X-Ray and Neutron Science
7th ESRF/ILL International
Student Summer Programme
5th September – 1st October 2021

Principles of synchrotron radiation

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ESRF | The European Synchrotron

OUTLINE

Properties of radiation

Where x-rays come from

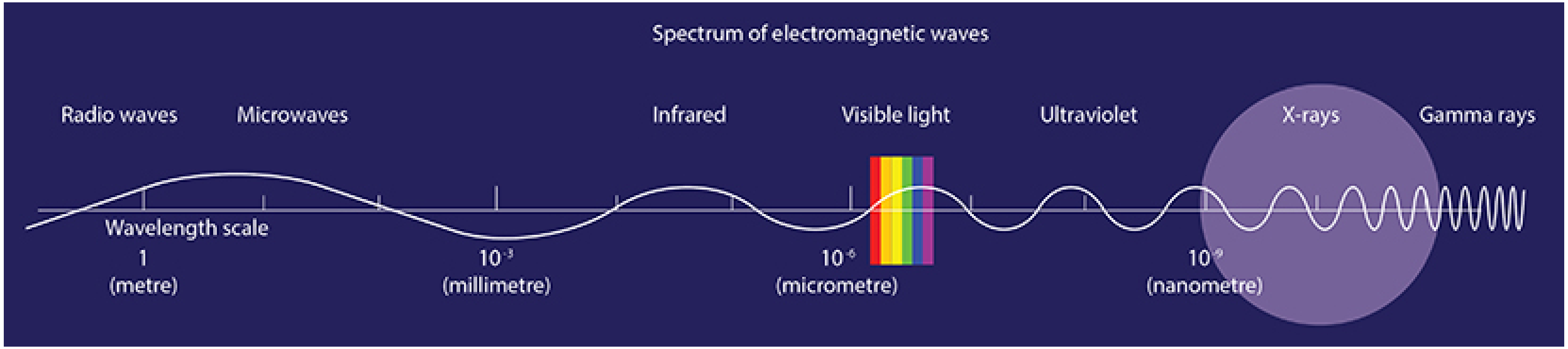
- review of relativity
- bending magnet radiation
- undulator radiation

Particle accelerators:

- how to store electron beams
- what is the distribution of the electron beam

ESRF upgrade: EBS

Spectrum of electro-magnetic radiation



$$\mathbf{f = c / \lambda}$$
$$\mathbf{E_\gamma = h f}$$

c = 299792458 m/s
h = 4.135667517 x 10^{-15} eV s

λ is the photon wavelength
f is the photon frequency
$\mathbf{E_\gamma}$ is the photon energy
c is the speed of light
h is the Planck constant

$\mathbf{E_\gamma}$ for X-rays is about 1-100 keV

PROPERTIES OF RADIATION

Flux of radiation:
number of photons per second

Brightness of radiation:
flux at a specific wavelength divided by source size and divergence

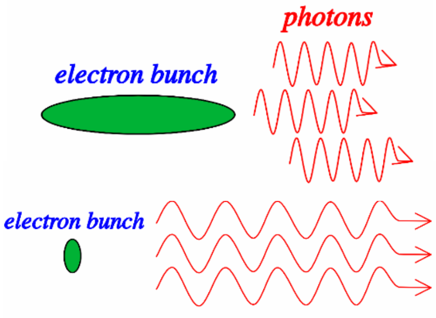
Low brightness: low photon density on sample

High brightness: high photon density on sample

Coherence of radiation:
are the photons produced in phase or not?

$\sigma_z > \lambda$

$\sigma_z < \lambda$



σ_z is the electron bunch length

Polarization of radiation:
directionality of radiation field
linear, circular partial/full polarization

Synchrotron light sources give some control over all these properties

WHERE X-RAYS COME FROM

Accelerating charged particles emit electro-magnetic radiation

Maxwell's equations:

$\nabla \cdot \mathbf{E} = 4\pi\rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$

Non-relativistic Larmor formula:

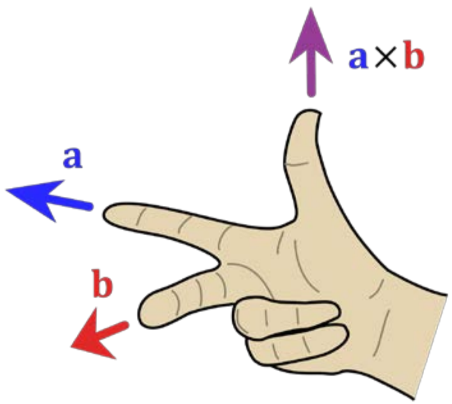
$$P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$$

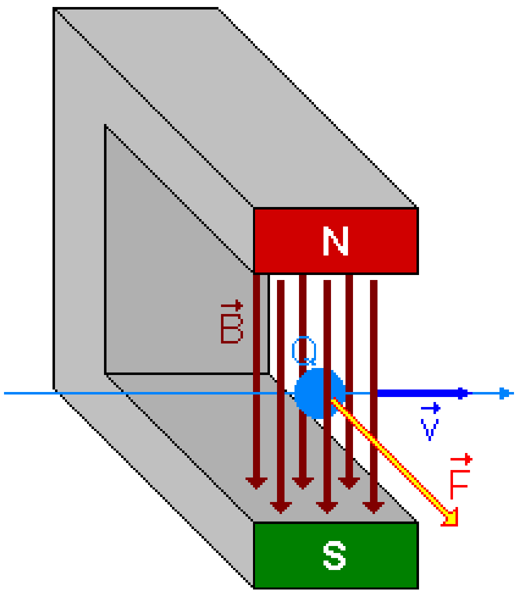
P is the power radiated
q is the electric charge
a is the acceleration

LORENTZ FORCE

We can move charged particles using the Lorentz force

$$\vec{F} = q \cdot \left(\vec{E} + \vec{v} \times \vec{B} \right)$$





The force, and so the acceleration, is perpendicular to the particle velocity. Particles travel on an arc of circumference inside the dipole magnets.

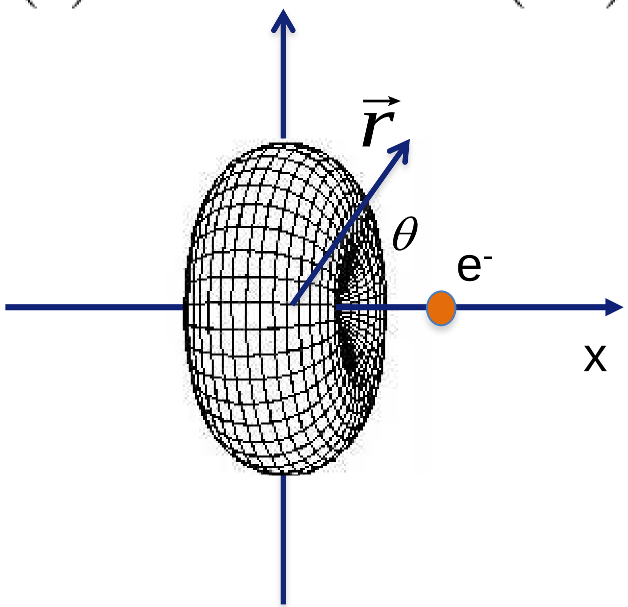
SINGLE PARTICLE IN A HARMONIC OSCILLATOR

Charge oscillating along x

$$x(t) = A\cos(\omega t)$$

$$v(t) = -A\omega\ \sin(\omega t)$$

$$a(t) = -A\omega^2\ \cos(\omega t)$$



It emits radiation in all directions

$$\langle S \rangle \propto \frac{\sin^2\theta}{r^2}\hat{r}$$

S is the Poynting vector (energy per unit of time and surface)

Frequency of radiation is $$f = \frac{\omega}{2\pi}$$

Wavelength of radiation is $$\lambda = \frac{c}{f}$$

BRIEF REVIEW OF RELATIVITY

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

c = 299792458 m/s
is the speed of light

Total energy $E = \gamma mc^2$

Kinetic energy $E_k = (\gamma - 1)mc^2$

$$E_k \rightarrow \frac{1}{2}mv^2, \quad for\ \beta << 1$$

For an electron with energy E:

$$\gamma = \frac{E}{mc^2} = \frac{E}{510998.9461\,\mathrm{eV}}$$

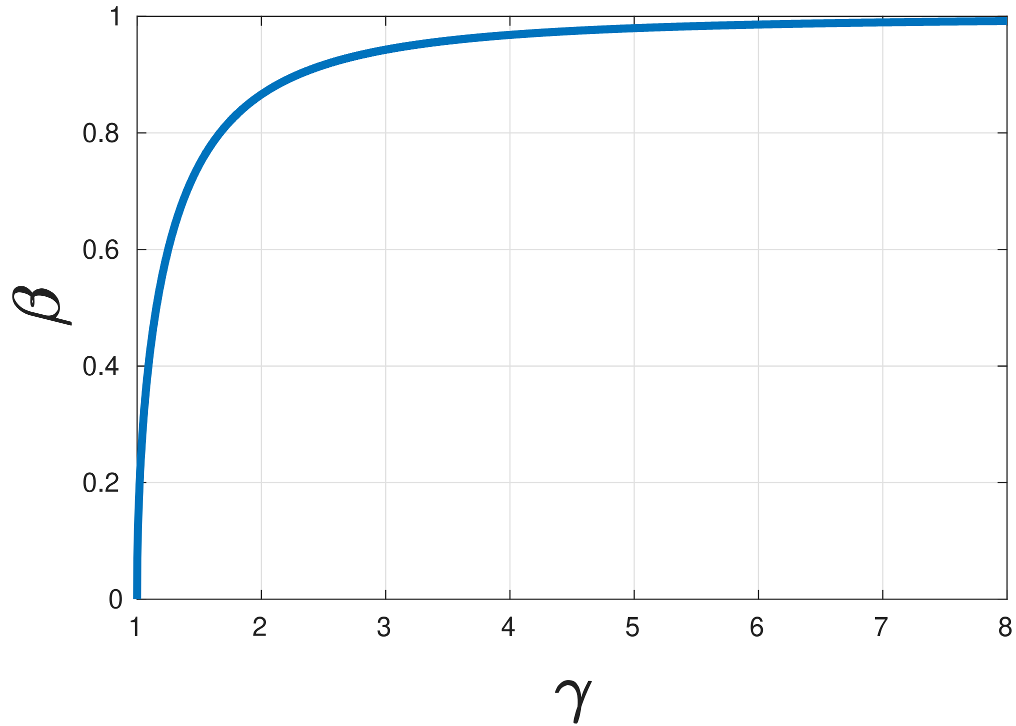
$$m_{e^-} = 510998.9461\,\frac{\mathrm{eV}}{c^2}$$

β is always less than 1

Velocity is almost c already for γ~5

At ESRF γ=11800

Increasing the electron energy from few MeV to 6 GeV does not change much the velocity.



