Influence of magnetic gradient fields on the Hall effect

B. Berkes
MCS Magnet Consulting Services
Wettingen, Switzerland

Summary

The behavior of the Hall effect in a Hall device exposed to a magnetic gradient field was already theoretically investigated several decades ago. However, due to the lack of very precise instrumentation at that time, particularly high resolution linear encoders and DVM's - both necessary to detect the small Hall voltage variations due to the magnetic gradient fields, prevented until now the experimental proof of the calculations.

For the first time measurements with Hall sensors with their supply current direction once perpendicular and then parallel to the magnetic gradient field direction were published recently [5]. Although these measurements were performed for different reason and on very low field and gradient level, their results gave some idea about the influence of magnetic gradient fields on the Hall voltage ([3], [4]) .

The basic equations from [3] - extended by additional calculations - were then used to determine the errors of the Hall voltage due to the influence of an undulator magnetic gradient field for point as well as for integral field measurements. In another example it was possible to show that due to the gradient fringe field of a dipole magnet, causing error terms with the first and second field derivatives, the measured effective field length will be always smaller than the true one.

1. Introduction (History)

The first theoretical investigation of the influence of magnetic gradient fields on the Hall voltage of a Hall sensor was made quite shortly after the introduction of the InSb and InAs Hall elements on the market [1]. However, the calculations were based on the assumption of constant internal resistance of the Hall element over its entire cross section and therefore did not reflect completely the actual physical situation.

Afterwards - and almost at the same time - more detailed calculations were presented independently by two authors ([2],[3]). While in [2] only the influence of a gradient field in the direction perpendicular to the Hall supply current was investigated, this influence was treated in [3] for both, gradient field acting perpendicular to the direction of the Hall supply current as well as parallel to it.

For reason of better understanding of what follows, the resulting equations 2.1 and 2.2 from [3] are reproduced here in their original form *). For the Hall supply current being orthogonal to the gradient direction one obtains

$$[\,U_h\,]_{ort} = U_{ho} \cdot [\,1-(b^2/12)\cdot 1/(B_o\cdot R_{so})\cdot(dB/dx)_o\cdot dR_s/dx)_o\,] = \qquad (1)$$

$$= U_{ho}\cdot[\,1+(F')_{ort}\,]$$

Replacing $(dR_s/dx)_o$ by $(dR_s/dB)_o\cdot(dB/dx)_o$ one shall get for the error term in (1) **)

$$(F')_{ort} = -(b^2/12)\cdot[(dB/dx)_o]^2\cdot[\,R_{so}(B=0)\cdot a_s\cdot 2\,)/R_{so}\,(B_o\,)] = \qquad (2)$$

$$= -(b^2/12)\cdot[(dB/dx)_o]^2\cdot a_s\cdot 2\,/\,(1+a_s\cdot B_o^2\,)$$

For the Hall current supply and gradient direction being parallel to each other, the resulting equation is

$$[\,U_h\,]_{par} = U_{ho}\cdot[\,1-(l^2/12)\cdot 1/(B_o\cdot R_{ho}\,)\cdot(dB/dx)_o\cdot(dR_h/dx)_o\,] = \qquad (3)$$

$$= U_{ho}\cdot[\,1+(F')_{par}\,]$$

$$(F')_{par} = -(l^2/12)\cdot[(dB/dx)_o]^2\cdot[\,R_{ho}(B=0)\cdot a_h\cdot 2\,)/R_{ho}\,(B_o\,)] = \qquad (4)$$

$$= -\,(l^2/12)\cdot[(dB/dx)_o]^2\cdot a_h\cdot 2\,/\,(1+a_h\cdot B_o^2\,)$$

---

*) The missing factor 2 in the error term of both equations in [3] has been already corrected.

