Non-linear modeling of storage and damping rings using symplectic integrators

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Outline

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Outline

- Symplectic integration
  - The SABA2 integrator
  - Application to the ESRF storage ring ideal lattice
- Compact Linear Collider (CLIC) damping rings
  - Design challenges
  - Sextupole scheme
  - Tune scans
  - Wiggler effect
  - Effect of radiation damping
Symplectic integration

Laskar and Robutel Cel. Mech Dyn Astr. 80, 39, 2001

- Symplectic integrators with positive steps for Hamiltonian systems $H = A + \epsilon B$ with both $A$ and $B$ integrable
- Consider Hamiltonian system $H(\vec{p}, \vec{q})$, with $N$ degrees of freedom
- A trajectory of the system in phase space is described by $\vec{x}(t) = (x_1(t), \ldots, x_{2N}(t))$, $x_i = p_i$, $x_{i+N} = q_i$, $i = 1, \ldots, N$
- Hamilton’s equations of motion take the form
  $$\frac{d\vec{x}}{dt} = \{H, \vec{x}\} = L_H \vec{x},$$
  with the usual Poisson brackets $\{f, g\} = \sum_{i=1}^{N} \left( \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} - \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} \right)$.
- The solution is formally written as
  $$\vec{x}(t) = \sum_{n \geq 0} \frac{t^n}{n!} L_H^n \vec{x}(0) = e^{tL_H} \vec{x}(0).$$
SABA$_2$ integrator

- A symplectic integrator of order $n$ from $t$ to $t + \tau$
  consists of approximating the operator $e^{\tau L_H} = e^{\tau (L_A + L_{eB})}$
  by products of $e^{c_i \tau L_A}$ and $e^{d_i \tau L_{eB}}$, $i = 1, \ldots, n$ which
  integrate exactly $A$ and $B$ over the time-spans $c_i \tau$ and $d_i \tau$

- The constants $c_i$ and $d_i$ are chosen for reducing the error

- The SABA$_2$ integrator is written as
  \[ SABA_2 = e^{c_1 \tau L_A} e^{d_1 \tau L_{eB}} e^{c_2 \tau L_A} e^{d_1 \tau L_{eB}} e^{c_1 \tau L_A}, \]
  with $c_1 = \frac{1}{2} \left( 1 - \frac{1}{\sqrt{3}} \right)$, $c_2 = \frac{1}{\sqrt{3}}$, $d_1 = \frac{1}{2}$.

- When $\{\{A, B\}, B\}$ is integrable, e.g. when $A$ is quadratic
  in momenta and $B$ depends only in positions, the accuracy
  of the integrator is improved by two small negative steps
  \[ SABA_2 C = e^{-\tau^3 \epsilon^2 \frac{c}{2}} L_{\{\{A,B\},B\}} \left( SABA_2 \right) e^{-\tau^3 \epsilon^2 \frac{c}{2}} L_{\{\{A,B\},B\}}, \]
  with $c = (2 - \sqrt{3})/24$. 
The accuracy of the SABA$_2$C was proved an order of magnitude more precise than the Forest-Ruth 4th order integrator.

Note finally that the usual “drift-kick” scheme corresponds to the 2nd order integrator of this class:

$$SABA_1 = e^{\frac{\tau}{2} L_A} e^{\tau L_B} e^{\frac{\tau}{2} L_A},$$
The accelerator Hamiltonian in the small angle, “hard-edge” approximation is written as \( H(x, y, l, p_x, p_y, \delta; s) = H_0 + V \),

with the unperturbed part \( H_0 = (1 + h x) \frac{p_x^2 + p_y^2}{2(1 + \delta)} \),

and the perturbation \( V(x, y) = \sum_{n \geq 1} \sum_{j=0}^{n} a_{n,j} x^j y^{n-j} \).