However, there are two effects as gamma gets large:

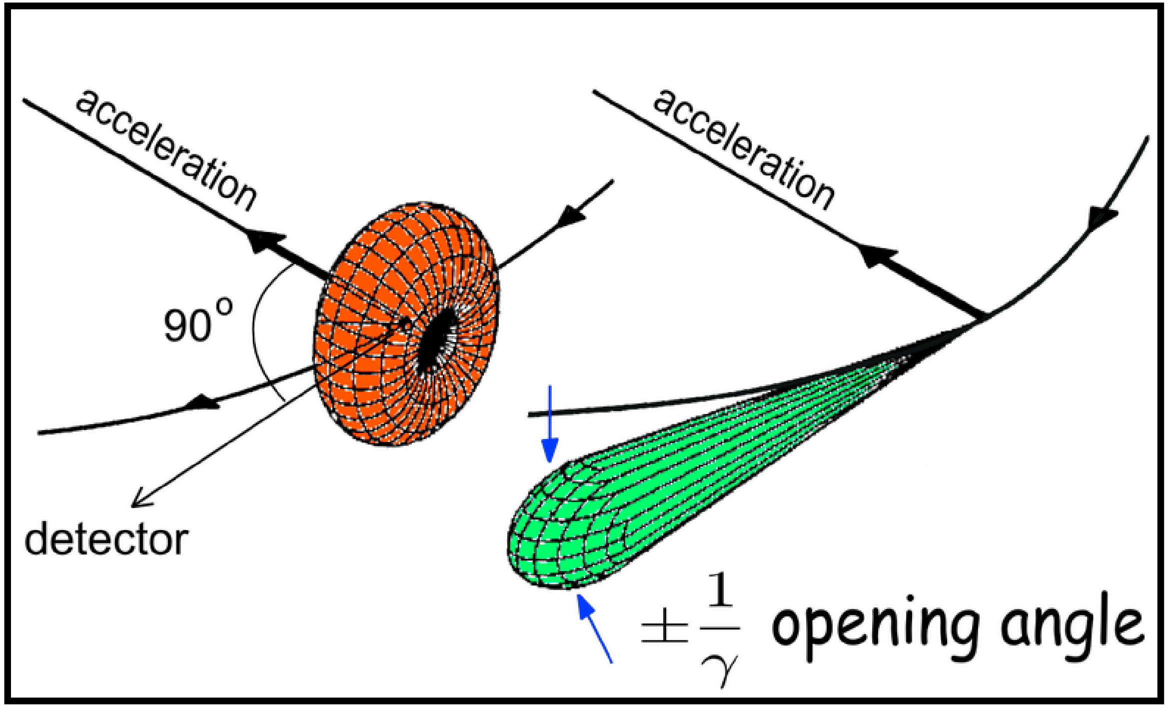
- length contraction
- time dilation

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = \frac{L_0}{\gamma}$$

$$T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}} = T_0\gamma$$

RADIATION EMITTED IN RELATIVISTIC CASE

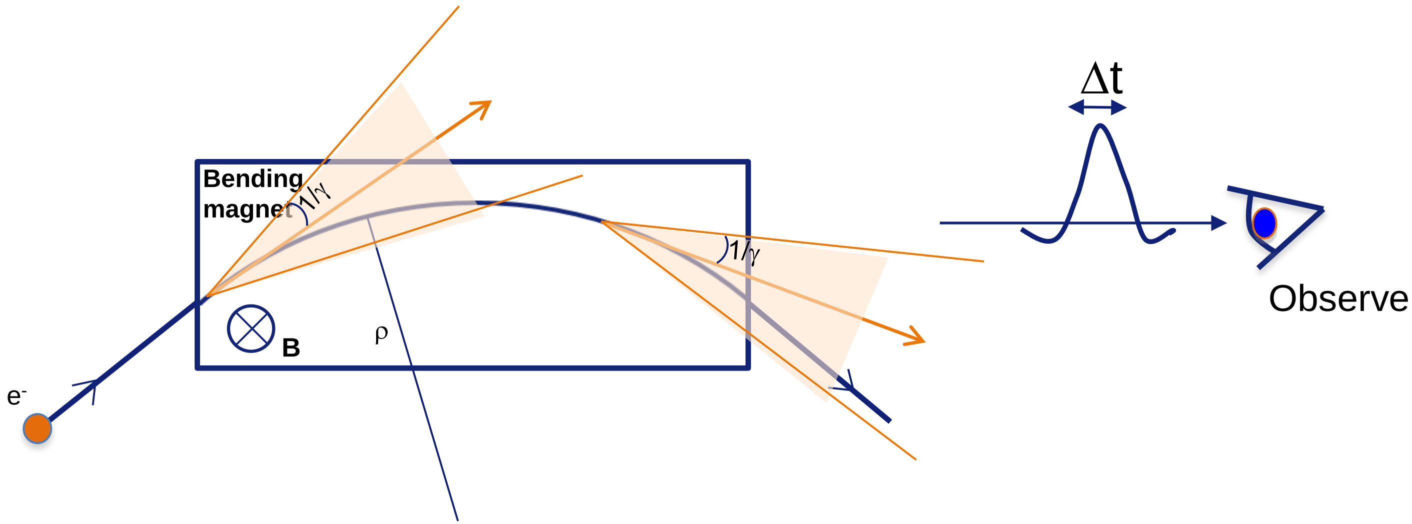
If the charged particle is moving very fast, the radiation is emitted in a cone, with aperture $1/\gamma$



BENDING MAGNET RADIATION: TYPICAL ENERGY

What is the **typical energy** of the radiation from a bending magnet?

First, let's see for how long an observer would see the radiation produced by an ultra-relativistic electron in a bending magnet.



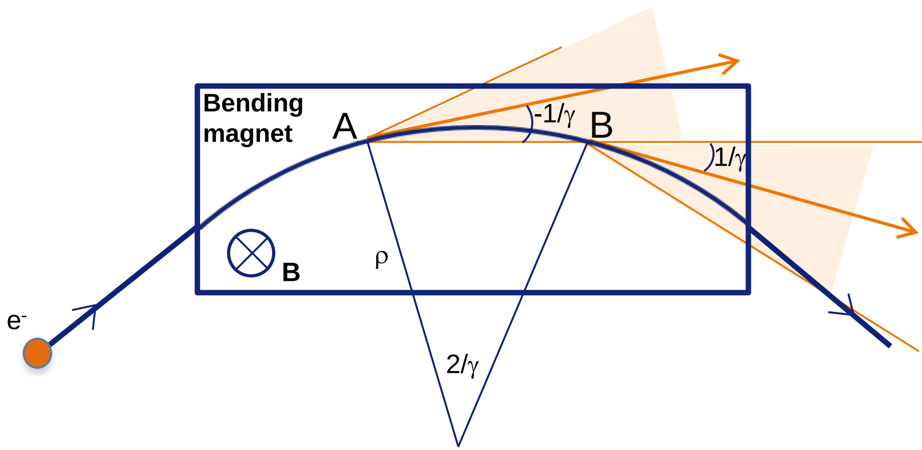
$f_{typ} = 1/\Delta t$

From the length Δt of the pulse we can estimate a typical energy E_{typ}:

$E_{typ} = h\, f_{typ}$

BENDING MAGNET RADIATION: TYPICAL ENERGY

How long does the observer see the radiation produced by an ultra-relativistic electron in a bending magnet?



The observer will first see the radiation produced in the point A, with an angle -1/γ. The last photon seen will be the one produced in B with an angle +1/γ. The observer will therefore see radiation for an amount of time equal to the difference between the time needed by the electron to go from A to B and the time needed by the photon to go from A to B.

$$\Delta t = t_{ABe^-} - t_{AB\gamma} = \frac{\overset{\frown}{AB}}{\beta c} - \frac{\overline{AB}}{c} = \frac{2\rho}{\gamma\beta c} - \frac{2\rho \sin(1/\gamma)}{c}$$

$$\sin(\theta) \simeq \theta - \frac{\theta^3}{6}, \quad for\, \theta << 1 \qquad \gamma >> 1, \;\; 1-\beta \simeq \frac{1}{2\gamma^2}$$

$$\Delta t \simeq \frac{2\rho}{\beta\gamma c} - \frac{2\rho}{c}\left(\frac{1}{\gamma} - \frac{1}{6\gamma^3}\right) = \frac{2\rho}{\beta\gamma c}\left(1 - \beta + \frac{\beta}{6\gamma^2}\right) = \frac{4}{3}\frac{\rho}{c\gamma^3}$$

$$\omega_{typ} = \frac{2\pi}{\Delta t} = \frac{3\pi c\gamma^3}{2\rho}$$

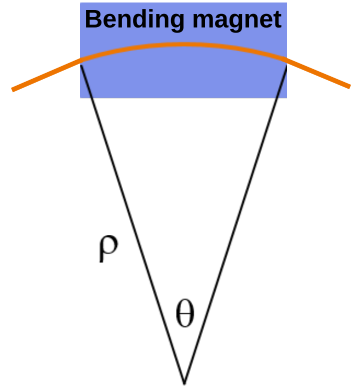
$$E_{typ} = \hbar\omega_{typ} = \frac{3\pi}{2}\frac{\hbar c}{\rho}\gamma^3$$

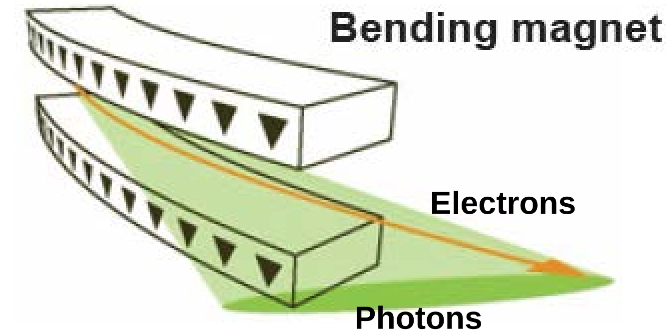
BENDING MAGNET RADIATION

Power emitted by an ultra-relativistic particle in a bending magnet

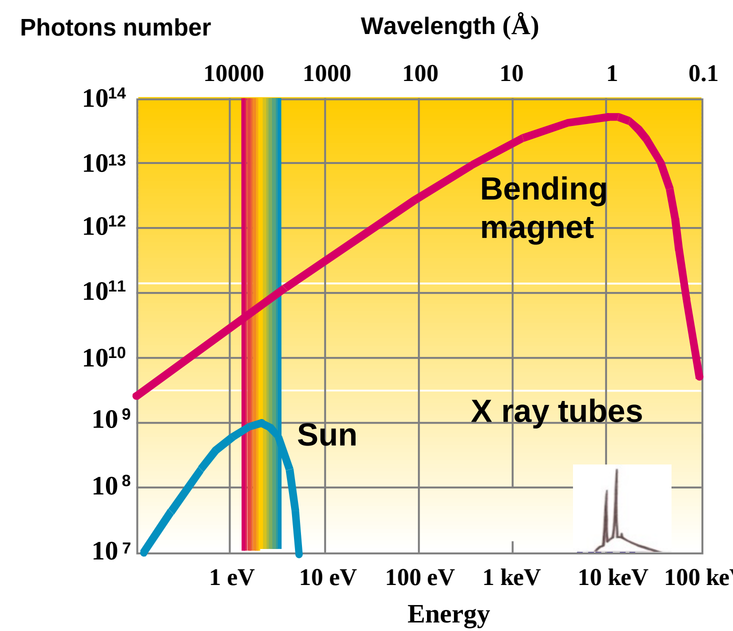
$$P = \frac{1}{6}\frac{e^2 c}{\pi \varepsilon_0}\frac{1}{\rho^2}\left(\frac{E}{mc^2}\right)^4$$