**) In both papers ([2], [3]) and in this report the nonlinearity between the Hall voltage and the induction was neglected.

with

$$U_{ho} = k \cdot i_s \cdot B_o \equiv U_{DC} \qquad (5)$$

where the indices "o" refer to the values of the field (induction) and the internal resistances as well as their derivatives at the center of the Hall sensor, while b and l represent its width and length. The quantities a_s and a_h are so called coefficients of magnetoresistance for the supply current and Hall voltage side of the Hall sensor, respectively. In (5) is the Hall supply current denoted with i_s and k is a constant of proportionality between the Hall voltage and the induction (see also footnote on p. 1). U_{DC} is the Hall voltage in a homogenous magnetic field, being identical with the Hall voltage U_{ho} at the center of the Hall sensor. The indices "ort" and "par" at the error terms F' and F" are referring to the orthogonality and parallelism between the Hall supply current and the gradient direction.

2. Extended calculations

The equation (1) is, apart from the intentionally missing second order and higher derivative terms in the expansions for the magnetic field and internal resistance of the Hall sensor, identical with the result in [2] (see also [4]). However, the implementation of the second order terms into the error calculations of the Hall voltage may also be of importance for measurements in magnetic gradient fields, as it will be shown later. Therefore, extended calculations of the Hall voltage errors with the above mentioned second order derivative terms were carried out and their results are presented below

$$(F')_{ort} = -(b^2/24)\cdot[\,a_s \cdot 2 / (1+a_s \cdot B_o^2)] \cdot [(dB/dx)_o]^2 \cdot \qquad (2a)$$

$$\cdot\, [(2 / (1+a_s \cdot B_o^2)) + (1-a_s \cdot B_o^2)]/(1+a_s \cdot B_o^2)]$$

$$(F'')_{ort} = +(b^2/24) \cdot ([d^2B/dx^2]_o/B_o) \cdot (1-a_s \cdot B_o^2)/(1+a_s \cdot B_o^2) \qquad (6)$$

The expression for the errors in case of the parallelism between the Hall supply current and the gradient directions have the same form

$$(F')_{par} = -(l^2/24)\cdot[\,a_h \cdot 2 / (1+a_h \cdot B_o^2)]\cdot[(dB/dx)_o]^2 \cdot \qquad (4a)$$

$$\cdot\, [(2 / (1+a_h \cdot B_o^2)) + (1-a_h \cdot B_o^2)]/(1+a_h \cdot B_o^2)]$$

$$(F'')_{par} = +(l^2/24)\cdot ([d^2B/dx^2]_o/B_o) \cdot (1-a_h \cdot B_o^2)/(1+a_h \cdot B_o^2) \qquad (7)$$

At this point a remark concerning the influence of the errors (2a) and (4a) as well as (6) and (7) on the Hall voltage seems to be necessary. As expected, the coefficients of magnetoresistance a_g and a_h are influencing the magnitude of the errors. While F' decreases linearly with a_g and a_h (which recommends the use of GaAs or Si as well as of high-doped InAs Hall sensors), F'' remains almost unaffected by their magnitude. However, F'' still depends on the size (length or width) of the Hall sensor and therefore it is desirable to use for precision measurements in magnetic gradient fields a Hall sensor with dimensions as small as possible, what would also be in favor of smaller F'-values.

3. Experimental results and data analysis

It must be noted that for more than 30 years there were made no efforts to prove the theoretical results, obtained in [3], though they might have been at some magnetic field measurements of importance. Therefore, it seems to be necessary - especially with respect to measurements of strong gradient fields - to revitalize the problem .

For the mentioned purpose the results of the recent measurements [5], used also in this report,offered an opportunity for better understanding the magnetic gradient field influence on the Hall voltage. However, those results were obtained as a side effect, investigating in fact a completely different topic, namely material defects and related phenomena in high-temperature superconducting tapes.

The report [5] explains the measurements of inhomogenous magnetic fields produced by a copper meander sample, simulating the magnetic field profile generated by the magnetization currents in high-temperature superconducting tape. Two InSb Hall sensors with following data were used for the measurements:

Table 1.

