# Some aspects of the use of the Vibrating Wire Technique for a wiggler magnetic field measurement.

Alexander Temnykh

Cornell University, USA

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# Introduction

- Conventional techniques used for wiggler/undulators field and field integral measurement:
  - Hall probe
  - Short searching coils
  - Long flipping coils
- Field measured along strait lines is different from field seen by particles moving along wiggling trajectory. The difference may cause serious problems for beam dynamics:
  - J. Safranek et. al., Nonlinear Dynamics in SPEAR Wigglers, Proc. of PAC 1999, New York, pp. 157-161.
  - D. Alesini et. al., Beam-Beam Experience at DAΦNE, Proc. of PAC 2001, to be published.
- Can we measure field along a wiggling path?

  Yes, we can if we use Vibrating Wire Technique.

# Theory

- Beam trajectory and displacement of the taut wire with DC current in magnetic field are similar (a well known fact).

$$\frac{\partial^2 X_b}{\partial z^2} = \frac{q}{P} B(z); \quad T\frac{\partial^2 X_w}{\partial z^2} = -I_{dc} B(z)$$

$X_b$ – beam trajectory; $X_w$ – taut wire displacement;

$q, P$ – particles charge and momentum; $T, I_{dc}$ – the wire tension and DC current

if $\frac{I_{dc}}{T} = -\frac{q}{P}$ the wire will imitate beam trajectory.

# Theory

- Vertical wiggler magnetic field: $B(z) \approx B_w(z)\sin(\frac{2\pi z}{d})$
- The wire wiggling: $X(z) \approx A_w \sin(\frac{2\pi z}{d})$ where $A_w = \frac{I_{dc} B_w}{T}(\frac{2\pi z}{d})^2$
- Effect of the horizontal field appearance due to the wiggling (x=0)

$$B_x(x=0,y,z) = B_z(y,z)X'(z) \approx B_w A_w (\frac{2\pi z}{d})^2 \sin(\frac{2\pi z}{d})^2 \mathrm{y}$$

$$\delta I_x(x=0,y) = \int B_x dz = \frac{B_w A_w L_w}{2}(\frac{2\pi z}{d})^2 \mathrm{y}; \qquad \delta I_y(x=0,y) = 0$$