The unperturbed part of the Hamiltonian can be integrated

\[
\begin{align*}
 e^{sL_A} : & \quad x^f = \frac{1}{h} \left\{ (1 + hx) \left( \cos \phi + \frac{p_x}{p_y} \sin \phi \right)^2 - 1 \right\} \\
 & \quad y^f = y^i + \frac{1 + hx^i}{h} \left\{ \frac{p_x^i + p_y^i}{p_y^i} \frac{p_y^i}{2p_y^i} \sin(2\phi) + 2 \frac{p_x^i}{p_y^i} \sin^2 \phi \right\} \\
 & \quad p_x^f = p_x^i \frac{p_y^i - p_y^i \tan \phi}{p_y^i + p_x^i \tan \phi} \\
 & \quad p_y^f = p_y^i
\end{align*}
\]

with \( \phi = \frac{p_y hs}{2(1 + \delta)} \).
The perturbation part of the Hamiltonian can be integrated

\[
e^{sL_B} : \begin{cases} 
{x^f} = x^i, \quad p^f_x = p^i_x - \frac{\partial V}{\partial x} \bigg|_i^s \quad \text{with} \quad \frac{\partial V}{\partial x} \bigg|_i^s = \sum_{n \geq 1} \sum_{j=1}^{n} j a_{n,j}(x^i)^{j-1}(y^i)^{n-j} \\
y^f = y^i, \quad p^f_y = p^i_y - \frac{\partial V}{\partial y} \bigg|_i^s \quad \text{with} \quad \frac{\partial V}{\partial y} \bigg|_i^s = \sum_{n \geq 1} \sum_{j=0}^{n} (n-j) a_{n,j}(x^i)^j(y^i)^{n-j-1}
\end{cases}
\]

The corrector is expressed as

\[
C = \{\{A, B\}, B\} = \frac{1 + hx}{1 + \delta} \left[ \left( \frac{\partial V}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 \right],
\]

and the operator for the corrector is written as

\[
e^{sL_C} : \begin{cases} 
{x^f} = x^i \\
y^f = y^i \\
p^f_x = p^i_x - \frac{1}{1 + \delta} \left\{ h \left[ \left( \frac{\partial V}{\partial x} \right)^2 \bigg|_i + \left( \frac{\partial V}{\partial y} \right)^2 \bigg|_i \right] + 2(1 + hx^i) \left[ \frac{\partial V}{\partial x} \bigg|_i \frac{\partial^2 V}{\partial x^2} \bigg|_i + \frac{\partial V}{\partial y} \bigg|_i \frac{\partial^2 V}{\partial x \partial y} \bigg|_i \right] \right\}^s \\
p^f_y = p^i_y - \frac{2(1 + hx^i)}{1 + \delta} \left\{ \frac{\partial V}{\partial x} \bigg|_i \frac{\partial^2 V}{\partial x \partial y} \bigg|_i + \frac{\partial V}{\partial y} \bigg|_i \frac{\partial^2 V}{\partial y^2} \bigg|_i \right\}^s
\end{cases}
\]
Consider the old ESRF “ideal” lattice, i.e. perfectly symmetric (periodicity of 16) with the only non-linearity coming from the sextupoles

Integrate the equations of motions with three different methods

- “Drift-Kick” method by splitting the 0.4m sextupoles in a drift+kick+drift
- Splitting the sextupoles in 10*(drift+kick)+drift
- Using the SABA$_2$C symplectic integrator

Produce frequency maps by using Laskar’s NAFF algorithm and compare
Comparison between frequency maps produced by “drift-kick” 1 kick versus 10 kicks
Frequency map using the SABA$_2$C symplectic integrator reproduces the “10-kick” case
Diffusion maps

Diffusion map using the SABA\textsubscript{2}C symplectic integrator shows lower horizontal and slightly higher vertical DA than 1-kick integrator.

Colour coding following the logarithm of the diffusion vector amplitude

\[ D_{t=\tau} = \nu_{t\in(0,\tau/2]} - \nu_{t\in(\tau/2,\tau]} \]

26/05/2008
The CLIC Project

- **Compact Linear Collider**: multi-TeV e-p collider for high energy physics beyond the LHC
- Center-of-mass energy from 0.5 to 3 TeV
- RF gradient and frequencies are very high
  - 100 MV/m in room temperature accelerating structures at 12 GHz
- **Two-beam-acceleration concept**
  - High current “drive” beam, decelerated in power extraction structures (PETS), generates RF power for main beam.
- **Challenges**:
  - Efficient generation of drive beam
  - PETS generating the required power
  - 12 GHz RF structures for the high gradient
  - Generation/preservation of small emittance beam
  - Focusing to nanometer beam size
  - Precise alignment of the different components
Damping ring design goals