Hall sensor	Active area (W x L) [μm^2]	Sensitivity [mV/T]	Nominal current [mA]
HP1	50 x 50	195.4	20
HP2	80 x 460	166.1	100

With each sensor two sets of measurements were made above the copper sample: a)with the Hall supply current direction perpendicular, and b) parallel to the gradient direction (moving direction *of the Hall sensors*)) . The measurement data with both Hall sensors are visible on Figs. 1 and 2.*

Analyzing the data vs. (2a) and (4a) one can immediately figure out that due to very small gradient values there will be, in spite of large a_s- and a_h-values for InSb, no detectable "gradient" errors at measurements with HP1 and HP2. Furthermore,one can observe that there is practically no difference in the field profiles obtained by the above mentioned two directions for the HP1 sensor. Comparing the experimental results in Fig. 1 with the equations (2a) and (4a) it is obvious that due to the equal dimensions (length and width) of the HP1 sensor the two sets of measurements shall be (except for the influence of a small difference between the values of the input and output coefficients of magnetoresistance) practically equal. In addition, the second order errors according to (6) and (7) will be, apart from the negligible differences between their magnitudes due to the above mentioned influence of the coefficients of magnetoresistance, the same.

The situation concerning HP2 measurements is, however, a more complex one.To understand fully the reason for the change of the amplitude and the profile of the curve obtained for the case with the Hall supply current and gradient direction being parallel, it is necessary to make use of both types of errors F' and F".

Because at $x = 0$ the field gradient is 0 , the error according (4a) will be also equal to 0 . However, the difference in the dimensions between the length and the width of the Hall sensor (see Table 1.) leads in these two cases to different results for the field amplitudes. In order to prove this statement, one can represent the field profile for HP1 (here are the errors extremely small, so the measurement data are - based on the Hall sensor calibration in a homogenous field - to a large extent correct) with the so called Glaser's "bell shape" function

$$B(x) = B_o \cdot [\, 1 + (x / x_o)^2 \,]^{-n} \qquad (8)$$

Based on the field data for HP1 from Fig. 1 one can evaluate x_o and n within a few percent as

