A Tool for Simulating Rotating Coil Magnetometers

L. Bottura, M. Buzio, P. Schnizer, N. Smirnov

2. October 2001

Introduction

- Series magnet measurement systems at CERN are mainly based on the rotating coil technique.
- Magnets are extremely large and measurement requirements are demanding.
- The aim is to identify the weak spot in the measurement system and to improve the measurement quality of the system.
- The calculation is based on first principles.
- The description of imperfections is kept as simple as possible.

Contents

Measurement of Field Components

Magnetic Field B(z):

$$B(z) = B_y + iB_x = \sum_{n=1}^{\infty} [B_n + iA_n] \left(\frac{z}{R_{Ref}} \right)^n$$

$B_n + A_n \cdot i = C_n$ …multipoles

R_{Ref} …reference radius

Flux Φ seen by a coil:

$$\Phi(t) = \sum_{n=1}^{M} K_n C_n \cos\left[n\beta(t)\right].$$

$\beta(t)$…angular position of the coil

The sensitivities K_n:

$$K_n = \left(\frac{N_w L R_{Ref}}{n}\right)\left[\left(\frac{z_2}{R_{Ref}}\right)^n - \left(\frac{z_1}{R_{Ref}}\right)^n\right]$$

N_w …number of windings

z_2, z_1…the inner and outer radii respectively of the coil

L …Length of the Coil

Voltage $V(t)$ induced into the coils:

$$V(t) = -n\,\dot{\beta}(t)\,\sum_{n=1}^{N} K_n C_n \sin\left[n\beta(t)\right].$$

The harmonics are given as:

$$C_n = \frac{1}{K_n} FT\left(\int_0^P V(t)dt\right).$$

$FT\ldots$Fourier Transform

$P\quad\ldots$time of one rotation

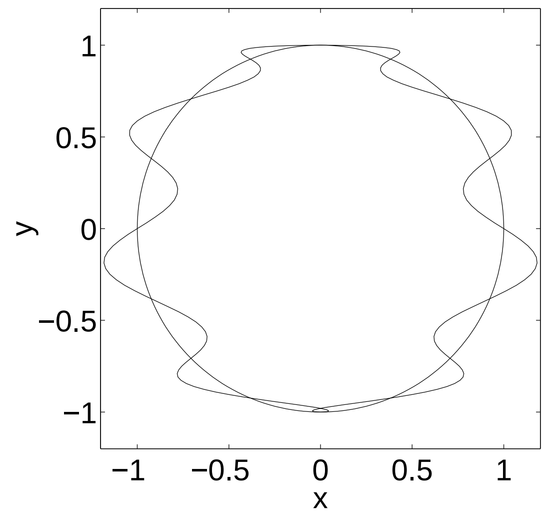
Mathematical Approaches

- In Fourier Space
- In Time Space purely numerical
- In Time Space semi numerical

Fourier Space

- The magnetic field is described as Harmonics.
- The induced voltage could be described as a spectrum of n.
- Electronic Noise has a spectrum.
- Each component is effecting the spectrum of the previous one.
- An missing mark of the angular encoder, a drifting power supply is difficult to model.