ρ is the radius of curvature





Radiation spectrum



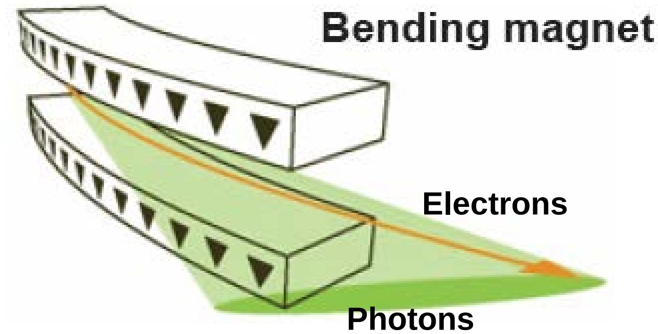
Critical photon energy: half of the power is radiated at larger energy and half at lower

$$\varepsilon_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$

BENDING MAGNET RADIATION

Parameters for the EBS bending magnet beamlines

E = 6 GeV
γ = 11800
ρ = 23.3 m
ε_c = 21 keV

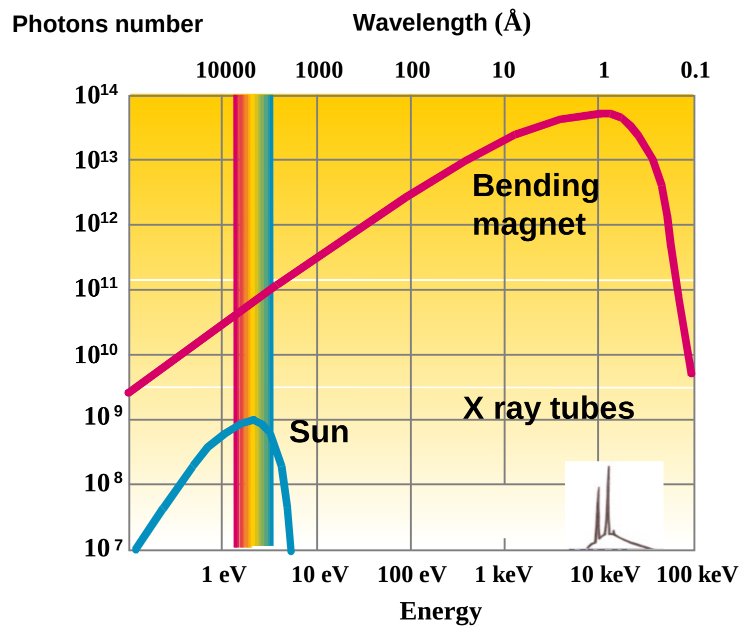


If we put protons instead of electrons?

m_p=938 MeV/c2
m_e=0.511 MeV/c2

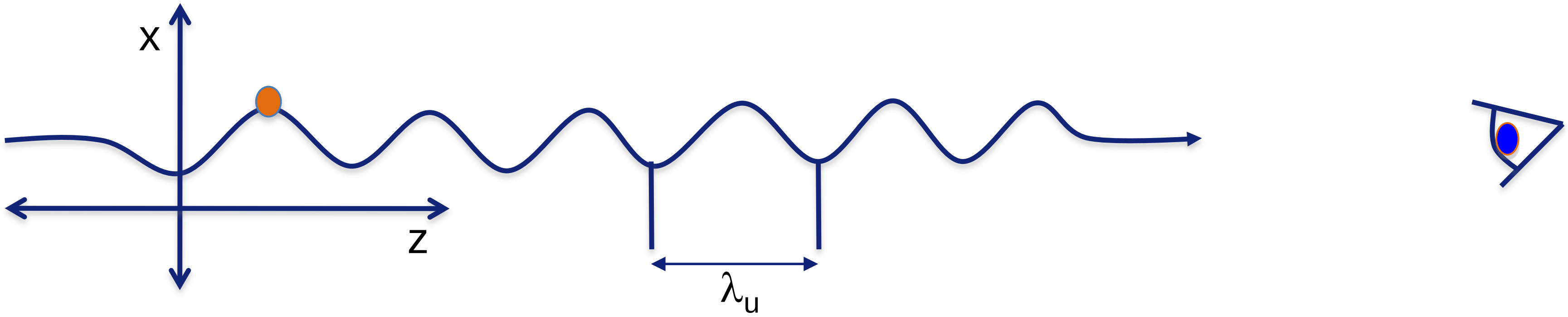
Radiated power would be 10^{13} times smaller
ε_c = 3.3 μeV

Radiation spectrum



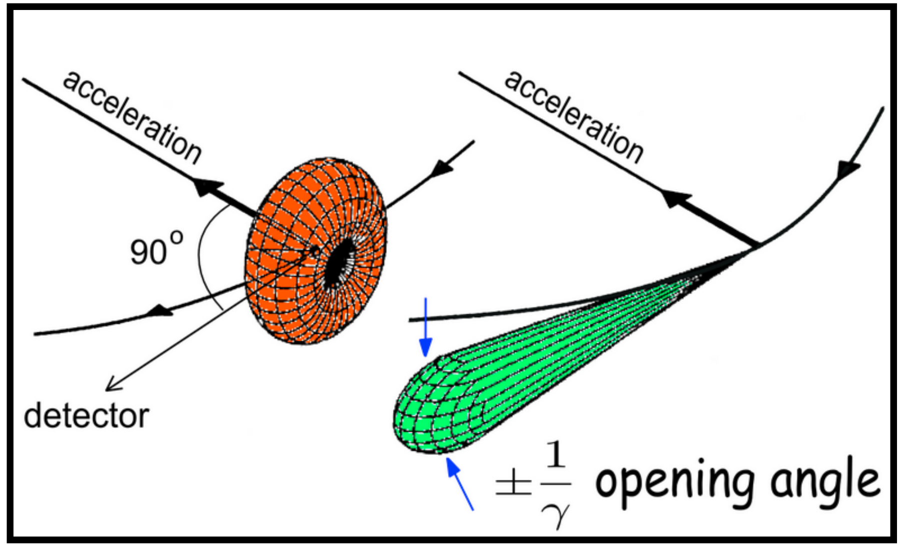
UNDULATOR RADIATION

Consider now a single charge oscillating on x and also moving along z direction



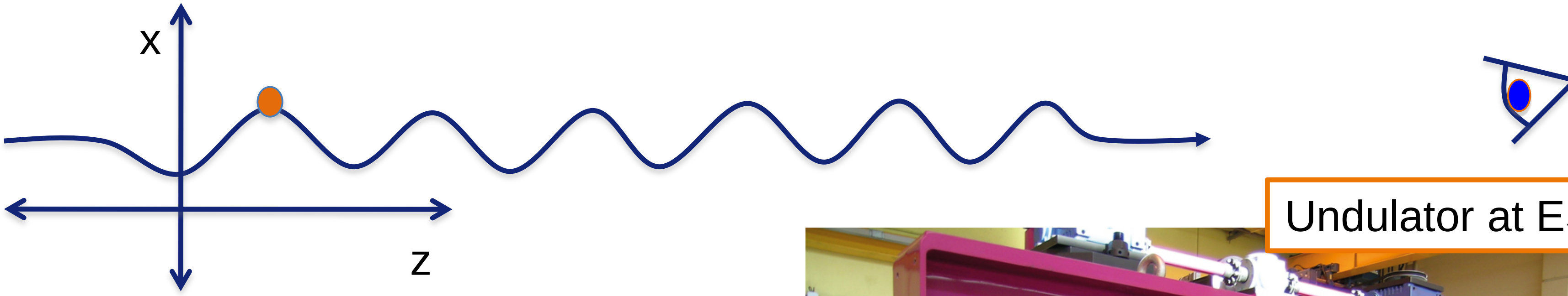
1. The wavelength is shifted by Doppler effect and, in case of relativistic speed, there is a time dilation effect. $$\lambda = \frac{\lambda_u}{\gamma^2}$$

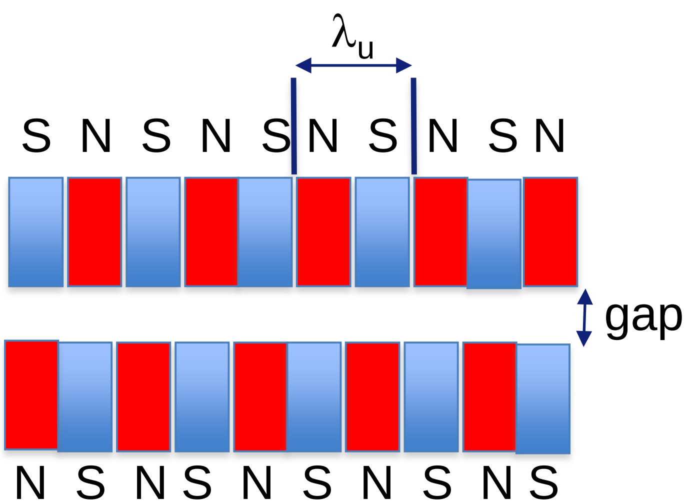
2. Pattern of radiation gets distorted from motion. For high energy gets bent into cone of angle $1/\gamma$.



UNDULATOR RADIATION

We can build magnets to have this kind of trajectory inside: undulators and wigglers





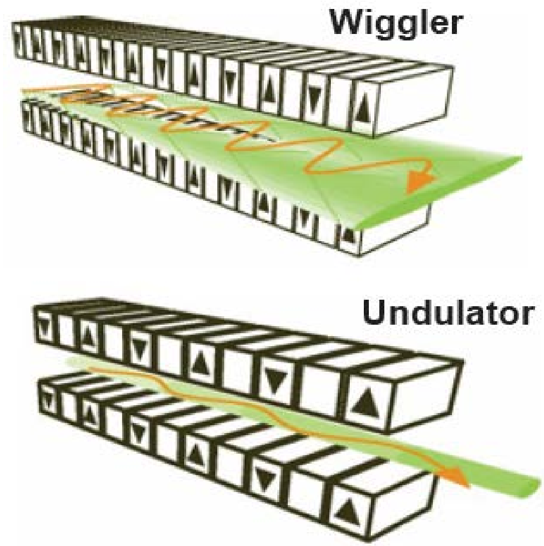
Undulator at ESRF

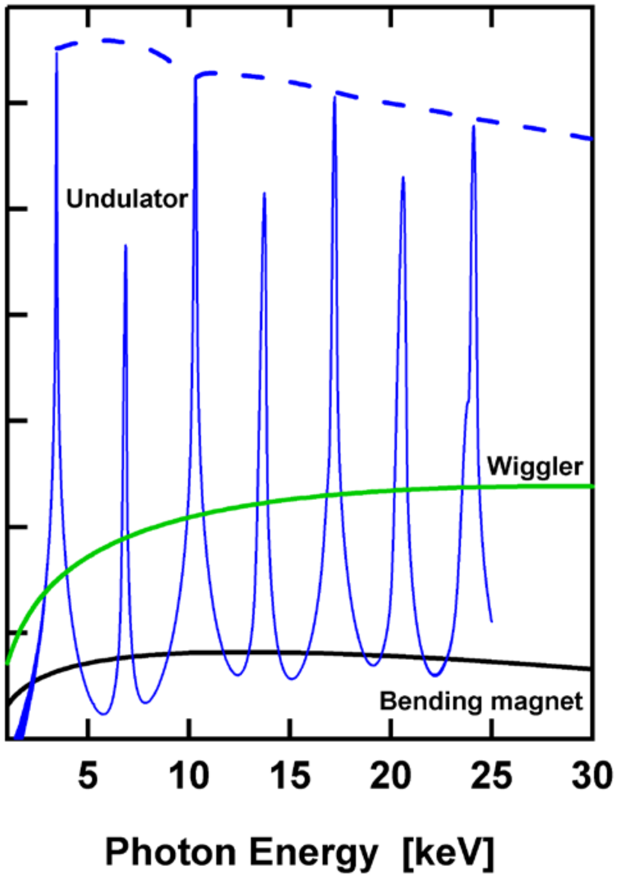
UNDULATOR RADIATION

Undulator parameter K

$$K = \frac{eB_0\lambda_u}{2\pi mc}$$

If K is large (K>>1), the device is no longer called undulator, but wiggler.
K can be changed by varying the gap of the undulator.