$$x_o = 0.427 \text{ mm} \,, \; n = 1.336$$

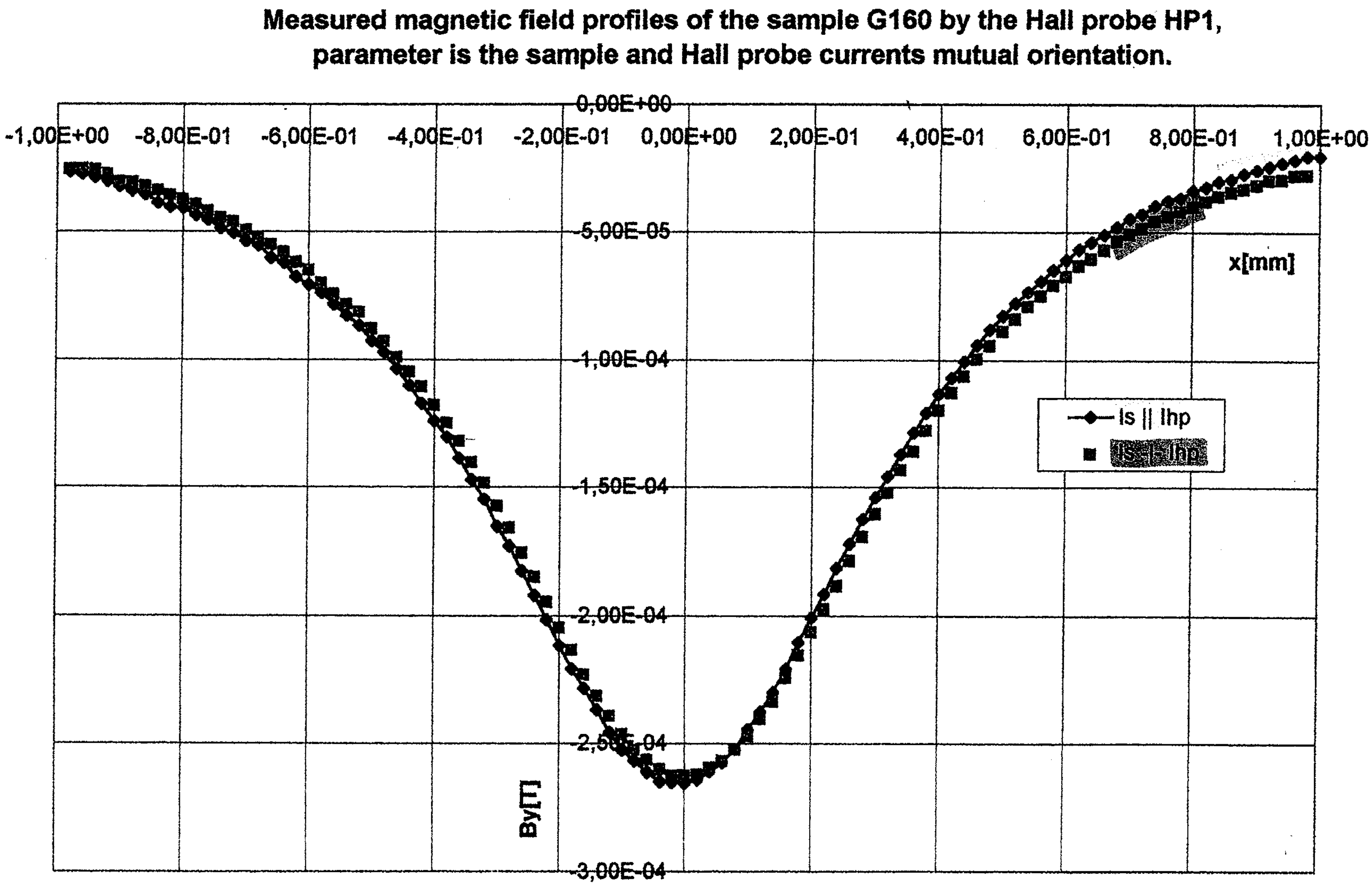
With these values it follows

$$[d^2B / dx^2]_o / B_o = -14.65 \text{ mm}^{-2}$$

**) The resolution of the positioning system was 0.5 μm .*

Fig. 1

(by courtesy of J. Kvitkovič)



