Finite Element analysis in X-ray optical systems

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Introduction

X-ray optics
- Monochromator crystal
- First mirror (HHL)
- Bent focusing mirror (KB, …)
- Compound refractive lens (CRL)
- Sagittal focusing mirror
- Piezo bimorph mirror
- Bending devices
- …

Analyses
- Cooling and geometry optimization
- Thermal deformation
- Performance vs heat load
- Thermal stress analysis
- Surface shape and profile optimization
- Mechanical stress analysis
- Performance vs energy tuning (bending forces)
- Bent shape
- Multi-electrodes application (gap, voltage distribution)
- …

Guidelines, data for
- Design, Manufacturing, Mounting, Operation

Data for
- Ray-tracing
- Dynamic diffraction simulation

Boundary conditions (P, hcv, Tf, F, D, V, …)
Mirror - FE model (ID20 mirror)

- **Geometry of the mirror LxWxT**: 1000x70x50 mm³
- **Absorbed power**: 50 W
- **Primary slits HxV**: 1.5x0.8 mm²
- **Grazing angle**: 3.5 mrad
- **Side cooling by water at 290 K**

1998
Mirror - FEA results (ID20 mirror)

Temperature in K

Vertical displacement
Uy (mm)

\[
\frac{1}{R_{\text{thermal}}} = \frac{1}{R_{\text{beamOFF}}} - \frac{1}{R_{\text{beamON}}}
\]

<table>
<thead>
<tr>
<th>bending force F (N)</th>
<th>45</th>
<th>80</th>
<th>150</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{\text{thermal}} ) (km)</td>
<td>23.2</td>
<td>21.2</td>
<td>25.6</td>
<td></td>
</tr>
<tr>
<td>( R_{\text{thermal}&amp;FEA} ) (km)</td>
<td>22</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hcv (FEA) (W/m²/K)</td>
<td>500</td>
<td>5000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

measurement #: "ID20-Test of mirror1 curvature", 1/04/98, by Ch. Vettier
Monochromator crystal - FEA vs Test results

- Channel-cut Si crystal monochromator
- Liquid nitrogen cooling from 2 sides
- Beam size 10.35 mm (H) x 2.3 mm (V)
- Bragg angle 14°
- Heat load from undulators U46 + U17: Gaussian distribution, volume absorption

\[
FWM_{th} = \sqrt{(\theta_{th} + \theta_0)^2 + FWHM_{intr}^2}
\]

- Good agreement between the calculation and experimental results
- Local minimum in thermal slope error

Zhang L. et al., JSR (2003). 10, 313-319
Monochromator crystal - FEA results

Thermal slope error versus absorbed power in 4 different cooling coefficients (W/m²/°C)

Slope error independent of the cooling coefficient in the linear region \( \Rightarrow \) direct cooling (high \( h \)) is not always necessary

Slope error varies significantly with the cooling coefficient in the non-linear region.

Slope \(~\) P curve can be divided into 3 regions:

- Linear region: \( \text{slope} \sim P \)
- Transition region: a local minimum
- Non-linear region: \( \text{slope} \sim P^{4.6} \)
Piezoelectric active mirror

- **Piezoelectric bimorph: spherical shape**
- **Active mirror**
  - Spherical shape (mono-electrode)
  - Toroidal shape (multi-electrodes)
  - Active: variation of the radius of curvature by changing electric voltage
- **FEA key points**
  - electrode distribution, gap effects for a required shape
  - Piezoelectric matrix, elastic coefficient matrix

Comparison with experiments


Piezoelectric active mirror (2)

- Multi-electrodes → elliptical shape
- Gap between electrodes, gap induced residual slope error

**Slope deviation** = slope_{FEA} - slope_{ref}

![Electric field distribution (Volt)](image)

- **Parameters:**
  - gap = 3 mm, rod length = 22.5 mm, p = 0.3 m, q = 3 mrad
  - ANSYS 5.0, time = 1, VOLT = 2000
  - Color scale: 0, 222, 444, 666, 888, 1111, 1333, 1555, 1777, 2000

![Graph](image)

- PV and RMS slope deviation vs gap (mm)
Bending devices - Flexor bender

- Flexor bender widely used in KB micro-focusing mirror device
- FEA used to
  - optimize the position and the size of mirror
  - evaluate bending forces
  - Simulate the performance

\[
\frac{d^2 z}{dx^2} = \frac{h}{EI} \left( \frac{(F_1 + F_2)}{2} (1 - \delta) + \frac{(F_2 - F_1) x}{L} (1 - \delta_x) \right)
\]

