X-ray optics
Crystal optics

Jürgen Härtwig

ESRF X-ray Optics Group, Crystal Laboratory
What was already presented (among others)?

Physics of the electron beam source (Boaz Nash)

Physics of X-ray radiation production and transport (Manuel Sánchez del Río)

Multilayers in synchrotron optics (Christian Morawe)

Energy resolving detectors for X-ray spectroscopy (John Morse)

So we continue today with the X-ray optics
Outline

1. Introduction
2. Monochromators
3. Some properties of asymmetrical reflections
4. Shortly about high energy resolution
5. Crystal quality and how to measure it
6. Plane or divergent, monochromatic or polychromatic waves in our experiments?
1. Introduction

Some questions I plan to discuss and maybe to answer:

Which kind of monochromators are used? How may I change the energy resolution, beam divergence, beam dimension?

May the Bragg diffraction geometry have an influence on the coherence?

Role of source size and angular source size. Influence on transversal coherence, resolution etc.?

Are “Imaging quality”, “focusing properties”, “coherence preservation” related?

What is a “highly perfect” crystal? What are the lowest strains that we can measure?

Is there a “highly parallel (monochromatic)” beam? Is there a “nanometric parallel” beam?

We need to define what a “plane” or a “monochromatic” wave could be in the real experimental life. How may we approximate them? etc.
Optical system / experimental set-up:

Source ⇒ optical elements ⇒ sample ⇒ optical elements ⇒ detector

The task of the optics:

To transform the beam to obtain the best matching with the experiment; not loosing the good properties of the beam after its creation.

It acts on:
- shape
- wavelength/energy
- divergence
- polarisation
- coherence
“No optics is the best optics”!? 

Yes, but ...

In principle possible - all in vacuum, working with non-modified “white” or “pink” beam. However, not very useful. We may need e.g. monochromatisation, focussing, ... .
A whole zoo of optical elements

- slits, pinholes
- filters, windows
- mirrors (reflectivity based)
- beam splitter monochromators (crystals)
- monochromators/collimators/analysers (crystals, multi-layers - often also called “mirrors”)
- phase plates (phase retarder, polarizer) (crystals)
- lenses, zone plates
- combined elements (ML gratings, Bragg-Fresnel-lenses)
- etc.

Mirrors and monochromators, collimators, analysers
flat, but also bent for collimation, focussing, image magnification, ...
Main physical effects used in X-ray optics were discovered in the first years starting from the discovery of X-rays by C. W. Röntgen:

- absorption (Röntgen 1895/96 → filters)
- Bragg-diffraction (Laue 1912 → monochromators, etc.)
- specular reflection (Compton 1922 → mirrors)
- refraction (Larsson, Siegbahn, Waller 1924 → later lenses)

Properties of double and many-crystal set-ups:
Jesse W. M. DuMond, Physical Review, 52, 872-885 (1937)
DuMond graphs, dispersive and non-dispersive set-ups, channel cut crystals (later invented as “Bonse-Hart-camera”), four crystal spectrometer (later invented as “Barthels monochromator”), etc.

But - many newer developments
Quite a lot of literature

Overviews:


Short remark concerning beam dimensions

Few years ago – micro-beams were modern,
now – nano-beams are in vogue.

But - we need all kind of beams:

large beams (decimetre sized)
and small ones (nanometre sized),
“parallel”, divergent and focussed beams.
Beam dimensions - example: paleontology

Examples from Paul Taftoreau
Nearly 4 orders of magnitude in dimension

Without scanning

Without magnification

Multi scale experiments!
Further large scale objects:
   the monochromator crystals

For their tests we like to use:
wide beams, if possible at least 10 × 45 mm² (V × H),
with a “good” spatial resolution ~ 1 μm

The above field of view and resolution needs sensors with:
10,000 × 45,000 pixels, this is 450 Mega pixels

Not yet on the market (?)
2. Monochromators

but also by: filters +
source spectrum +
s螀ntillator screen
response spectrum
(Paul Tafforeau/ID19)

E = 8 keV

forbidden area

R = 1

Crystals

High-resolution ML's
Low-Z
High-Z
ML's
depth-graded ML's
(Mirrors)

Integrate

High resolution
10⁻⁸ and below

Ch. Morawe
Bragg diffracting X-ray optical elements like monochromators, analysers, etc.