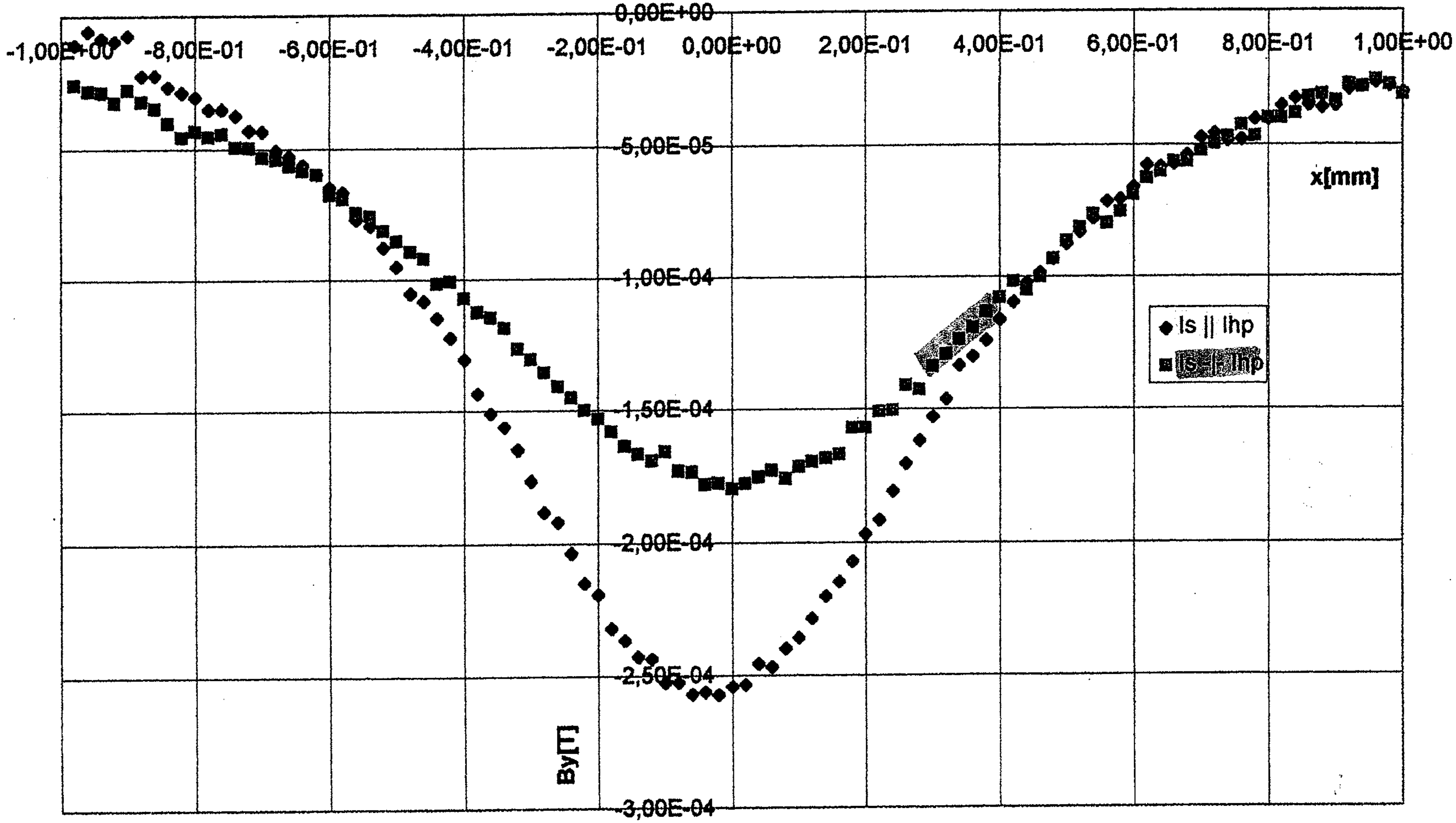
***green** - Hall supply current direction perpendicular to the field gradient*

***red** - Hall supply current direction parallel to the field gradient*

Fig. 2

(by courtesy of J. Kvitkovič)

Measured magnetic field profiles of the sample G160 by the Hall probe HP2, parameter is the sample and Hall probe currents mutual orientation.



green** - **Hall supply current direction perpendicular to the field gradient

red** - **Hall supply current direction parallel to the field gradient

and further, with the magnetoresistive term in (7) being practically equal to 1 (see next paragraph),

$$(F'')_{par} = +(0.46)^2 \cdot (-14.65) / 24 = -0.1292 \qquad (9)$$

*The above result means that in the case of parallel Hall supply current and gradient direction the field amplitude for x = 0 will be approx. 13% lower than at measurements with HP1 *). However, the above result (13%) is by more than a factor 2 smaller than the value (approx. 30%) obtained from the Fig. 2 . Because the influence of the magnetoresistive term in the equation (7) on the calculation result in (9) is - in spite of a large a_h, but due to very low B_o-values - negligible, the difference between the experimental data and the calculation result may be explained only by a larger value of the length l of the Hall sensor than that one used in the evaluation of (9).*

At the same time, one can show that in case of Hall supply current and gradient directions being orthogonal to each other, the error for HP2 sensor will amount approx. −0.4%. Because the corresponding error for the HP1 sensor amounts only −0.15%, the difference between the field amplitudes at x = 0 for measurements with HP1 and HP2 must be visible, which can be proven by comparison of the values on Figs. 1 and 2 .

*Following the field profile obtained by HP1 to both sides of x = 0 , one shall cross the points, where the second derivative will change its sign. Therefore, starting from this points the measurement values in Fig. 2 (HP2 sensor) will become - due to the bigger dimensions of HP2 compared to HP1 sensor - larger than at HP1. This phenomenon can be very well observed by comparison of the field profiles on Figs. 1 and 2 **).*

4. Hall voltage in an undulator gradient field

One of the most interesting questions in connection with the influence of the magnetic gradient field on the Hall voltage is, whether the measurements of undulator fields with strong gradients - as performed until to day - do exactly reproduce the field pattern of the undulator.