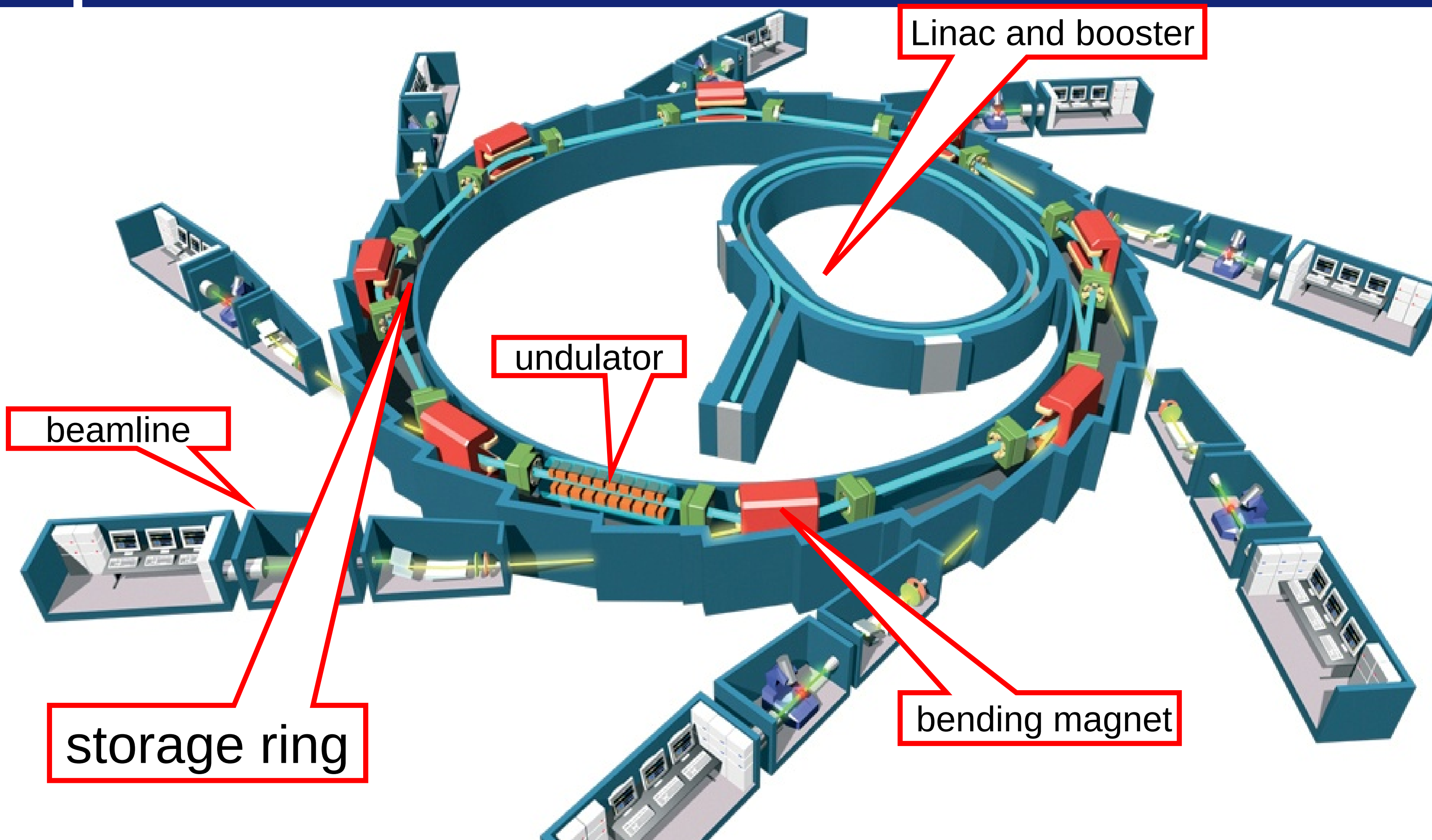




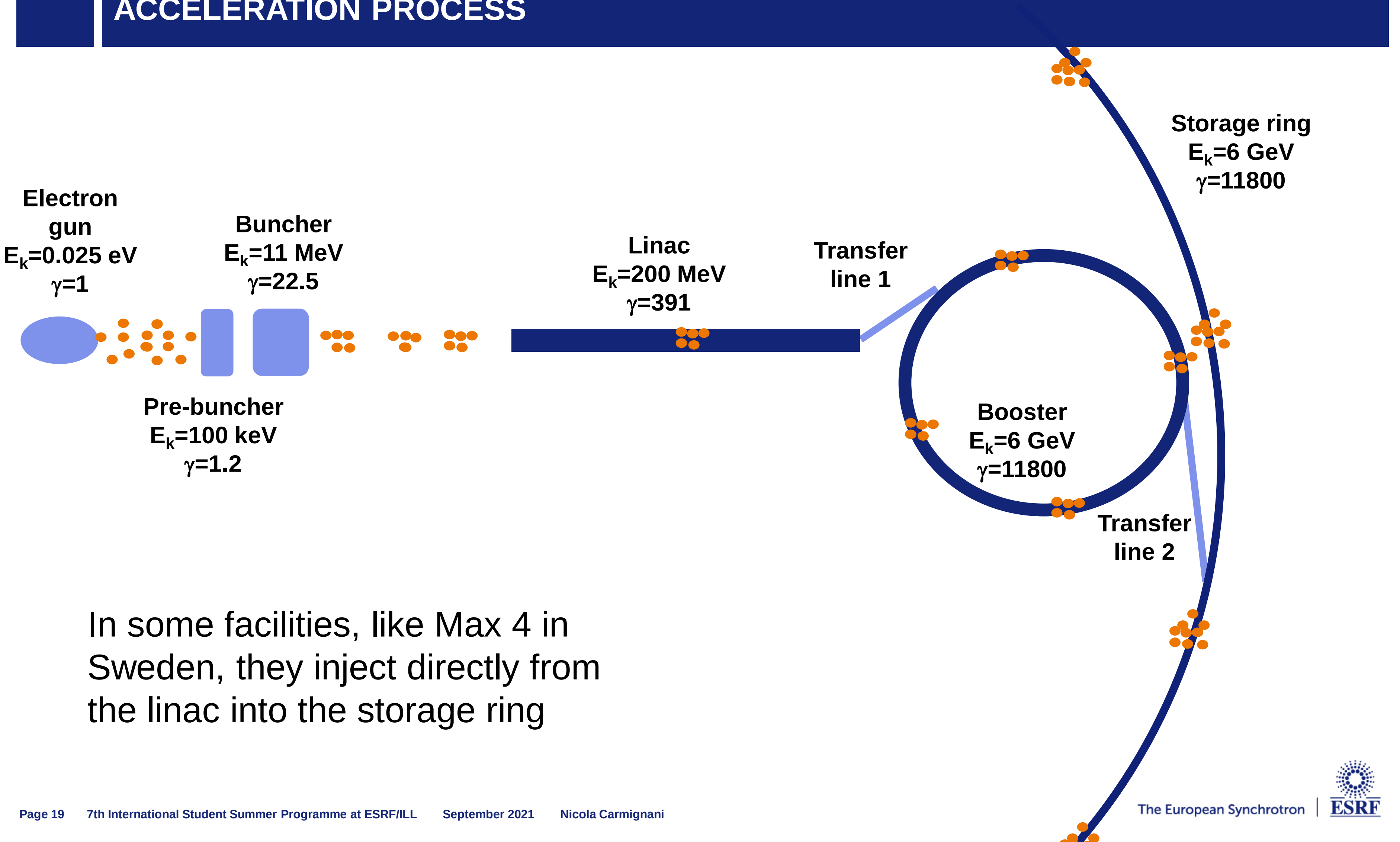
Fundamental undulator wavelength λ_1

$$\lambda_1(\theta) \approx \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + (\gamma\theta)^2\right)$$

A TYPICAL SYNCHROTRON LIGHT SOURCE USER FACILITY



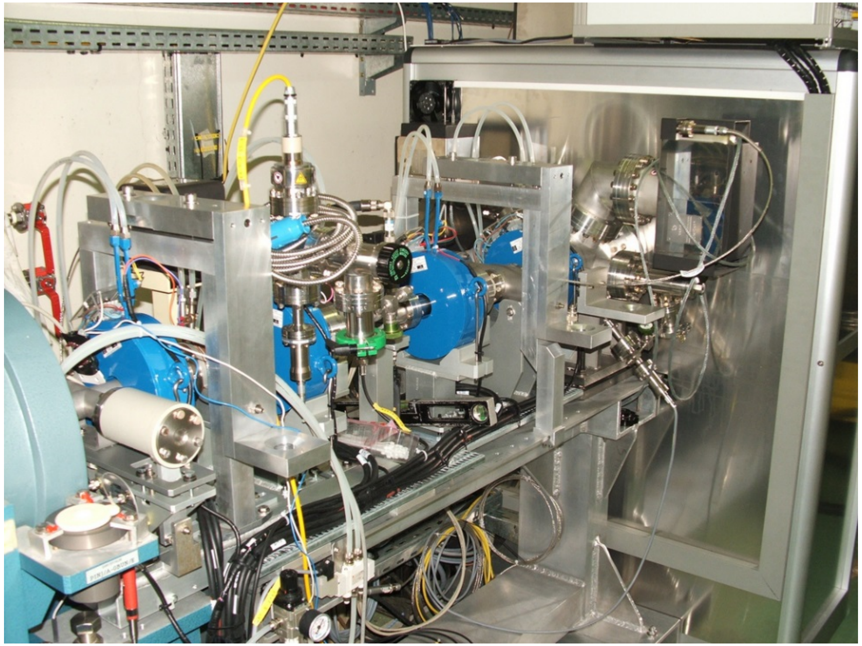
ACCELERATION PROCESS



In some facilities, like Max 4 in Sweden, they inject directly from the linac into the storage ring