Manufactured from dislocation free crystals.

Mostly used: Si, Ge, C* (locally dislocation free diamond).

We mainly use silicon.

They must be tailored into monochromators etc.

Orientation, cutting, lapping, polishing and etching.

Strain free crystal preparation.

Accurate and stable mounting.

Adequate cooling scheme.
Crystal laboratory:

Manufacturing of nearly all perfect Si (and few Ge) crystal monochromators & analysers, etc for all ESRF beam lines, CRG beam lines and external laboratories.

Silicon pieces are made from float zone silicon ingots with 100 mm diameter (Wacker).

More than 1.5 tons of silicon single crystal material has been processed in more than twenty years.
Which types of monochromators are used?
Single crystal monochromators - beam splitter monochromators

Double crystal monochromators

Reflection (Bragg) and transmission (Laue) geometry used
Single crystal monochromators -
beam splitter monochromators

Double crystal monochromators

Reflection (Bragg) and
transmission (Laue)
geometry used
Few theory and definitions

Reflectivity (and transmissivity) curve of a crystal

**Theory**
Plane & monochromatic incoming wave, varying the angle of incidence counting the diffracted photons

**Real situation - experiment**
Convolution (autocorrelation) of reflectivity (or transmissivity) curve with other reflectivity curves, or/and wavelength (energy) distribution, or/and divergence distribution (instruments/apparatus function)

\[ R + T + A = 1 \]
Numerical example (using XOP):
**double monochromator non-dispersive (+,-)**
double crystal set-up in Bragg case

two 111-Si reflections

known since: C. G. Darwin,
*The theory of X-ray reflection*,
Phil. Mag. 27, 315, 675 (1914)

However, in operation - mostly fixed angle
Reflectivity curves, with stronger absorption

Prins-Darwin or reflectivity curve
Si 111, 8keV, 5cm thick plate

$w^\theta_n = 7.5''$

How to measure it? Not so easy! See later.
Double crystal monochromators

Incident and exit beams have the same direction

In working position: fixed angle (mostly); multiplication of two reflectivity curves
Two variants

Fixed exit monochromator
- more than one movement necessary
Two variants

Fixed exit monochromator
- more than one movement necessary
Two variants

Fixed exit monochromator
- more than one movement necessary

Channel-cut monochromator
- NOT fixed exit
- naturally aligned
- weak link plus piezo movement for detuning etc.
Two variants

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**Fixed exit monochromator**
- more than one movement necessary

**Channel-cut monochromator**
- NOT fixed exit
- naturally aligned
- weak link plus piezo movement for detuning etc.
“Generic” cryogenically cooled channel-cut double crystal monochromator

no fixed exit

high heat-load applications

Crystal assembly (simplified for visibility)
Possible problem – “higher harmonics”

if \((h,k,l)=(nh',nk',nl')\)

\[ 2d_{nh',nk',nl'} \sin \theta_B = \lambda/n \]

\[ \text{Si} \ (2n,2n,0), \ \lambda_1 = 1.6 \text{Å}, \ \lambda_2 = 0.8 \text{Å}, \text{etc.} \]

Refraction correction (middle of the reflection domain):

\[ \Delta \theta_0 = \frac{r_0 \lambda^2 F_0}{2 \pi V_{uc} \sin 2\theta_B} \left( 1 - \frac{\gamma_h}{\gamma_0} \right) \]

\[ \gamma_0 = \sin(\alpha + \theta_B), \quad \gamma_h = \sin(\alpha - \theta_B) \]
Reduction of higher harmonics (if no mirrors may be used) 
monochromator detuning

At detuned position 
(slightly misaligned)

Smaller band in 
angle and energy
Synchrotron optics: Multilayer high flux monochromators

- Two bounce optics
- 100x larger bandwidth compared with Si(111)
- Harmonics suppression due to refraction and filling factor
How may I change the beam dimension, the beam divergence, the energy resolution?
3. Some properties of asymmetrical reflections

Up to now we looked at symmetrical cases of Bragg diffraction

- $\alpha = 0^\circ$
  - symmetrical Bragg case (reflection case)
- $\alpha = 90^\circ$
  - symmetrical Laue case (transmission case)

$\alpha$ - angle between lattice planes and surface
With asymmetric reflections - possibilities to change the beam width and the divergence for a single crystal reflection.
Of course, the same works also other way around.