**) One has always to bear in mind that the Hall sensor is calibrated in a homogenous magnetic field.*

***) There is a slight difference between the scales of the abscissas on Figs. 1 and 2 .*

To understand what happens during the magnetic measurements of an undulator field by a Hall device (sensor), one shall anticipate a field function of the form

$$B_o = B_{max} \cdot \sin(2\pi x / X) \qquad (10)$$

with x being now the moving direction of the Hall sensor along the undulator; B_{max} is the field amplitude, X the field periodicity *) and the 1st and 2nd derivatives of (10) are

$$(dB/dx)_o = B_{max} \cdot (2\pi / X) \cdot \cos(2\pi x / X) \qquad (10a)$$

$$(d^2B / dx^2)_o = -B_{max} \cdot (2\pi / X)^2 \cdot \sin(2\pi x / X) \qquad (10b)$$

According to (10a) and (10b) it is obvious that in a magnetic gradient field each of the measured voltage data will be burdened with errors according to (2a) and (6) as well as to (4a) and (7) .

Contrary to the above statement, the field integrals over the full period will be free from both types of errors. To prove the above statement it is first of all necessary to convert the Hall voltage into the induction values. Due to the anticipated linear relation between the induction and the Hall voltage, one can deal directly with the above mentioned equations simply by exchanging U_h with B_{mea} and U_{ho} with B_o , which leads to

$$[B_{mea}]_{ort} = B_o + B_o \cdot (F')_{ort} + B_o \cdot (F'')_{ort} \qquad (11)$$

Introducing (10) through (10b) into (11) and using the equations for F' and F'', all three terms on the right side will contain a multiplier of the form sin $(2\pi x / X)$. For $B_{max} \leqslant 2T$ and small values of a_s, leading to $a_s \cdot B_o^2 \leqslant 1$, all other multipliers in (2a) and (6) will be always positive. Therefore, due to the sine-term multiplier, the integral values of the errors within the period [0, X] will be 0 .

As an example, we shall assume an undulator with X = 5 cm and B_{max} =1 T ; the field profile is measured with a SIEMENS cross-type GaAs Hall sensor KSY14 [6] (chip-dimensions: 370 x 370 μm^2, active area: approx. 200 x 200 μm^2 [7] , a_s = 4% / T^2 [B1]); the Hall supply current and the gradient directions are mutually orthogonal . Because within the half-period [0, X/2] the term $B_o \cdot [(dB/dx)_o]^2$ has two maxima at approx. X/10 and 4X/10 , while the maximum of the term $B_o \cdot [d^2B/dx^2]_o / B_o$ appears at x = X/4 ,the corresponding maximum errors with the above undulator and Hall sensor data will amount to

*) In the following considerations the fringe field areas of the undulator are omitted.

$[B_o \cdot (F')_{ort}]_{max} = -2.43 \cdot 10^{-6}$ T and $[B_o \cdot (F'')_{ort}]_{max} = -2.63 \cdot 10^{-5}$ T

The above results show a negligible influence of both type of errors on the measurement, which is mainly due - as already stated above - to the small relevant sensor's data.

Because within the half-period [X/2, X] both terms, $B_o \cdot [(dB/dx)_o]^2$ and $B_o \cdot [d^2B/dx^2]_o/B_o$, will change the signs, but their amplitudes will remain the same, the sum of the maximum point errors within [0, X] will be 0 . Based on this fact, one can conclude that within [0, X] the sum of all errors in B , occurring at individual measurement points, will be 0 .

5. Effective field length of a dipole magnet

The advance in the production technology of the Hall sensors over the past 30 to 40 years enabled among other improvements of various sensor's parameters also the reduction of sensor's size. However, in the 60 th and 70 th of the past century, the common Hall sensor dimensions (length x width) were in the range of several mm^2 *). Therefore, it can be expected that in such a case the errors according to (2a) and (6) as well as to (4a) and (7) will not be anymore negligible (Par. 4.), but will (based on the measurement data) lead to false conclusions.