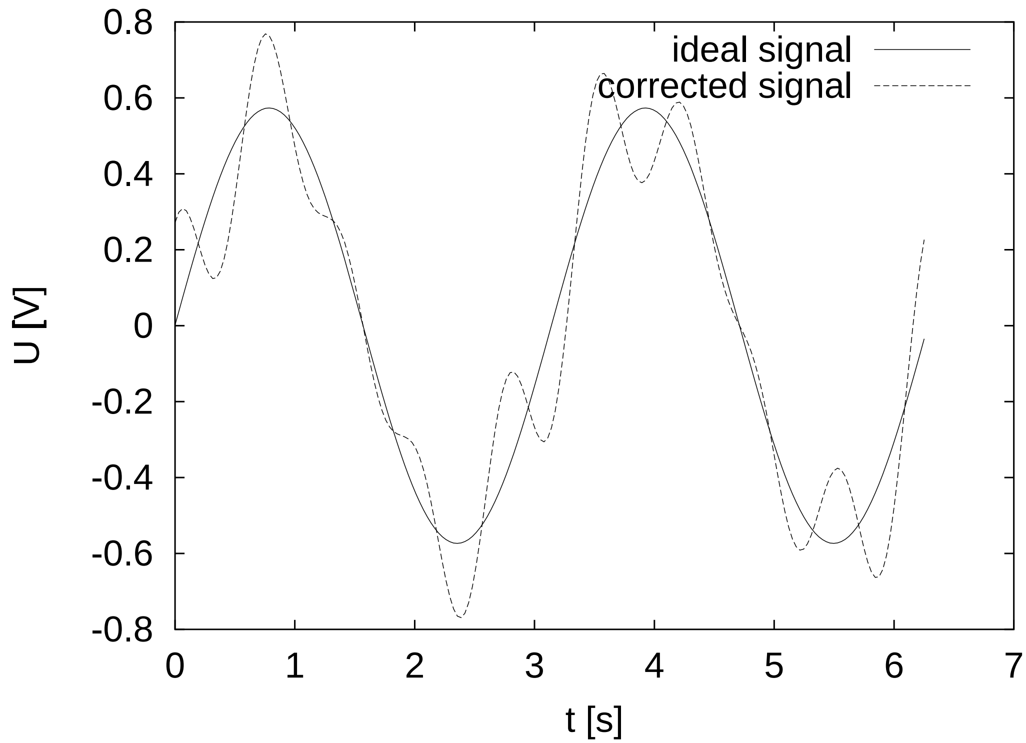
Time Space purely numerical



Deviation of the coils movement from the ideal line

Follow the deviation from the ideal line and change the sensitivities accordingly. They must be differentiated numerically.

Time Space semi numerical



Coil imperfections are modelled by correcting the induced voltage for each time step using analytical formulae.

- The sensitivities K_n are well known but the formalisms are more complex.
- This was considered a sufficient advantage to accept the tradeoff of the more complicated formalism.
- The usage of the sensitvities allows the simple modelling of a bucked system.
- The frequency space approach was not considered further as some components are difficult to model.

Physical Models

- First principles are used to describe each element and its imperfections.
- The description of the imperfection is kept as simple as possible.
- They still allow the input of experimental experience e.g. the torsional vibrations of mechanical connections are modelled using a series of decaying frequencies.

- *motor*
  - *ideal* : ωt
  - *imperfection* : $\sum_{i=1}^{N} A_i \sin\left(\alpha_i t + \phi_i\right)$
- *mechanical connections*
  - *ideal* : $\theta(t)$
  - *imperfection* : $\sum_i e^{-d_i t} a_i sin(c_i t + \psi_i)$
- *rotating coils*:
  - *ideal*:

$$V(t) = -n\,\dot{\beta}(t)\,\sum_{n=1}^{N} K_n C_n \sin\left[n\beta(t)\right].$$

  - *imperfection* : transversal vibrations

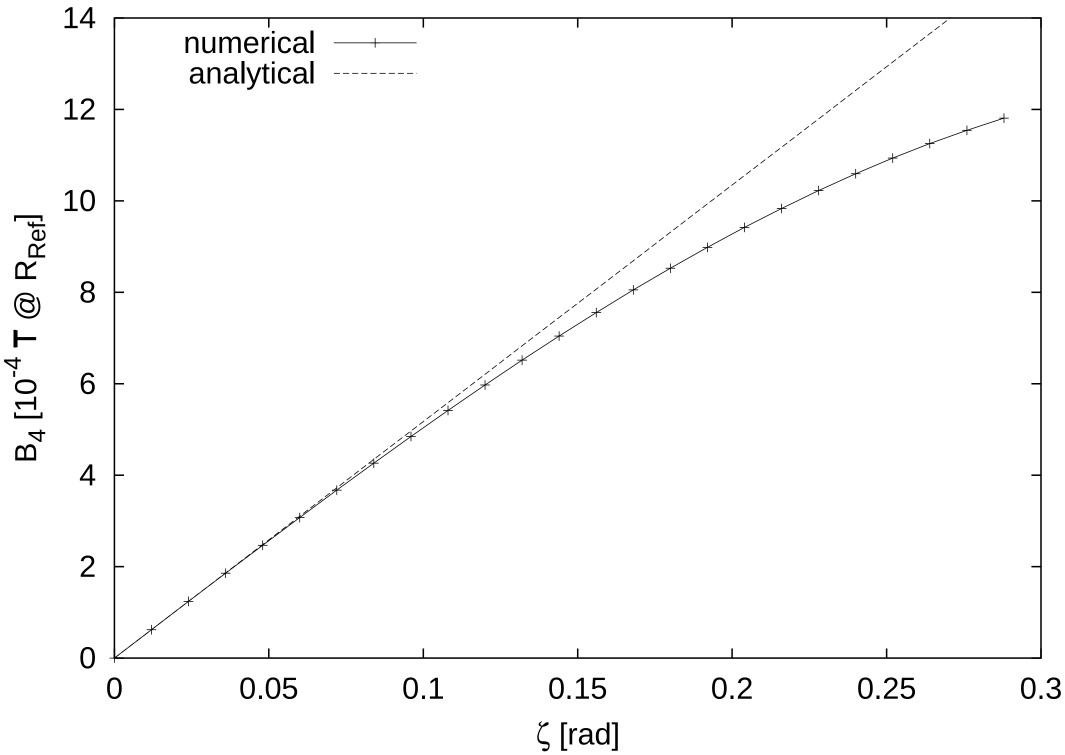
- *the angular encoder:*
  - *ideal* : triggers the measurement.
  - *imperfection* : Spurious marks. Sinusoidal errors
- *cables:*
  - *ideal* : transfer the signal from source to receiver.
  - *imperfection* : pickup of electromagnetic noise, thermal voltages
- *electronics:* compensation circuitry, preamplifier, integrator
  - *ideal:* $U_{OE} = \int_{t'_0}^{t'_0+P} G_P \sum_i G_{C_i} U_{IC_i} dt$
  - *imperfections:* Input - Output Offset voltages, non linear amplification.