So we have possibilities to act on the beam dimension (expansion, compression), as well as on the beam divergence (smaller, larger)
But all is related and things have their price

Asymmetry factor:
\[ b = \frac{\sin(\theta_B + \alpha)}{\sin(\theta_B - \alpha)} = \frac{\gamma_0}{\gamma_h} \]

This example: \( \alpha > 0, \ |b| < 1 \):
- \( L_{\text{in}} > L_{\text{out}} \) and \( w_{h,\text{in}} < w_{h,\text{out}} \)
- \( w_{h,\text{in}} L_{\text{in}} = w_{h,\text{out}} L_{\text{out}} = \text{constant} \)
- Right asymmetry (\( \alpha < 0 \)) - less divergence possible to obtain

Relation to symmetrical reflections
- \( w_{h,\text{in}} = |b|^{-1/2} w_{h,\text{sym}} \)
- \( w_{h,\text{out}} = |b|^{1/2} w_{h,\text{sym}} \)
Some “philosophy”

With flat crystals we may change the divergence. We may decrease (or increase) it. Not “more parallel”, but less divergent! This is collimation*).

Focussing needs convergent beams. We can not focus with flat crystals in a classical way. It works with bent crystals. Often other X-ray optical elements are more efficient for this (future lectures?!).

*) The word "collimate" comes from the Latin verb collimare, which originated in a misreading of collineare, "to direct in a straight line". (Wikipedia)
BUT ...

\[ w_h^{in} L_{in} = w_h^{out} L_{out} = \text{constant} \]

This is too simple, hand-waving “derivation”. We need an additional dimension for the phase space. Besides size and angle also wavelength (or energy) is needed

Let us start from the basics
\[ \Psi_0 = \theta_B + \alpha - \pi/2 + \Delta\theta_{\text{in}} \]
\[ \Psi_h = \theta_B - \alpha + \pi/2 \pm \Delta\theta_{\text{out}} \]

From electrodynamics (and dynamical diffraction theory) we know that:

For the wave vectors outside the crystal:
\[ \tilde{K}_h \neq \tilde{K}_0 + \tilde{h} \]

But for the \textit{tangential components} - continuity:
\[ \tilde{K}_{ht} = \tilde{K}_{0t} + \tilde{h}_t \]

And remember, wave vectors depend on wavelength: \[ K = f(\lambda) \]
For small $\Delta\theta_{\text{in}}$, $\Delta\theta_{\text{out}}$ and $\Delta K$ we obtain (for Bragg and Laue case!):

$$|\Delta\theta_{\text{out}}| = \frac{\sin(\theta_B + \alpha)}{\sin(\theta_B - \alpha)} \Delta\theta_{\text{in}} + \frac{\cos(\theta_B - \alpha) - \cos(\theta_B + \alpha)}{\sin(\theta_B - \alpha)} \frac{\Delta K}{K}$$

The divergence $\Delta\theta_{\text{in}}$ and polychromaticity $\Delta K/K$ of the incoming beam contribute to the divergence $\Delta\theta_{\text{out}}$ of the outgoing beam.

An increased divergence $\Delta\theta_{\text{out}}$ of the outgoing beam (with respect to that if the incoming beam) means:

the source is virtually closer, or the source size is virtually larger, or the angular source size is virtually larger.

**Special case 1**

Monochromatic, divergent incoming radiation

$$\Delta K = 0, \Delta\theta_{\text{in}} \neq 0$$

$$\Delta\theta_{\text{out}} = |b| \Delta\theta_{\text{in}}$$

or

$$w_{h_{\text{out}}} = |b| w_{h_{\text{in}}}$$
Special case 2

Polychromatic, parallel incoming radiation

\[ \Delta K \neq 0, \Delta \theta_{in} = 0 \]

\[ \Delta \theta_{out} = \frac{\cos(\theta_B - \alpha) - \cos(\theta_B + \alpha)}{\sin(\theta_B - \alpha)} \frac{\Delta K}{K} \]

Remember, “our” beams often are rather close to plane waves, but rather polychromatic

2.1. \( \Delta \theta_{out} = 0 \) if \( \cos(\theta_B - \alpha) = \cos(\theta_B + \alpha) \) if \( \alpha \equiv 0 \) (sym. Bragg case!)