As an example , one can assume a dipole magnet with an airgap of $g = 5$ cm and a magnetic induction of $B_{DC} = 1$ T in the homogenous field region. The induction is measured with a SIEMENS InAsP Hall sensor of the type FC34 (sensor-dimensions: 22 x 12 mm^2, active area: 15 x 6 mm^2 [B2], $a_s = a_h = 60\ \% / T^2$). while in the homogenous field region there will be no measurement errors due to the field gradient of the first and second order, those errors will - however - appear at the magnet ends. Combining the measurement data from both magnet fringe field regions, one may describe the function obtained that way with

$$B(x) = B_{DC} \cdot e^{-a**2.x**2} \qquad (12)$$

Using now (11), with $B_o = B(x)$ in the fringe field region, the equation for the effective magnetic field length can be written as

---

*) Even at present time, such Hall sensors are still in use in many laboratories around the world.

$$\int_{-\infty}^{+\infty} B_{mea} \cdot dx = [\, B_{DC} \cdot L_{eff} \,]_{virt} = [\, B_{DC} \cdot L_{eff} \,]_{true} +$$

$$+ \int_{-\infty}^{+\infty} B(x) \cdot F' \cdot dx + \int_{-\infty}^{+\infty} B(x) \cdot F'' \cdot dx \qquad (13)$$

With the abbreviation

$$\int_{-\infty}^{+\infty} [\ldots] \cdot dx = INT$$

and neglecting in the first approximation the magnetoresistive terms in (2a) and (6) as well as in (4a) and (7) it follows with the corresponding first and second order derivatives of (12) for the effective magnetic field length errors *)

$$[B_{DC} \cdot (F')_{ort}]_{int} = INT - (b^2/24) \cdot a_s \cdot 2 \cdot B_{DC} \cdot (dB/dx)_o]^2 \cdot 3 = \qquad (13a)$$

$$= -(B_{DC})^3 \cdot (b^2/4) \cdot a_s \cdot 2 \cdot (\pi \cdot \ln 2)^{1/2} / (3^{3/2} \cdot g) =$$

$$= -0.06134 \cdot (B_{DC})^3 \ mm \cdot T$$

$$[B_{DC} \cdot (F'')_{ort}]_{int} = INT + (b^2/24) \cdot [d^2B/dx^2]_o = \qquad (13b)$$

$$= -B_{DC} \cdot (b^2/12) \cdot (\pi \cdot \ln 2)^{1/2} / g =$$

$$= -0.83746 \cdot B_{DC} \ mm \cdot T$$

$$[B_{DC} \cdot (F')_{par}]_{int} = -0.38339 \cdot (B_{DC})^3 \ mm \cdot T \qquad (13c)$$

$$[B_{DC} \cdot (F'')_{par}]_{int} = -5.23415 \cdot B_{DC} \ mm \cdot T \qquad (13d)$$

Dividing each of the above results by B_{DC} one shall obtain the effective magnetic field length errors (dimensions in mm) for mutually orthogonal and parallel directions between the Hall supply current and the field gradient. Because of the negative sign in all 4 cases it is obvious that the measured effective magnetic field length will be always smaller compared to the true one, i.e. neglecting (13a) through (13d), respectively.

****) For the evaluation of various integrals of (12) one may consult ([B3], p. 66 / formulae 2., 3. and 4.).***

From the above calculation one can conclude that - depending of the orientation of the Hall sensor vs. the field gradient and the effective magnetic field length - errors in the order of 0.1% to 2% may be expected. This is further not sensational, because the reduction in the effective field length can be under beam operating conditions compensated by slight increase of the gap induction.