INJECTORS

Electron gun, pre-buncher



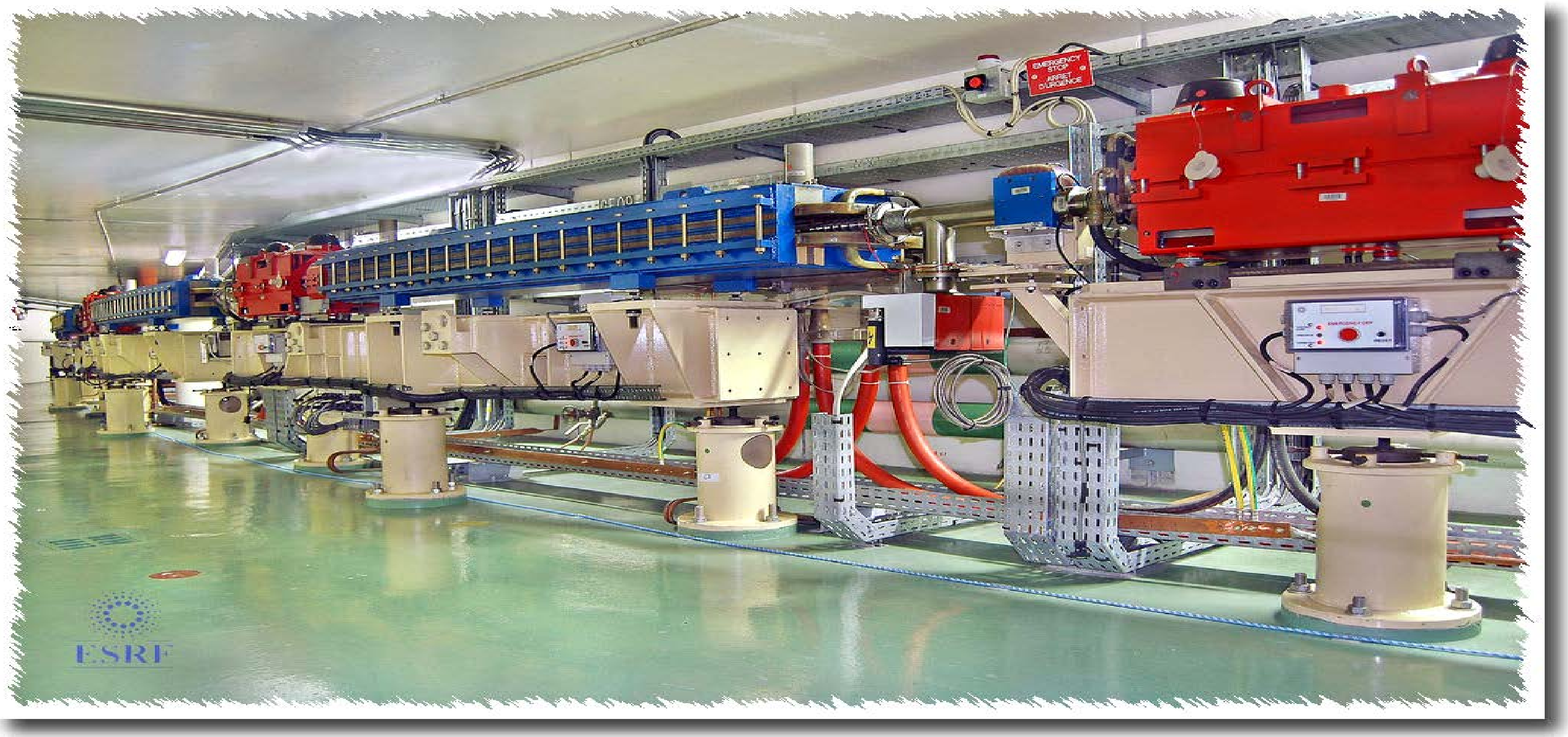
Linac

INJECTORS: TRANSFER LINES

Linac to booster transfer line (TL1)

INJECTORS

Booster



The European Synchrotron | ESRF

HOW DOES A STORAGE RING WORK: MAGNETS

128 at ESRF

Bending magnets (dipoles) have uniform constant vertical magnetic field $B_y = B_0$

They bend the beam and they define the circular trajectory.

At ESRF, each of the 5 pieces have a different magnetic field.

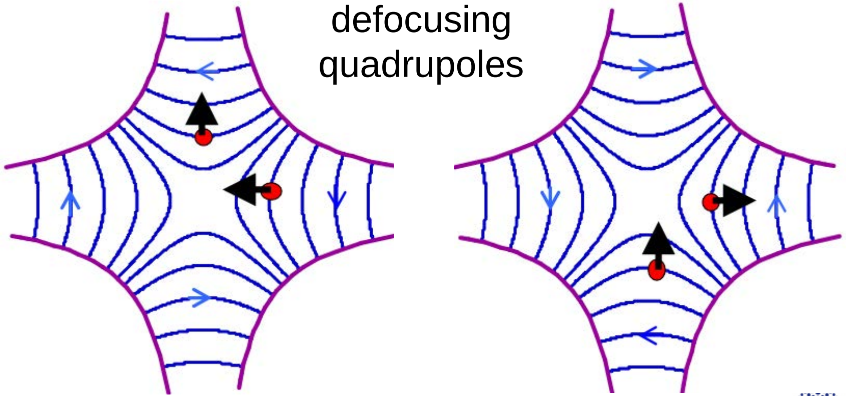
HOW DOES A STORAGE RING WORK: MAGNETS

512 at ESRF

Quadrupoles:
magnetic field is linear with the distance from the center
$B_y = K_1 x$
They are used to focus the beam.

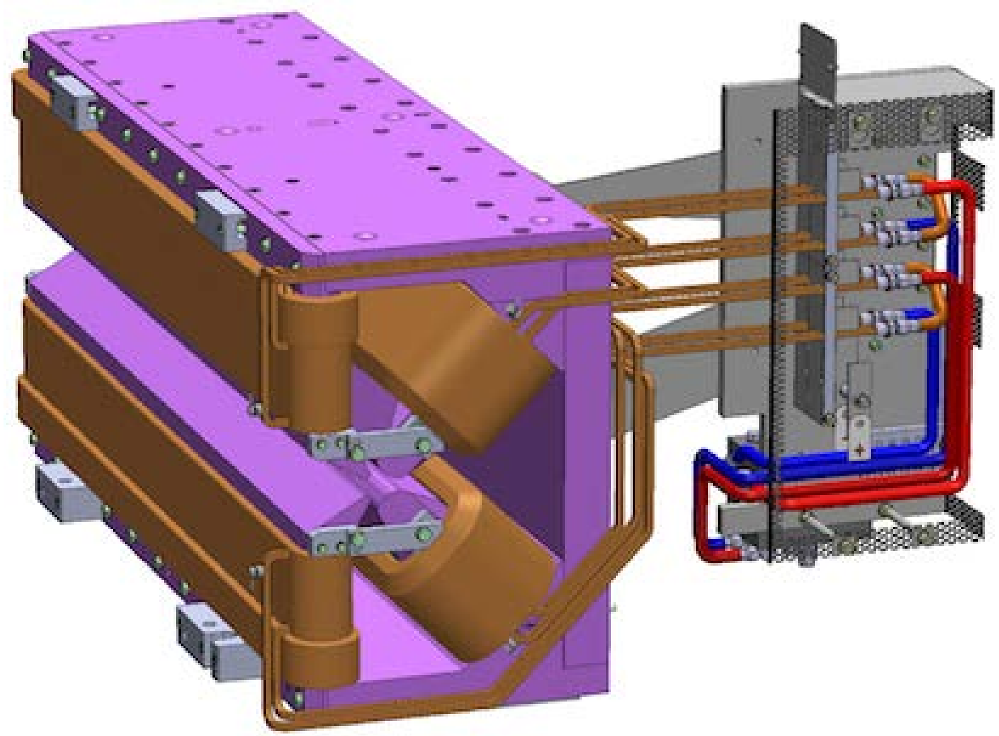
There are focusing or defocusing quadrupoles: if they focus horizontally, they defocus in vertical and vice versa (like astigmatic lenses in optics).

Focusing or defocusing quadrupoles



HOW DOES A STORAGE RING WORK: MAGNETS

96 at ESRF



Dipole-Quadrupoles (DQ): magnetic field is the one of a dipole plus the one of a quadrupole:

$B_y=B_0+K_1x$

They are used to bend and focus the beam.

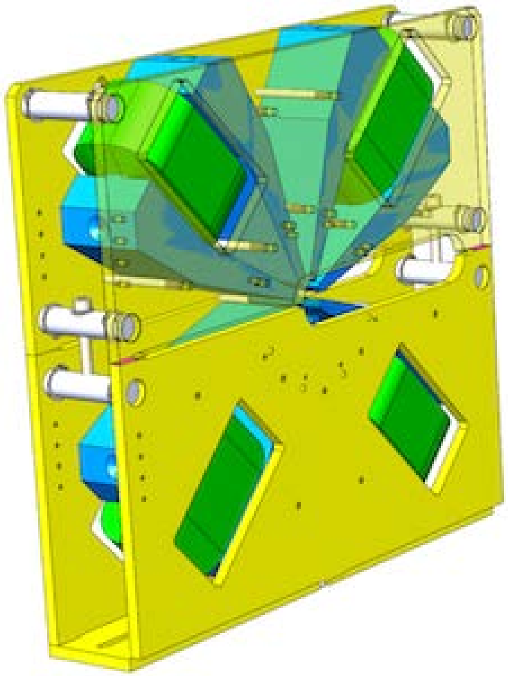
HOW DOES A STORAGE RING WORK: MAGNETS

192 at ESRF

Sextupoles: magnetic field is quadratic with the distance from the center
$B_y = K_2 x^2$
They are used to correct chromatic effects.

If correctly placed, they focus more the particles with higher energy and less the particles with lower energy. They correct chromatic effects.

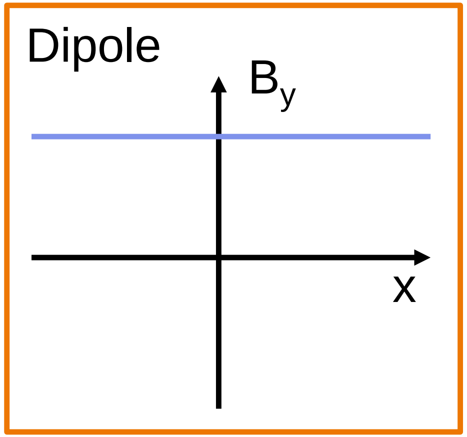
HOW DOES A STORAGE RING WORK: MAGNETS

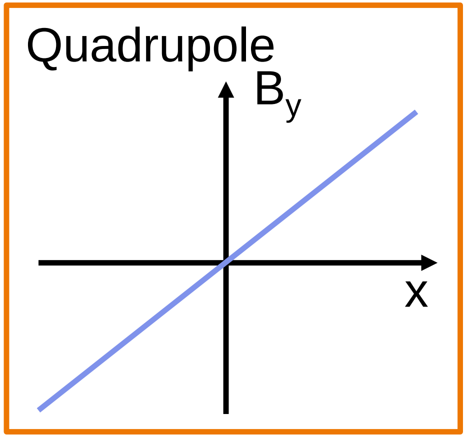


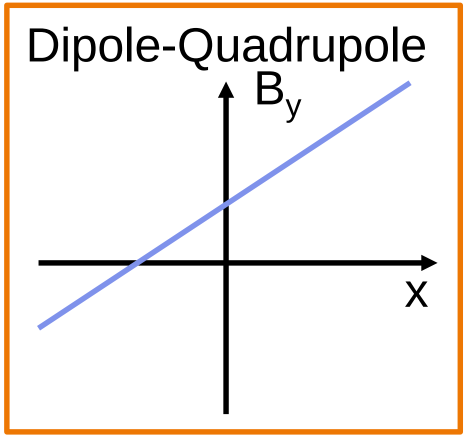
Correctors:
they can provide horizontal dipole field, vertical dipole field, quadrupolar field.