# Theory

- Effect of the wiggling on field components at z = 0

$$B_p(x) - \text{vertical field variation across single pole}$$

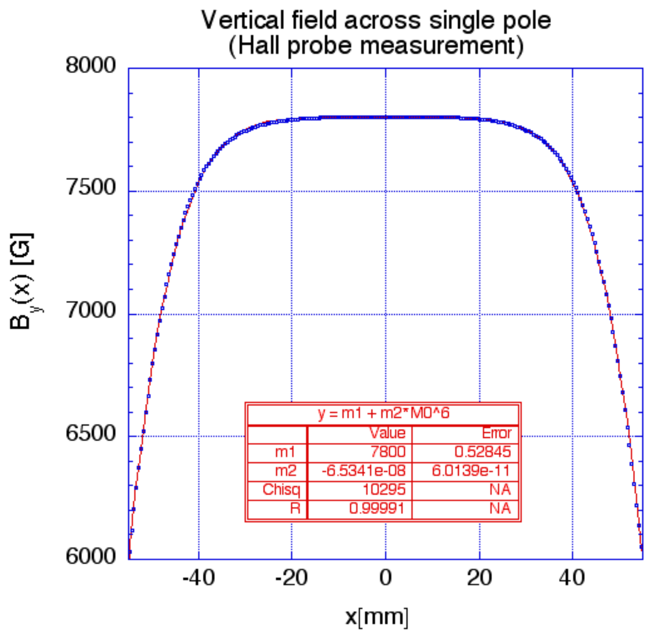
$$\delta I_x(x, y = 0) = 0;$$

$$\delta I_y(x, y = 0) = \frac{A_w L_w}{2} \frac{\partial B_p(x)}{\partial x}$$

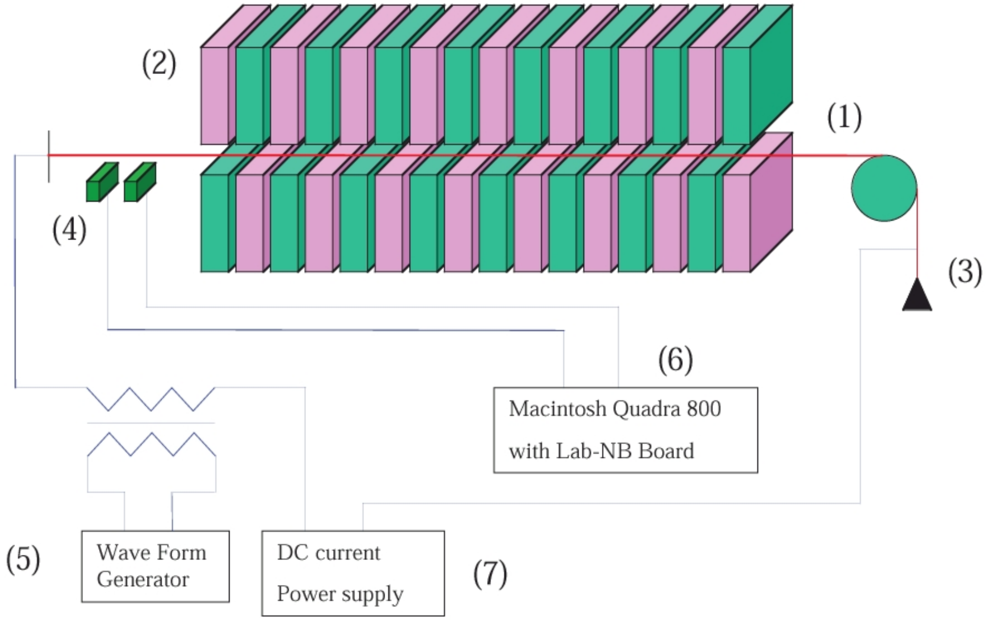
- For experimental condition:

$$B_p(x)[G] = 7.8 \times 10^3 - 6.53 \times 10^{-8} \cdot x^6 [mm]$$

$$\delta I_y(x, y = 0)[Gm] = 3.56 \times 10^{-8} \cdot x^5 [mm]$$



# Measurement Setup




(1) 100 micron copper-beryllium wire 382.2 cm in length
(2) G-line CHESS wiggler, Bmax=0.780T, d=12cm, L=3m
(3) Tension mechanism
(4) Horizontal and vertical wire position detectors

# Results

- Wiggling amplitude calibration with optical means
  - Fit:

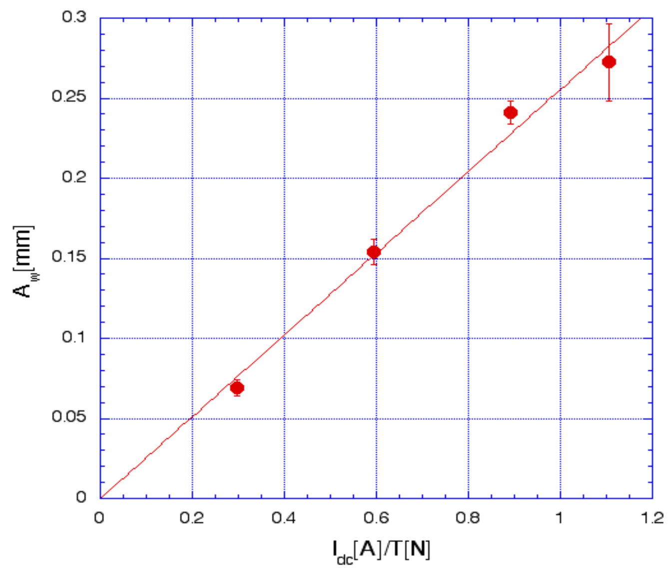
$$A_w[mm] = (0.255 \pm 0.006) \cdot I_{dc}[A]/T[N]$$

