\mathcal{F}_{fg}(\mathbf{k}, \mathbf{k}', \boldsymbol{\epsilon}, \boldsymbol{\epsilon}', \omega_{\mathbf{k}}, \omega_{\mathbf{k}'}) = \sum_{n} \frac{\langle f | \mathcal{D}' | n \rangle \langle n | \mathcal{D} | g \rangle}{E_{g} + \hbar \omega_{\mathbf{k}} - E_{n} - i\Gamma_{n}}$$
Inela

Dipole transition operator
$$\mathcal{D} = \boldsymbol{\epsilon} \cdot \mathbf{D}$$
 $\langle n | \mathbf{D} | \mathbf{g} \rangle = \sum_{i=1}^{N} e^{i \mathbf{k} \cdot \mathbf{R}_{i}} \langle n | \mathbf{r} | \mathbf{g} \rangle$

nelastic

REXS scattering amplitude:

$$\mathcal{F}_{\rm gg}(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}', \omega_{\mathbf{k}}) = \sum_{n} \frac{\langle \mathbf{g} | \mathcal{D}' | n \rangle \langle n | \mathcal{D} | \mathbf{g} \rangle}{E_{\rm g} + \hbar \omega_{\mathbf{k}} - E_{n} - i\Gamma_{n}}$$
Elastic



Resonant elastic magnetic X-ray scattering

Scattering length for dipolar transitions:

Hill and McMorrow 1996

$$\begin{aligned} \mathcal{F}_{E1} &= \frac{1}{2} (\boldsymbol{\epsilon}' \cdot \boldsymbol{\epsilon}) (F_{1,1} + F_{1,-1}) - \frac{i}{2} (\boldsymbol{\epsilon}' \times \boldsymbol{\epsilon}) \cdot \hat{\mathbf{z}} (F_{1,1} - F_{1,-1}) \\ & \text{Dispersion corrections} \\ &+ (\boldsymbol{\epsilon}' \cdot \hat{\mathbf{z}}) (\boldsymbol{\epsilon} \cdot \hat{\mathbf{z}}) (F_{1,0} - \frac{1}{2} F_{1,1} + \frac{1}{2} F_{1,-1}) \end{aligned} \qquad \begin{array}{c} \mathbf{REMXS+XMCD} \\ F_{l,m} \text{ resonant amplitudes} \end{aligned}$$

REMXS+XMLD+Anisotropic Tensor Scattering (ATS)

Expressed in orthogonal photon polarization states:

$$\mathcal{F}_{E1} = F_{E1}^{0} \begin{pmatrix} 1 & 0 \\ 0 & \cos 2\theta \end{pmatrix} - iF_{E1}^{1} \begin{pmatrix} 0 & z_{1}\cos\theta + z_{3}\sin\theta \\ -z_{1}\cos\theta + z_{3}\sin\theta & -z_{2}\sin 2\theta \end{pmatrix}$$
$$+ F_{E1}^{2} \begin{pmatrix} z_{2}^{2} & -z_{2}(z_{1}\cos\theta - z_{3}\sin\theta) \\ z_{2}(z_{1}\cos\theta + z_{3}\sin\theta) & -\cos^{2}\theta(z_{1}^{2}\tan^{2}\theta + z_{3}^{2}) \end{pmatrix}$$

M. Altarelli: Resonant X-ray Scattering: A Theoretical Introduction, Lect. Notes Phys. 697, 201-242 (2006) www.springerlink.com © Springer-Verlag Berlin Heidelberg 2006

What controls the REXMS scattering length?

The scattering length depends on terms of the form

$$\langle n | \mathbf{r} | g \rangle \propto \int_{0}^{\infty} r^{2} dr \underbrace{R_{nl}(r)}_{electrons} r \underbrace{R_{n'l'}(r)}_{valence}$$

Large resonance are therefore expected when there is a large overalp between the core and valence electron radial wavefunctions.

Transition Metals 3d	L edges, 2p->3d c. 1 keV	Strong	
Transition Metals 5d	L edges, 2p->5d c. 10 keV	medium	10 ⁵
Rare-Earths 4f,5d	L edges, 2p->5d c. 10 keV	Weak	K-edge L-edges M-edges
Rare-Earths 4f,5d	M edges, 3d->4f c. 1 keV	Strong	10 ² soft
Actinides 5f	M edges, 3d->5f c. 3 keV	Strong	Atomic Number Z

Giant resonances at the rare-earth $M_{4,5}$ edges (3p->4f)

J.M. Soriano, PhD thesis, University of Amsterdam



Experimental considerations



High flux beamlineTunable photon energy, 1-15 keV

•Well defined and variable incident polarization

- Versatile diffractometer
- Azimuthal degree of freedom
- Polarization analysis



Resonant X-ray Studies of 5d Oxides



5d Oxides

- Strong Spin-Orbit Coupling Limit
- Spatially extended d orbitals
- Significant correlation energy
- Mott-Hubbard physics in strong SOC limit
 - Novel MITs
 - Wavefunctions formed from entanglement of S and L
 - Interaction Hamiltonian takes on unique character (bond directional "Kitaev")
 - Predictions for novel quantum groundstates and excitations

Overview:

- Correlated electron systems in the strong spin-orbit coupling limit
- Perovskite iridates and the realisation of the J_{eff}=1/2 state
- Evidence for Kitaev physics in honeycomb iridates
- Slater metal-insulator transitions (MITs) in osmates

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