- Ultra-low emittance and high beam polarisation impossible to be produced by conventional particle source:
  - Ring to damp the beam size to desired values through synchrotron radiation
- Intra-beam scattering due to high bunch current blows-up the beam
  - Equilibrium “IBS dominated” emittance should be reached fast to match collider high repetition rate
- Other collective effects (e.g. e⁻-cloud) may increase beam losses
- Starting parameter dictated by design criteria of the collider (e.g. luminosity), injected beam characteristics or compatibility with the downstream system parameters (e.g. bunch compressors)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>CLIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>bunch population ((10^9))</td>
<td>4.1</td>
</tr>
<tr>
<td>bunch spacing ([\text{ns}])</td>
<td>0.5</td>
</tr>
<tr>
<td>number of bunches/train</td>
<td>312</td>
</tr>
<tr>
<td>number of trains</td>
<td>1</td>
</tr>
<tr>
<td>Repetition rate ([\text{Hz}])</td>
<td>50</td>
</tr>
<tr>
<td>Extracted hor. normalized emittance ([\text{nm}])</td>
<td>&lt;680</td>
</tr>
<tr>
<td>Extracted ver. normalized emittance ([\text{nm}])</td>
<td>&lt; 20</td>
</tr>
<tr>
<td>Extracted long. normalized emittance ([\text{eV m}])</td>
<td>&lt;5000</td>
</tr>
<tr>
<td>Injected hor. normalized emittance ([\mu\text{m}])</td>
<td>63</td>
</tr>
<tr>
<td>Injected ver. normalized emittance ([\mu\text{m}])</td>
<td>1.5</td>
</tr>
<tr>
<td>Injected long. normalized emittance ([\text{keV m}])</td>
<td>1240</td>
</tr>
</tbody>
</table>
Horizontal emittance vs. energy

Z. Zhao, PAC’07
Vertical emittance vs. energy

Emittance [pm]

Energy [GeV]

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CLIC damping ring layout

- Straight section including 38 wigglers: 96 m
- Extraction
- Injection
- Regular FODO cells with wigglers
- Dispersion suppressor & beta-matching section with two wigglers
- 48 TME cells
- ARC
- Dispersion suppressor & beta-matching section with RF cavities
- Regular FODO cells with wigglers
- Dispersion suppressor & injection/extraction region
- 27.53 m
- 48 TME cells
TME arc cell

- TME cell chosen for compactness and efficient emittance minimisation over Multiple Bend Structures (or achromats) used in light sources
- Large phase advance necessary to achieve optimum equilibrium emittance
- Very low dispersion
- Strong sextupoles needed to correct chromaticity
- Impact in dynamic aperture

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<table>
<thead>
<tr>
<th>Energy</th>
<th>2.42 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of the bending magnet, $B_a$</td>
<td>0.932 T</td>
</tr>
<tr>
<td>Length of the bending magnet</td>
<td>0.545 m</td>
</tr>
<tr>
<td>Bending angle</td>
<td>$2\pi/100$</td>
</tr>
<tr>
<td>Bending radius</td>
<td>8.67 m</td>
</tr>
<tr>
<td>Length of the cell, $L_{TME}$</td>
<td>1.73 m</td>
</tr>
<tr>
<td>Horizontal phase advance, $\mu_x$</td>
<td>210°</td>
</tr>
<tr>
<td>Vertical phase advance, $\mu_y$</td>
<td>90°</td>
</tr>
<tr>
<td>Emittance detuning factor, $\epsilon_x$</td>
<td>1.8</td>
</tr>
<tr>
<td>Horizontal chromaticity, $\partial\beta_x/\partial\delta$</td>
<td>-0.84</td>
</tr>
<tr>
<td>Vertical chromaticity, $\partial\beta_y/\partial\delta$</td>
<td>-1.18</td>
</tr>
<tr>
<td>Average horizontal beta function, $\langle \beta_x \rangle$</td>
<td>0.847 m</td>
</tr>
<tr>
<td>Average vertical beta function, $\langle \beta_y \rangle$</td>
<td>2.22 m</td>
</tr>
<tr>
<td>Average horizontal dispersion, $\langle D_x \rangle$</td>
<td>0.0085 m</td>
</tr>
<tr>
<td>Relative horizontal beta function, $\beta_x = \beta_x^{\ast}/\beta_x^{\ast}$</td>
<td>0.113/0.07 = 1.6</td>
</tr>
<tr>
<td>Relative horizontal dispersion, $D_x = D_x^{\ast}/D_x^{\ast}$</td>
<td>0.00333/0.00143 = 2.33</td>
</tr>
</tbody>
</table>
Wigglers’ effect with IBS

- For higher wiggler field and smaller period the transverse emittance computed with IBS gets smaller.
- The longitudinal emittance has a different optimum but it can be controlled with the RF voltage.