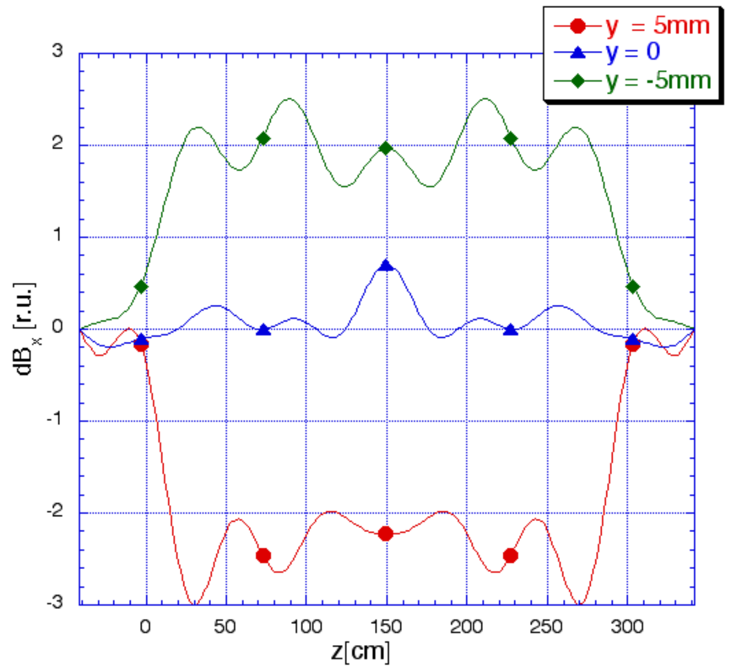
  - Model:

$$A_w[mm] = 0.28 \cdot I_{dc}[A]/T[N]$$

- Difference between horizontal field along z measured with (Aw=0.124mm) and without wiggling (Aw=0). 9 odd harmonics were used.

  Vertical position y= - 5, 0, 5 mm
  From model dBx ~ 0.8e-3Bmax
  1[r.u.] ~ 4e-4 Bmax



# Results (theoretical prove)

- If the function of field distribution along magnet is known, one can use only one wire vibrating mode to measure the field integral.

$$\delta B(z) = B_0 \sin\left(\frac{2\pi z}{d}\right)^2; \qquad \delta I = \int_0^L \delta B(z)\,dz = \frac{B_0 L_w}{2}$$

Harmonic measured by vibrating wire :

$$H_1 = \frac{2}{l}\int_0^l \delta B(z)\sin\left(\frac{\pi z}{d}\right)dz = B_0 \frac{2}{\pi}\sin\left(\frac{\pi L_w}{2l}\right)$$

The field integral will be :

$$\delta I = H_1 \frac{l}{2} \frac{\pi L_w}{2l} \sin^{-1}\left(\frac{\pi L_w}{2l}\right)$$

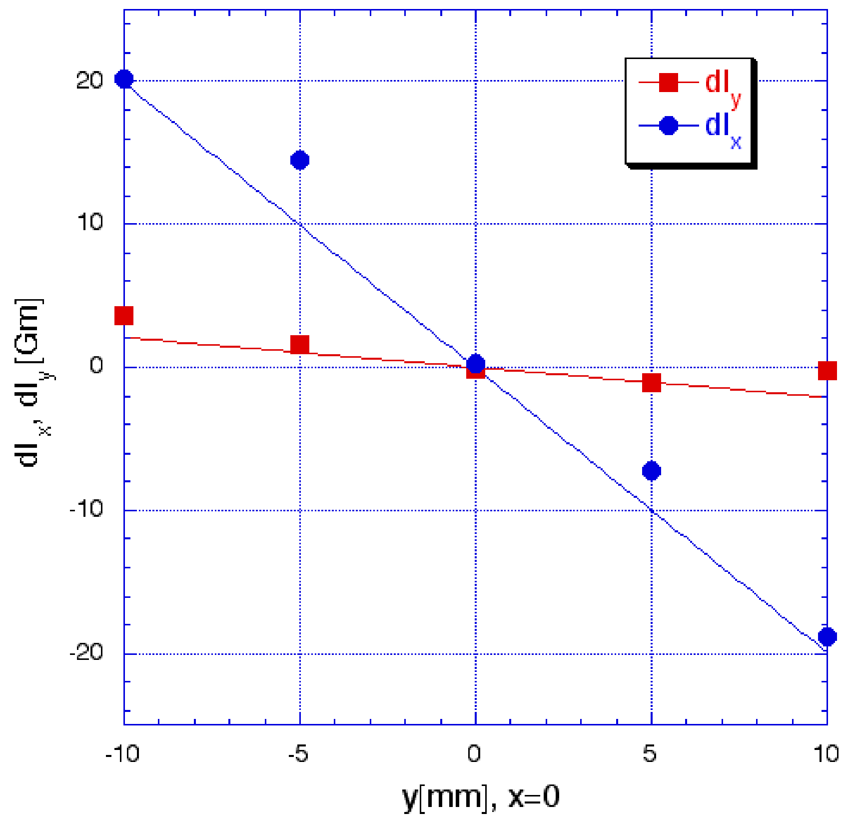
- Effect from small shim placed in the middle of magnet and measured with long flipping coil was used for calibration.