Finally, it is worth to mention that the above results may be improved by taking into considerations also the magnetoresistive terms in the error equations. As an example, one can make use of (7), where the term $[(1 - a_h \cdot B_o^2) / (1 + a_h \cdot B_o^2)]$ ***is simply replaced by*** $[1 - 2\, a_h \cdot (B_o)^2 + (a_h)^2 \cdot (B_o)^4]$. ***Inserting then for*** B_o ***the function*** $B(x)$ ***according to (12), one obtains***

$$[B_{DC} \cdot (F'')_{ort}]_{int} = INT + (l^2/24) \cdot [d^2B/dx^2]_o = \cdot [1 - 2\, a_h \cdot (B(x))^2 + (a_h)^2 \cdot (B(x))^4] \qquad (14)$$

The evaluation of (13) leads to

$$[B_{DC} \cdot (F'')_{par}]_{int} = -\, 2.45052 \cdot B_{DC}\ \ mm \cdot T \qquad (14a)$$

or, with other words, the error of the effective magnetic field length is now only approx. half as big as calculated by (12d).

6. Recommendation for measurements in gradient fields

Performing high-precision magnetic field measurements in gradient fields with Hall sensors having larger active areas and coefficients of magnetoresistance, one has to recalculate the raw data taken at discrete points in order to obtain the U_{ho}***-(***U_{DC}***-)values. This is primarily necessary in case of irregular gradient fields, where an integral result is required, because (as already mentioned previously in this report) the Hall sensors are calibrated in a homogenous field. Therefore, taking the raw data in gradient fields as correct one may have disregarded possible errors due to the first and second field derivatives.***

It shall to be pointed out here that the correct measurement data may be obtained (using the equations in par. 2.) only by an iterative process. At the beginning one shall set - by neglecting the error terms (zero's approximation) - $(U_{ho})_o$ ***equal to the raw Hall voltage*** U_h ***. Converting now the Hall voltage into the induction*** $(B)_o$ ***and implementing it into the equations with the error terms one shall obtain the first approximation of the*** $(U_{ho})_1$***, which leads to*** $(B)_1$ ***and so on. For the final result only one or two iteration steps shall be necessary, if the error terms are not too big.***

- 10 -

Acknowledgments

The author is very much indebted to Mr. J. Kvitkovič from the Slovak Academy of Sciences in Bratislava (SK) for his consent to use the measurement data and figures from his report. Best thanks deserves also Dr. W. Joho from the Paul Scherrer Institute in Villigen (CH) for the discussion concerning the influence of the undulator gradient field on the Hall sensor.

IMMW-12
ESRF, Grenoble
October 2001

REFERENCES

[1] Brunner, J.: *Der Halleffekt im inhomogenen Magnetfeld, Solid-State Electr. 1960, Vol. 1, pp. 172-175*

[2] Robins, J. K.: *Linearization of Hall probes by a new method, and an estimate of errors from spatial field variations, Internal Report NIRL/M/81, Rutherford High Energy Laboratory, March 1965*

[3] Berkes, B.: *The behaviour of Hall probes in magnetic gradient field, Proc. Int. Symp. on Magnet Technology, SLAC / Stanford University, Sep. 1965, pp. 483-486*

[4] Clark, D.J. et al.: *The criteria and technique of magnetic field measurement on UCLA 50 MeV spiral ridge cyclotron, Appendix I, Nucl. Instr. & Methods 14, 1962, p. 333*

[5] Kvitkovič, J. - Majoroš, M.: *Hall magnetometry of BiSrCaCuO high temperature superconducting tapes, Proc. Int. Symp. on Cryogenics in Sci. and Industry, CERN, Geneva, Oct. 1996*

[6] Wolfrum, M.: *KSY14 - The ultra-flat, versatile Hall sensor, Siemens Components XXV, no. 5, 1990, pp. 167-172*

[7] Eberle, W. *Private communication (Infineon)*

BIBLIOGRAPHY

*[B1] ***** :* *Magnetic sensors data book, Siemens AG, München 1989*

*[B2] ***** :* *Siemens Hall Generators, Siemens & Halske AG, München 1959*

[B3] Bronstein, I.N.- Semendjajev, K.A.: *Taschenbuch der Mathematik, 22. Auflage, Verlag Harri Deutsch, Thun u. Frankfurt 1985*