The choice of the wiggler parameters is finally dictated by their technological feasibility.

- Normal conducting wiggler of 1.7T can be extrapolated by existing designs.
- Super-conducting options have to be designed, built and tested.

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ESRF workshop 20
Two sextupole schemes
- 2 and 9 families of sextupoles
- For 2nd scheme sextupoles are separated by a -I transformer (2nd order achromat)

Dynamic aperture is $9\sigma_x$ in the horizontal and $14\sigma_y$ in the vertical plane (comfortable for injection)

Korostelev, PhD thesis 2006
Optimising the CLIC DR tune by scanning the tune space for maximum horizontal and vertical dynamic aperture

- DA limited by \((1, \pm 2)\) and integer resonance line
Effect of broken periodicity

Levichev, Piminov (2005)

With horizontal or vertical beta beating of 5%, appearance of large amount of non-systematic resonances, shrinking the DA and the optimal tune areas
DA for Damping wigglers

- Particles through 3D wiggler field tracked with symplectic integrator (Verlet scheme)
- Linear optics distortion corrected with quadrupole magnets in the dispersion suppressor

Levichev, Piminov (2005)

- Longitudinal field variation contributes to an octupole-like tune-spread
- Effect of wiggler in DA quite small
Effect of COD and coupling

- Several alignment errors considered introducing closed orbit distortion and dispersion variation
- Correction with dispersion free steering (orbit and dispersion correction)
- Skew quadrupole correctors for correcting dispersion in the arc and emittance minimisation
- Even after correction, reduction of the DA, especially in the vertical plane

<table>
<thead>
<tr>
<th>Imperfections</th>
<th>Symbol</th>
<th>1 r.m.s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrupole misalignment</td>
<td>$\langle \Delta Y_{\text{quad}} \rangle$, $\langle \Delta X_{\text{quad}} \rangle$</td>
<td>90 $\mu$m.</td>
</tr>
<tr>
<td>Sextupole misalignment</td>
<td>$\langle \Delta Y_{\text{sext}} \rangle$, $\langle \Delta X_{\text{sext}} \rangle$</td>
<td>40 $\mu$m</td>
</tr>
<tr>
<td>Quadrupole rotation</td>
<td>$\langle \Delta \Theta_{\text{quad}} \rangle$</td>
<td>100 $\mu$rad</td>
</tr>
<tr>
<td>Dipole rotation</td>
<td>$\langle \Delta \Theta_{\text{dipole arc}} \rangle$</td>
<td>100 $\mu$rad.</td>
</tr>
<tr>
<td>BPMs resolution</td>
<td>$\langle R_{\text{BPM}} \rangle$</td>
<td>2 $\mu$m.</td>
</tr>
</tbody>
</table>
Effect of radiation damping

- Including radiation damping and excitation shows that 0.7% of the particles are lost during the damping.
- Certain particles seem to damp away from the beam core, on resonance islands.

Levichev (2007)
Perspectives

- Optimise the TME cell for realistic magnet parameters
- Reiterate sextupole optimisation and non-linear dynamics including magnet and wiggler field errors
- Include space effect: in fact the space charge tune-shift at injection is negligible but when the beam shrinks it becomes quite large

\[ \Delta \nu_y = \frac{N_{bp} \gamma_0}{(2\pi)^{3/2} \gamma^3 \sigma_s} \oint \frac{\beta_y}{\sigma_y (\sigma_x + \sigma_y)} ds \approx 0.15 \]

and should be taken into account at least in the tune optimisation

- Include effect of radiation damping, excitation and IBS in the non-linear optimisation process
- Use symplectic integrators + resonance analysis (frequency and diffusion maps)