# Results

- Vertical / horizontal field integrals versus y, Aw = 0.060mm,
  - Measurement

$$\delta I_x(x=0,y)[Gm] = (-1.99 \pm 0.17)\cdot y[mm]$$

$$\delta I_y(x=0,y)[Gm] = (-0.21 \pm 0.08)\cdot y[mm]$$




  - Model:

$$\delta I_x(x=0,y)[Gm] = -1.92\cdot y[mm]$$

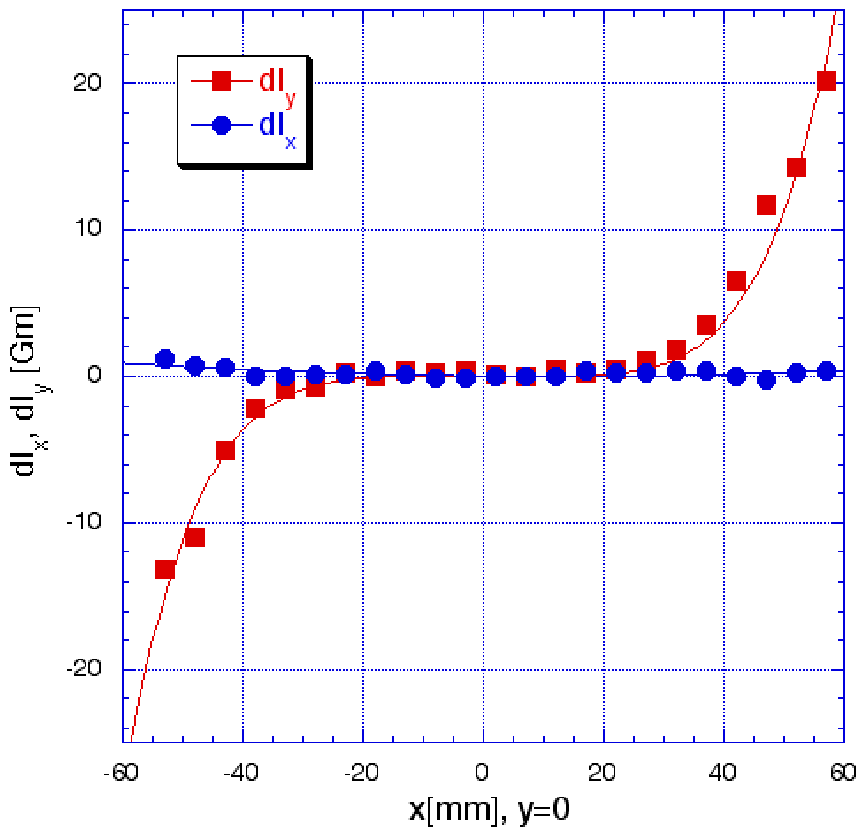
$$\delta I_y(x=0,y)[Gm] = 0$$

# Results

- Vertical / horizontal field integrals versus x, Aw = 0.060mm,
  - Measurement

$$\delta I_x(x, y = 0)[Gm] \approx 0$$

$$\delta I_y(x, y = 0)[Gm] = (3.60 \pm 0.12) \cdot x^5 [mm]$$




  - Model:

$$\delta I_x(x, y = 0)[Gm] = 0$$

$$\delta I_y(x = 0, y)[Gm] = 3.56 \times 10^{-8} \cdot x^5 [mm]$$

# Conclusion

- Vibrating Wire Technique was used to measure field distribution and field integrals along a wiggling path imitating beam trajectory.
- Obtained data is in excellent agreement with model calculation.
- The Vibrating Wire Technique has unique features which can be effectively exploited in the Insertion Devices magnetic field measurements.