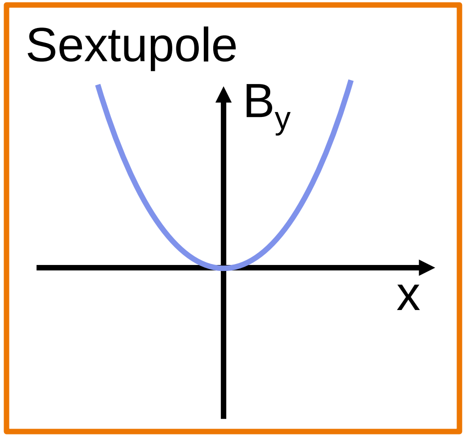
Octupoles:
the field is cubic with distance:
$B_y = K_3 x^3$

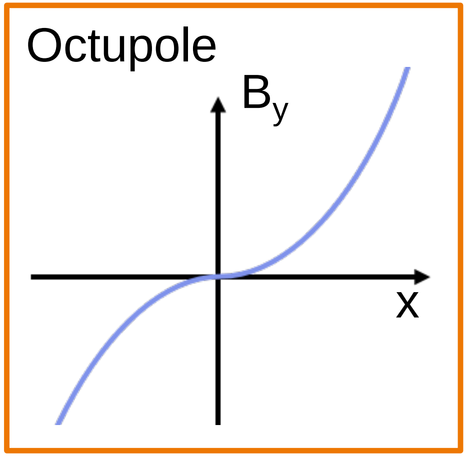
HOW DOES A STORAGE RING WORK: MAGNETS



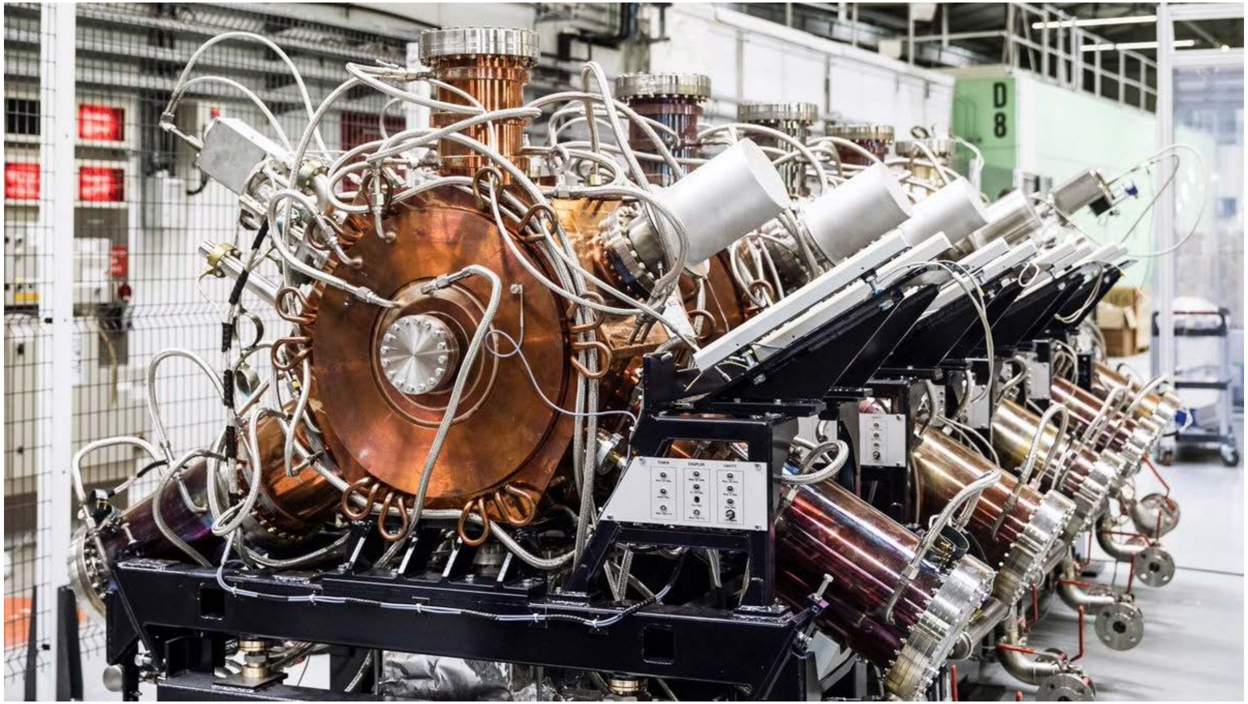








HOW DOES A STORAGE RING WORK: ACCELERATING RF CAVITIES



Radio Frequency cavities
They give to the beam the energy lost in synchrotron radiation and they provide longitudinal focusing.

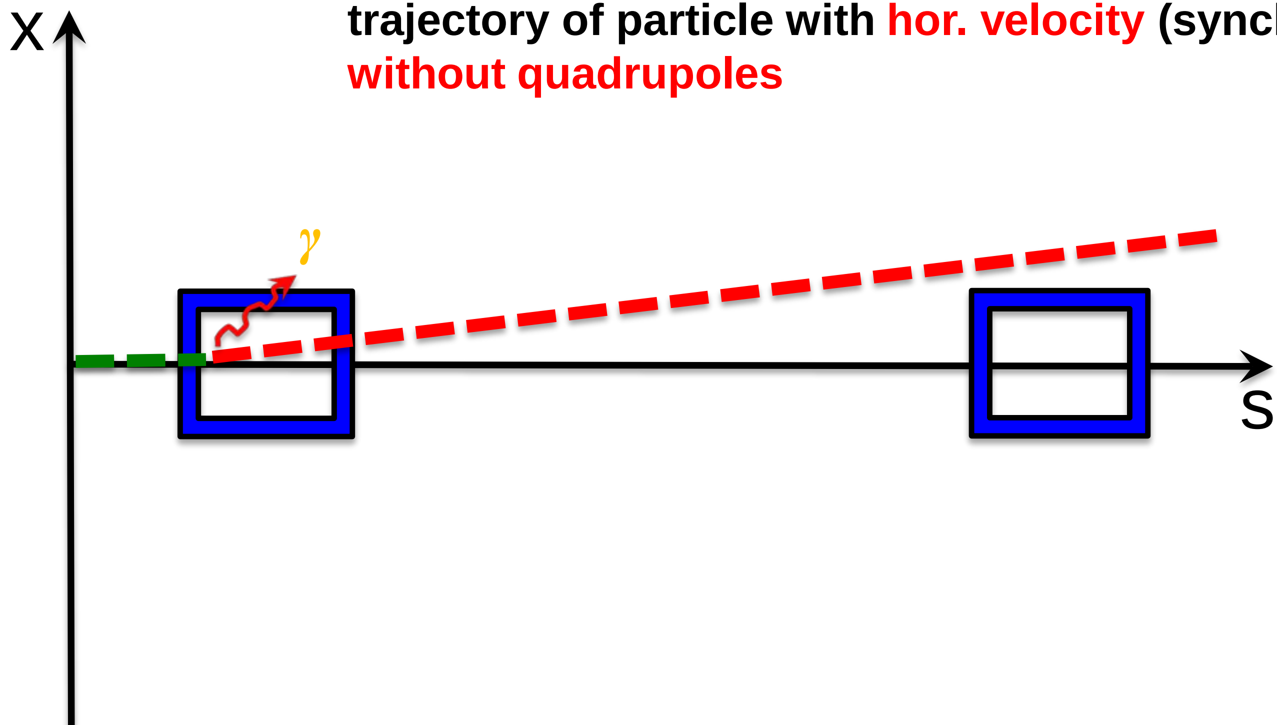
They produce an oscillating longitudinal electric field
$V(t)=V_{RF}\sin(h\omega_0 t)$
$\omega_0=2\pi\ f_{rev}$