Bending devices - Flexor bender (2)

- 2 different bending moments / rectangular mirror → $1/R(x) \sim P1(x)$ linear function of $x$
- Rotation axis should be on the neutral plane of the mirror - to avoid bending capability loss
Sagittal focusing crystal - anticlastic deformation - ribs

1999
Compound Refractive Lens - failure analysis

- Beryllium CRLs installed in 1997 in FE
- CRL with 4 holes of 1mm in diameter to focus 8 keV X-ray beam
- Bonded to a water cooled copper block
- Failure observed on Dec-2003: sudden change of focusing capability

FEA results:
- $T_{\text{max}} = 873 \, ^\circ \text{C}$
- $\sigma_{V&M\text{max}} = 564 \, \text{MPa} \rightarrow \text{high stress, large plastic deformation}$
- Thermal fatigue failure

Total absorbed power: 139 W (-20% ?)
Cooling coefficient $h_{\text{eff}} = 0.005 \, \text{W/mm}^2/{^\circ \text{C}}$

Phase contrast images
- Damaged lens
- New lens

### CRL - Design optimization

<table>
<thead>
<tr>
<th></th>
<th>present</th>
<th>optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>width</td>
<td>mm</td>
<td>2</td>
</tr>
<tr>
<td>$t_{\text{thin}}$</td>
<td>mm</td>
<td>0.1</td>
</tr>
<tr>
<td>$V_{\text{bm}}$</td>
<td>mm</td>
<td>4</td>
</tr>
<tr>
<td>$h_{\text{eff}}$</td>
<td>W/mm²/°C</td>
<td>0.005</td>
</tr>
<tr>
<td>$T_{\text{max}}$</td>
<td>°C</td>
<td>873</td>
</tr>
<tr>
<td>$\sigma_{\text{VMmax}}$</td>
<td>MPa</td>
<td>564</td>
</tr>
</tbody>
</table>
- N number of parabolic lenses (Be, Al, ..., 15um, 0.56, 1.4W)
- Cooled or not cooled?
- How cooled?

- Copper plate
- Indium foil
- Bronze frame
- Be or Al lens

- Outer ring cooling
KB mirror profile optimization

Grazing angle $\theta=8\text{mrad}$

- Aspheric shape: $R(x)$ varies strongly with $x$
- Highly bent:
  - Radius of curvature reaches $R_{\text{min}}=11.2\text{m}$ at $x=28\text{mm}$
  - Slope of bent mirror in the range of a few of mrad

- Which profile of the mirror?
- How to determine the profile?
KB mirror profile optimization - algorithm

From beam theory:

\[
W(x) = \frac{1}{2} \left( F_1 \cdot L_{arm-1} + F_2 \cdot L_{arm-2} \right) \left( F_2 \cdot L_{arm-2} - F_1 \cdot L_{arm-1} \right) \cdot \frac{x}{L}
\]

\[
\frac{1}{12} E \cdot t^3 \cdot \frac{1}{R(x)}
\]

1. Initial width calculated by Eq.(A): \( W_1(x) \)
2. bent mirror shape calculated by FEA as well as curvature along the axis \( x \) on the mirror surface \( f_n(x) = \frac{d^2 U}{dx^2} \)
3. Comparison with ideal shape \( f_{ref}(x) = \frac{1}{R(x)} \)
4. Correction of the mirror width as :

\[
W_{n+1}(x) = W_n(x) \cdot \frac{f_n(x)}{f_{ref}(x)}
\]

5. 4~5 iterations (repeat steps 2-4) give stabilized results
KB mirror profile optimization - results

- by beam theory
- by FEA
- Final proposed

Residual slope error of 3 profiles
(HFM: p=36m, q=83mm, θ=8 mrad, F1=F2=16N)

Residual slope error of mirrors with 3 profiles
(HFM: p=36m, q=83mm, θ=8 mrad, F1=F2=16N)
KB mirror profile optimization - summary

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FEA deals with following issues:

- Mirror width profile
- Residual slope error
- Sensitivity to the uncertainty of preload spring parameters
- Si crystal orientation
- Stress in the mirror
- Resolution requirement for the Pico motors
- Stress in the bender and glue layers
- Error analysis
- Mirror performance at different photon energy
- ...

24-25 Feb 2009
Summary

FEA

- Widely used in X-ray optic Design, manufacturing and operation
- Providing data for
  - Ray-tracing
  - Dynamic diffraction simulation