Implementation

- In a real system the behaviour and influence of each component is influenced by the attached devices and the surrounding environment.
- Software mirror → Object Oriented Hierarchy
- encapsulation of the key parameters
- 6 digits accuracy → possible using double precision
- integration using ODE solver

Validation

- pure ideal case
- Transversally vibrating coil
  $1\mu m$ equals measurement resolution
  6 digits match analytically and numerically
- Torsionally vibrating coil



Example of the influence of a torsional vibrating coil on the measurement of the octupole in a pure quadrupole field for a vibration $p = 2$.

Example of Applications

Search for a fake harmonic in a dipole:

- Magnet Calculation gave a small quadrupole harmonic in a dipole.
- A far too large quadrupole was measured.
- Study, what measurement system imperfection and which quantity gives such an error.

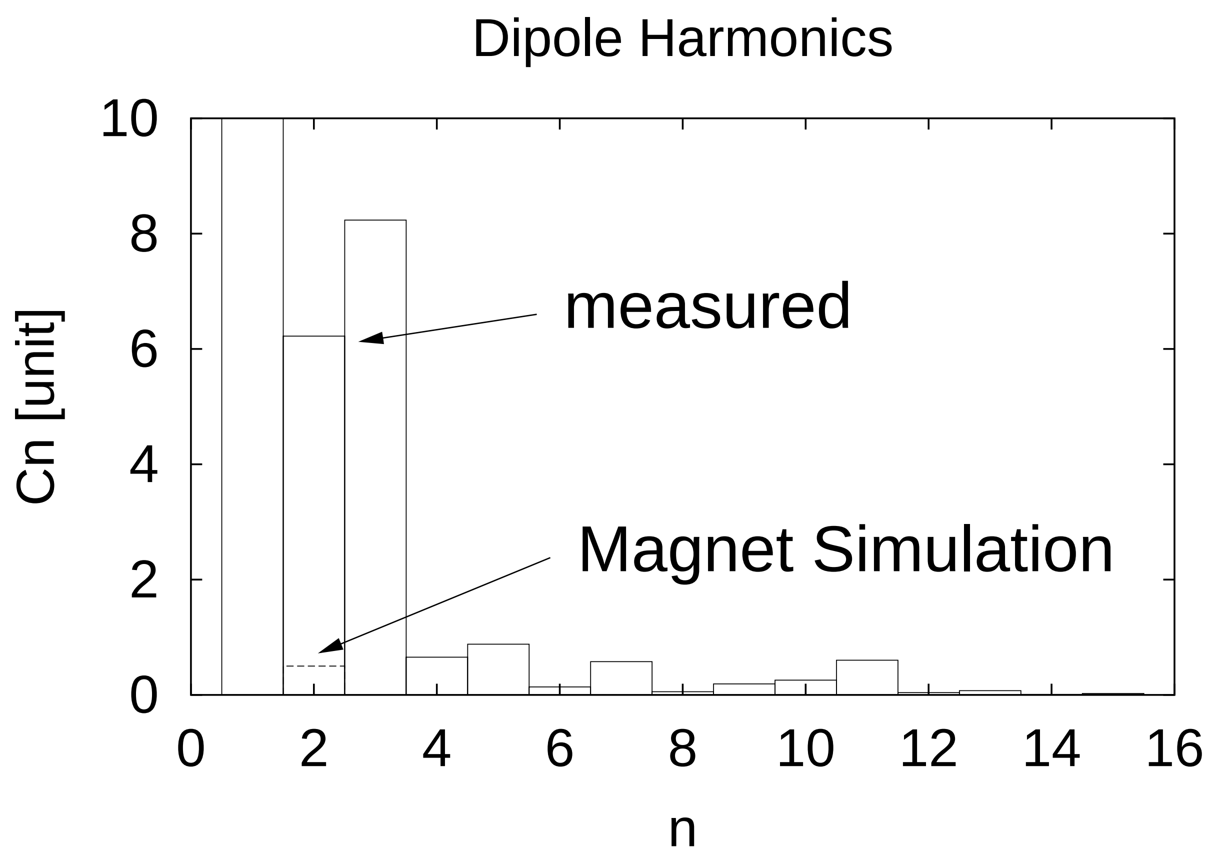


Figure 1: Harmonics found in a LHC dipole prototype. The quadrupole harmonic is rather large.

Noise coupled into the cable

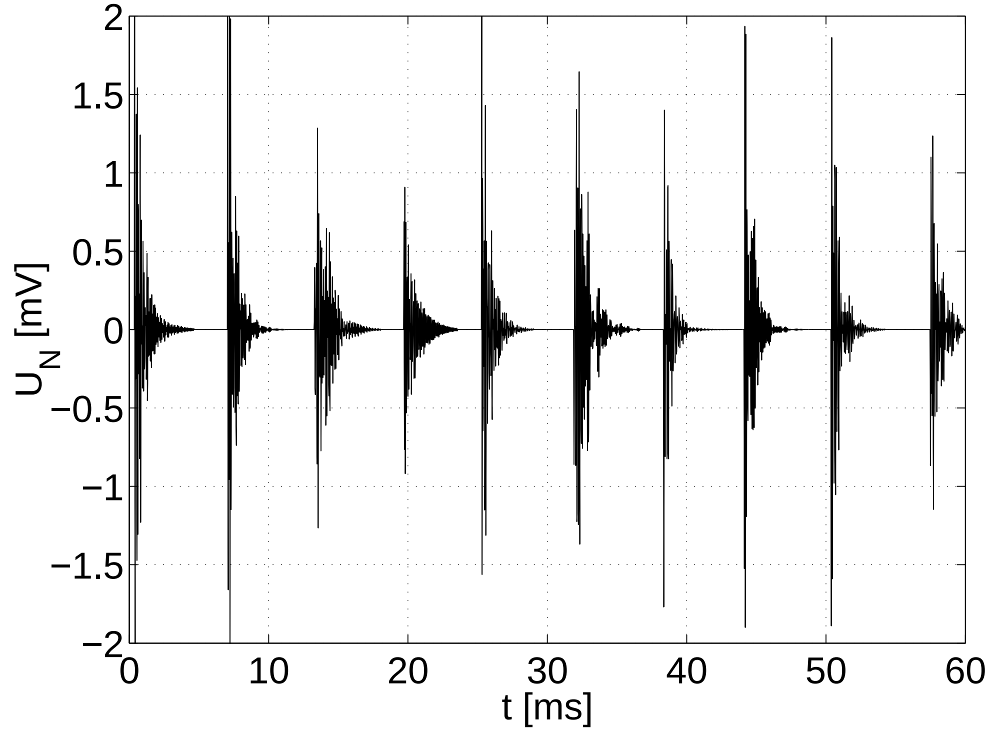
Noise model:

$$U_N(t) = \sum_{i=1}^{N} q_i e^{-r_i(t-t_{0i})} \sin\left[s_i\left(t - t_{0i}\right)\right].$$

$q_i,\ r_i,\ s_i\ldots$ vibration parameters

t_{0i} $\ldots$ start time of the peak

All parameters are set randomly.



Noise in the cable induced by the enviromental pollution of a switching power supply.

Table 1: Influence of the electrical noise on the harmonics

voltage q_{max} [mV]	0.1	1	10
$[Cn/C2 \cdot 10^4]$ $C_2 = 1.5\,T\,@\,R_{Ref}$	10^{-4}	10^{-3}	10^{-2}
$[Cn/C2 \cdot 10^4]$ $C_2 = 0.2\,T\,@\,R_{Ref}$	$7 \cdot 10^{-4}$	$7 \cdot 10^{-3}$	$7 \cdot 10^{-2}$

Conclusion

- A tool available allowing the simple investigation of imperfections
- Allows the study of noise
- The modularity allows to add new components