At ESRF:
f_{rev} = 355 kHz
h=992
$f_{RF}=hf_{rev}$= 352 MHz
V_{RF}=6.5 MV

Most of the ESRF energy use (around 1.5 MW of power) is in these cavities

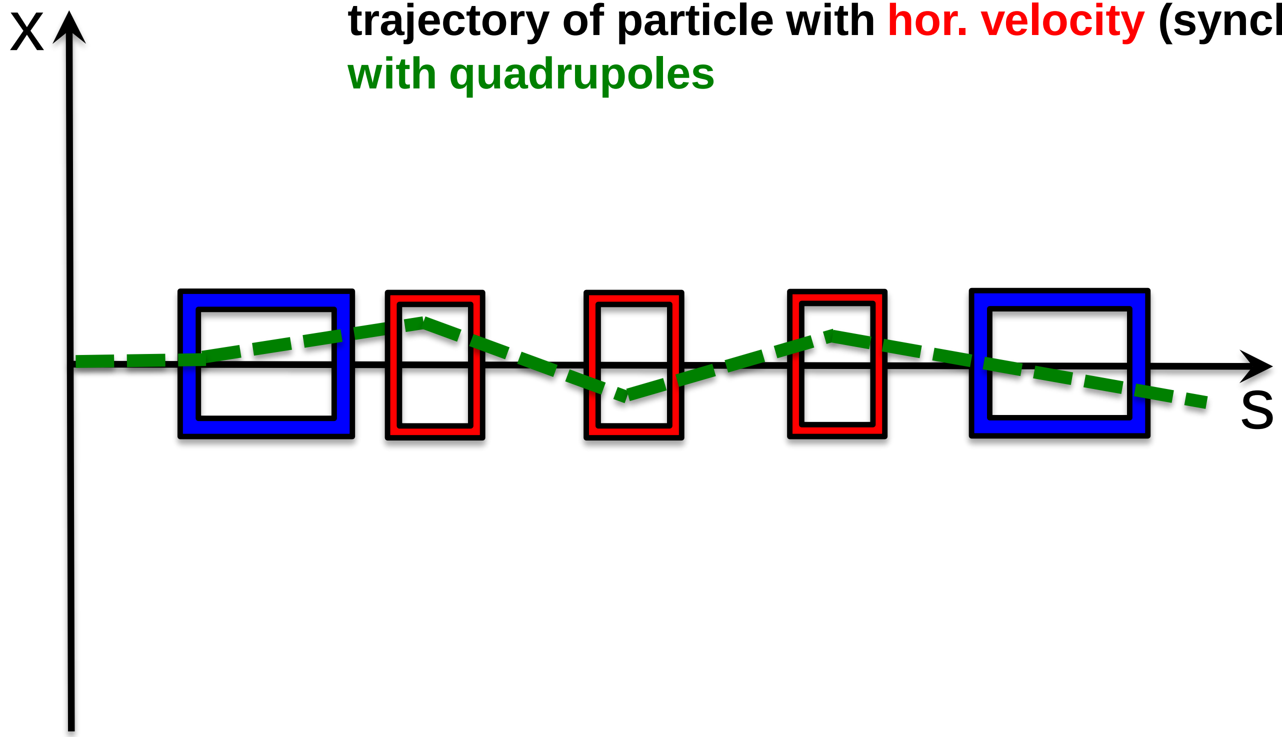
HOW DOES A STORAGE RING WORK: DIPOLES

trajectory of particle with **hor. velocity** (synch. rad.):
without quadrupoles

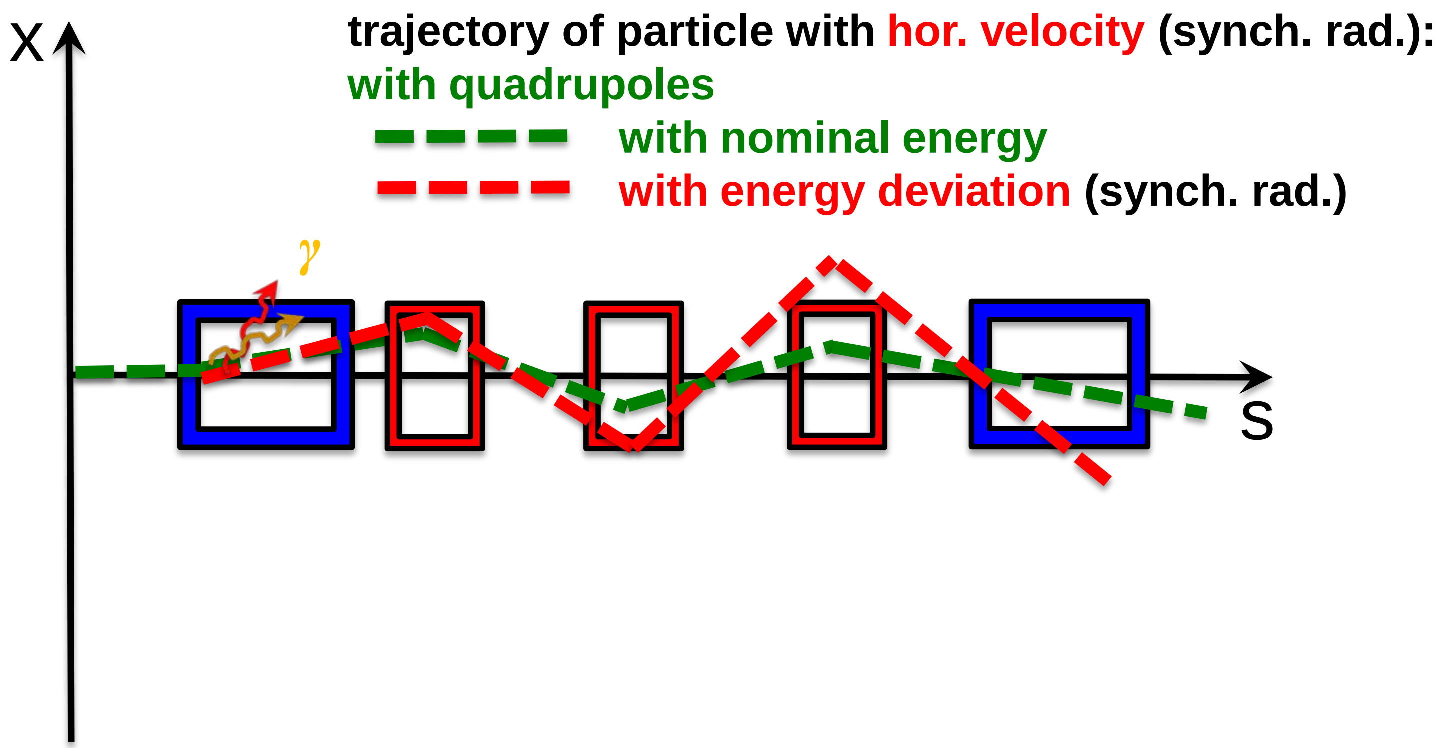


HOW DOES A STORAGE RING WORK: DIPOLES, QUADRUPOLES

trajectory of particle with hor. velocity (synch. rad.):
with quadrupoles

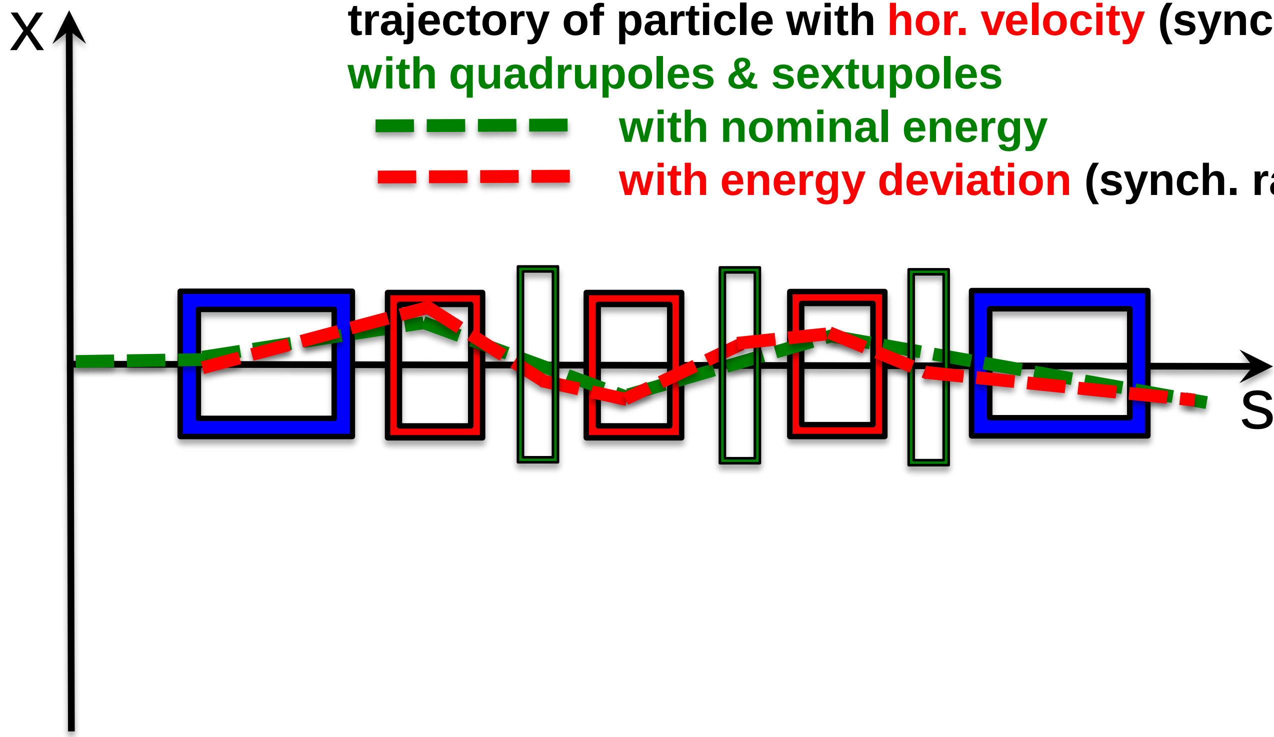


HOW DOES A STORAGE RING WORK: DIPOLES, QUADRUPOLES

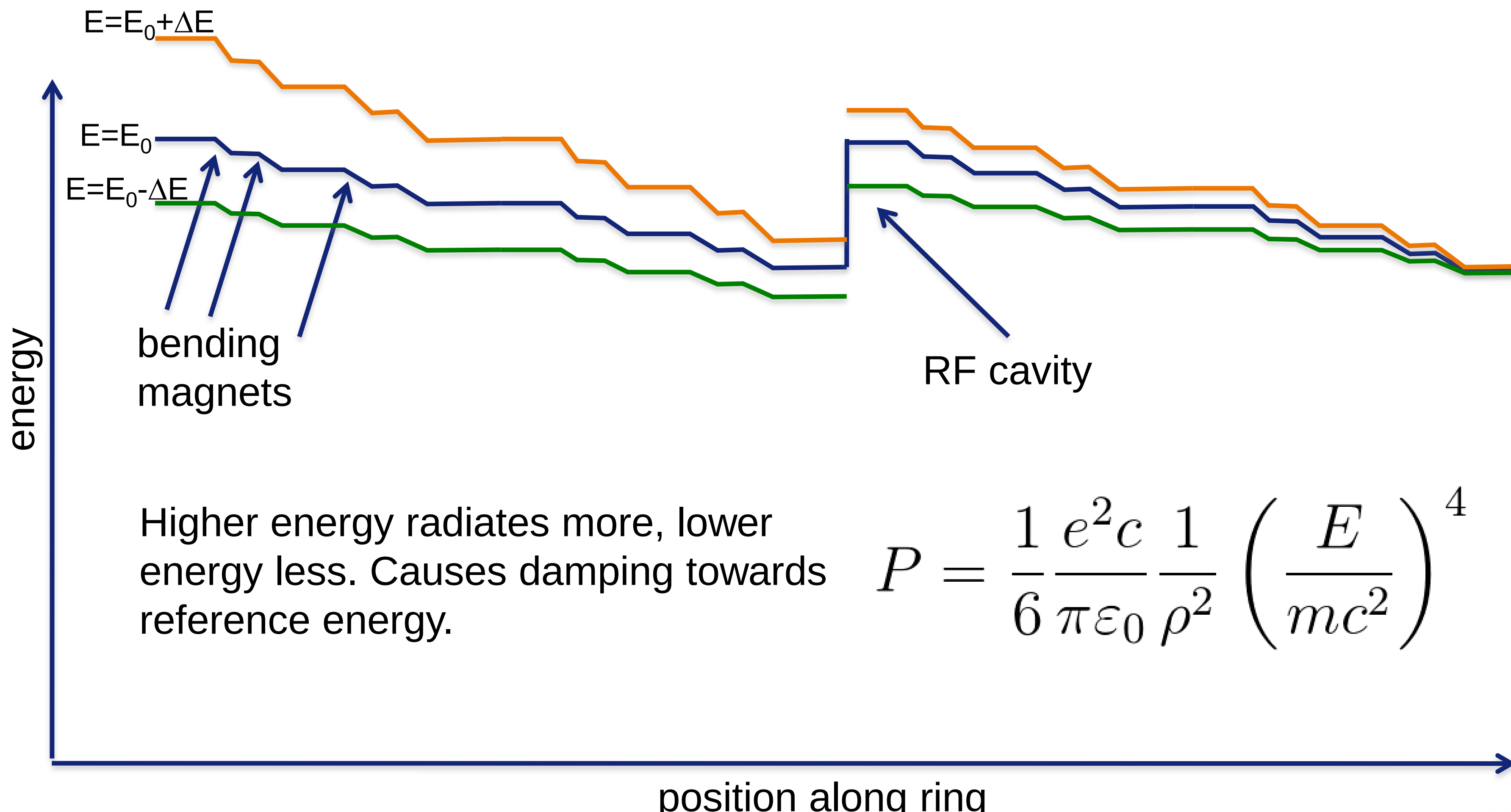


HOW DOES A STORAGE RING WORK: DIPOLES, QUADRUPOLES, SEXTUPOLES

trajectory of particle with hor. velocity (synch. rad.):
with quadrupoles & sextupoles
with nominal energy
with energy deviation (synch. rad.)



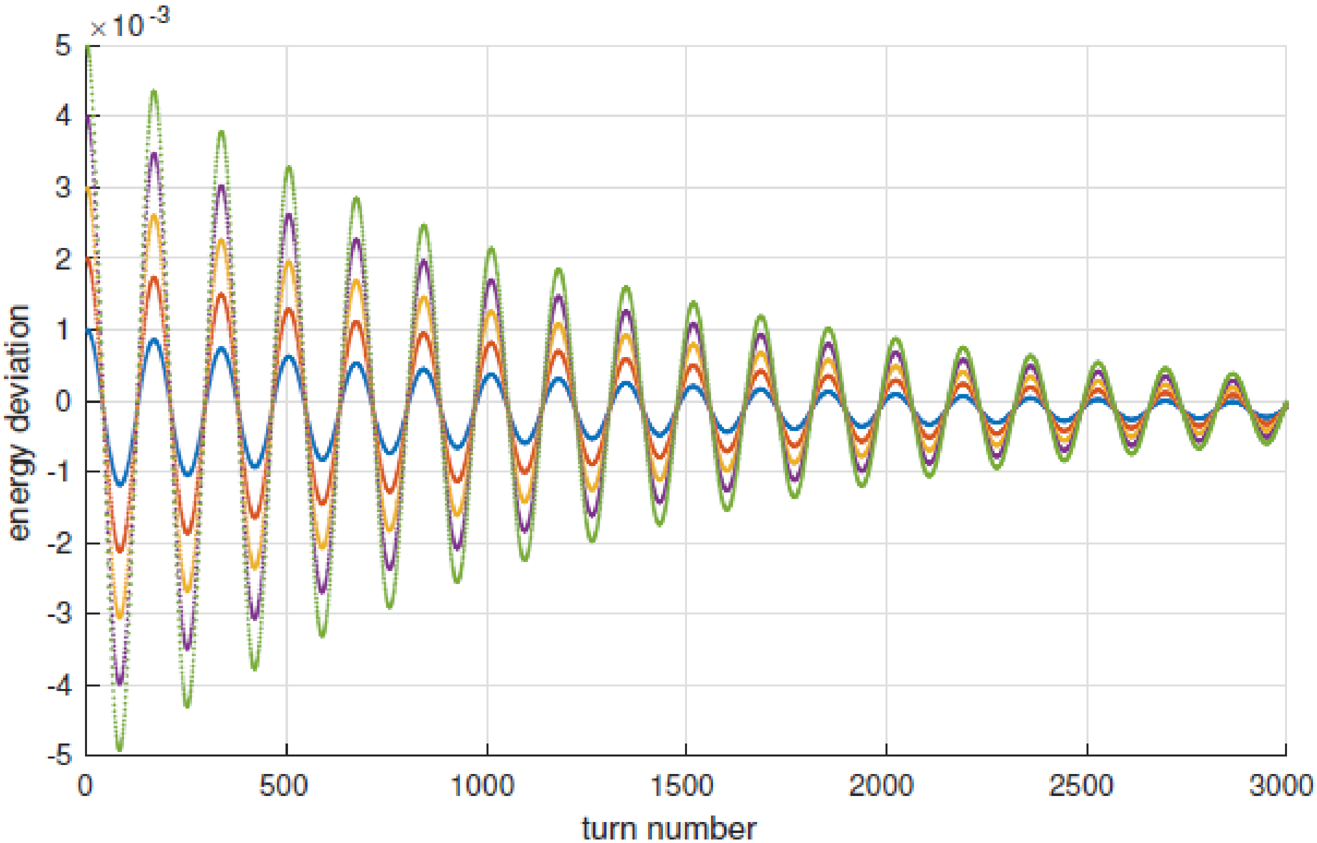
DISTRIBUTION OF THE STORED ELECTRONS



Higher energy radiates more, lower energy less. Causes damping towards reference energy.

$$P = \frac{1}{6}\frac{e^2 c}{\pi\varepsilon_0}\frac{1}{\rho^2}\left(\frac{E}{mc^2}\right)^4$$

DISTRIBUTION OF THE STORED ELECTRONS



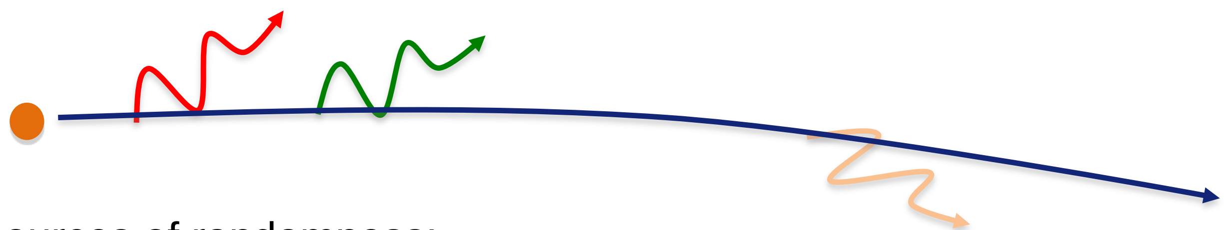
Energy oscillations (**synchrotron oscillations**) and radiation damping.

So what defines the size of the electron beam?

DISTRIBUTION OF THE STORED ELECTRONS

Question: if we have radiation damping, why isn't the electron beam size zero?

Answer: because of the graininess of the photon emission

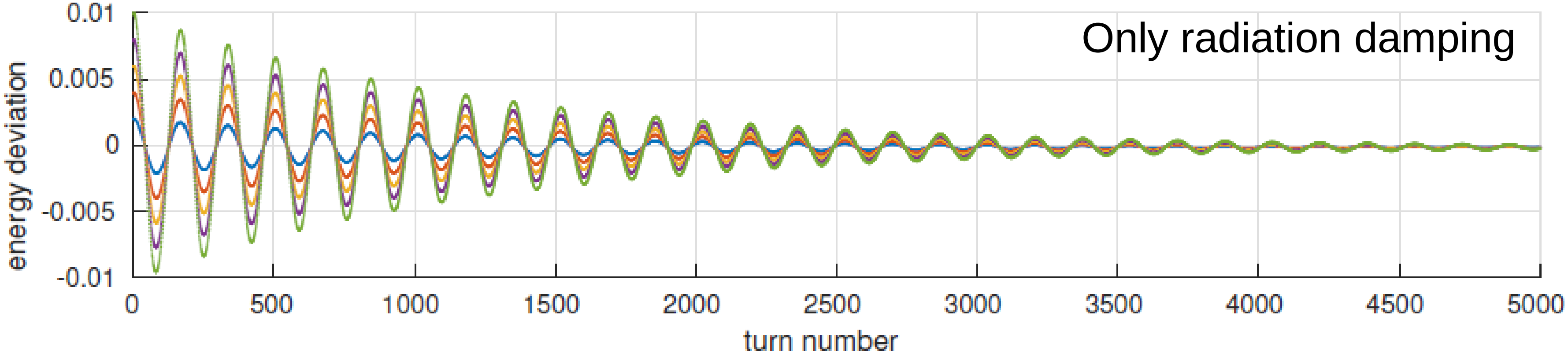


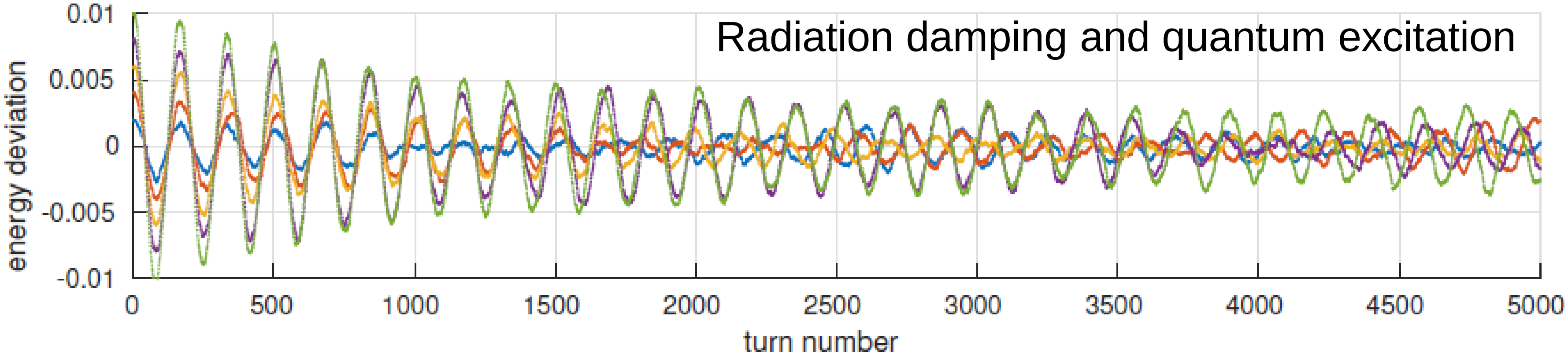
Two sources of randomness:
- emission time of photons: Poisson process
- energy of the photon emitted

This causes a diffusion process in energy, that is converted to a transverse diffusion by the dipoles (higher energy electrons are bended less than lower energy ones).

For ESRF, each electron emits only a few hundreds photons per turn.
Energy loss per turn U_0 = 2.53 MeV

DISTRIBUTION OF THE STORED ELECTRONS





Results of damping and diffusion

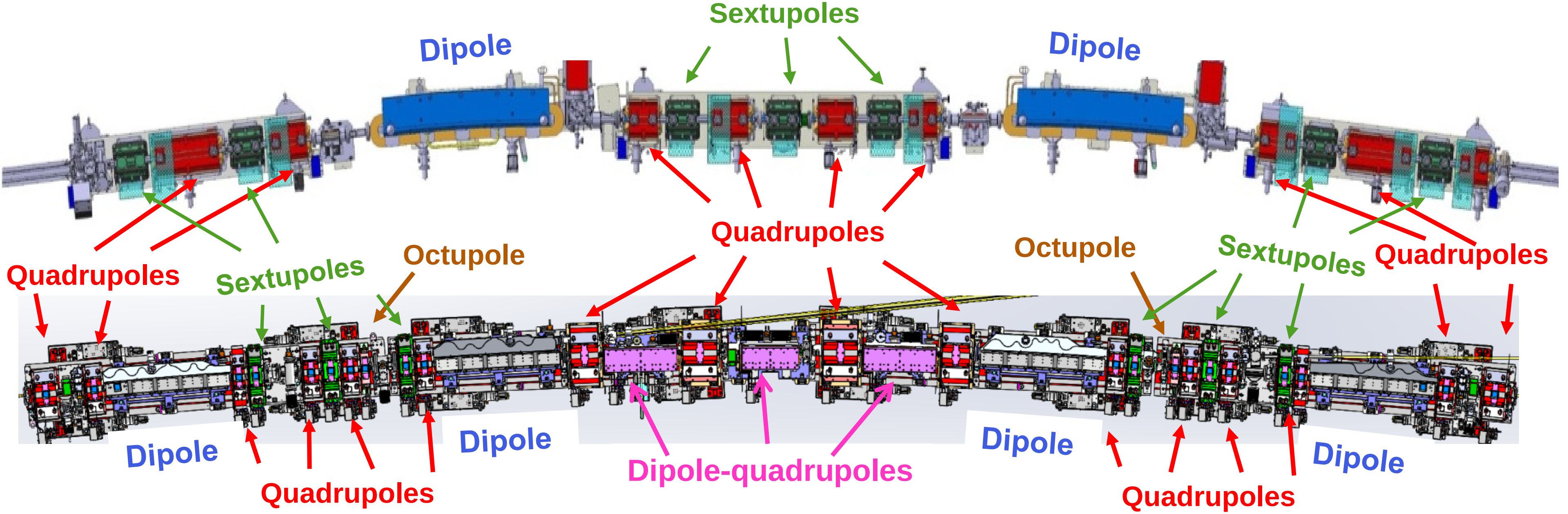
The electron beam reaches a unique Gaussian distribution, independent of how one injects into the ring.

This is a major difference between electron synchrotrons and proton synchrotrons (e.g. LHC)

By careful choice of where the dipoles and quadrupoles are, one can reduce the size of this equilibrium beam size (emittance = beam size in phase space).
So called “low emittance ring design”.

In fact, due to developments in lattice design, ESRF has completely replaced the storage ring in 2019 to reduce the electron beam emittance. 4nm rad -> 130 pm rad

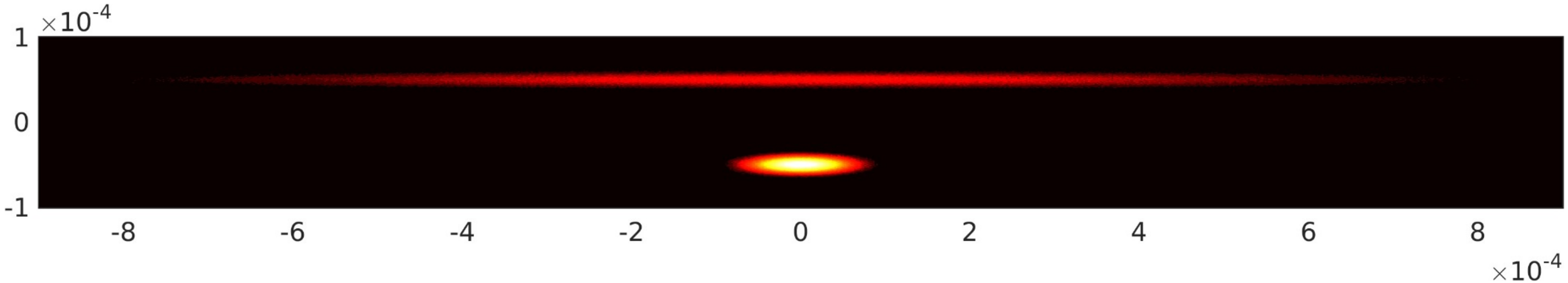
THE EBS UPGRADE AT ESRF



The ESRF **Extremely Brilliant Source** (EBS) has 31 magnets per cell instead of 17 of the old machine.
Free space between magnets (total for one cell): **3.4 m** instead of **8 m**!

Magnet type	ESRF	ESRF EBS
Dipoles	64	128
Dipole-Quadrupoles	0	96
Quadrupoles	256	512
Sextupoles	224	192
Octupoles	0	64
Correctors	0	96

x-y beam distribution for old and new machine.



Picture from
Simone Liuzzo

Electron beam horizontal emittance has been reduced by a factor 30 and the brightness is increases by the same factor.

$$\varepsilon_{ESRF} = 4 \text{ nm rad}$$

$$\varepsilon_{EBS} = 130 \text{ pm rad}$$

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MANY THANKS FOR YOUR ATTENTION