Only in the symmetric Bragg case the beam divergence is conserved for a polychromatic beam!!!
Only in that case coherence is conserved!!!
Only in that case focussing is not perturbed!
Only in that case highest geometrical resolution possible!

2.2 \( \Delta \theta_{out} \neq 0 \) for all other cases

A divergent, polychromatic beam is transformed in a even more divergent, polychromatic beam!
Source size and angular source size are crucial parameters with respect to the character of the wave “seen” by the sample.

\[ \text{angular source size } \delta \quad (\delta = s/p) \]

not source divergence!!

The angular source size is important for further physical properties:

- the geometrical resolution for imaging,
- the transversal coherence length,
- the demagnification limit of a “lens”.
Image blurring due to non-zero source size

angular source size: \( \delta = \frac{s}{\rho} \)

\[
\begin{align*}
\text{geometrical resolution: } \rho &= \frac{q}{s} \frac{s}{\rho} = q \delta \\
\end{align*}
\]

Spatial coherence

Transversal coherence length: \( l_T = \frac{1}{2} \lambda \frac{p}{s} = \frac{1}{2} \lambda / \delta \)
Magnification, demagnification, focussing properties/quality

Geometrical demagnification, source size limit:

\[ \rho_G = q \frac{s}{\rho} = q \delta \]

Diffraction limited focusing:

\[ \rho_{DL} = 1.22 \frac{\lambda}{\sin \alpha} \]

(graph: J. Susini)
4. Shortly about high energy resolution

One way:
Sophisticated many crystal set-ups.

Example ID18:
A two-step collimation with low-index asymmetric reflections followed by a two-step angular analysis with high-index asymmetric reflections.

Idea:
High-resolution optics for Nuclear Resonance Scattering

“0.5 meV” monochromator ($\Delta E/E \approx 3.5 \cdot 10^{-8}$)

(theoretically expected performance)

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1st crystal: Si(400)
$\Theta_B = 18.469^\circ$
asymmetry: $|b| = 0.18$
$\Theta_{in} = 5.4^\circ$
$\Theta_{out} = 31.6^\circ$
angular acceptances:
$\Delta\Omega_{in} = 20 \mu$rad
$\Delta\Omega_{out} = 3.6 \mu$rad
footprint: 6.4 mm

2nd crystal: Si(400)
$\Theta_B = 18.469^\circ$
asymmetry: $|b| = 0.18$
$\Theta_{in} = 5.4^\circ$
$\Theta_{out} = 31.6^\circ$
angular acceptances:
$\Delta\Omega_{in} = 20 \mu$rad
$\Delta\Omega_{out} = 3.6 \mu$rad
footprint: 33 mm

3rd crystal: Si(12 2 2)
$\Theta_B = 77.533^\circ$
asymmetry: $|b| = 9.8$
$\Theta_{in} = 27.6^\circ$
$\Theta_{out} = 2.7^\circ$
angular acceptances:
$\Delta\Omega_{in} = 0.72 \mu$rad
$\Delta\Omega_{out} = 7 \mu$rad
footprint: 41 mm

4th crystal: Si(12 2 2)
$\Theta_B = 77.533^\circ$
asymmetry: $|b| = 3.2$
$\Theta_{in} = 35.4^\circ$
$\Theta_{out} = 10.4^\circ$
angular acceptances:
$\Delta\Omega_{in} = 1.3 \mu$rad
$\Delta\Omega_{out} = 4 \mu$rad
footprint: 3.3 mm

---

acceptance: 130 nano-rad
divergence: 65 nano-rad
vertical size: 0.6 mm
divergence: 2 μrad
vertical size: 0.6 mm
Other possibility – **back scattering geometry**

(Bragg angle close to 90 deg)

with **high order reflections** (large $h, k, l$)

$$\lambda = 2d_{hkl} \sin \theta_B$$

$$\Delta \lambda = \Delta \theta \cdot 2d_{hkl} \cos \theta_B$$

$$\frac{\Delta \lambda}{\lambda} = \frac{\Delta E}{E} = \frac{\Delta \theta}{\tan \theta_B}$$

a) $\Delta \theta \rightarrow$ a few $10^{-8}$ rad

b) $\theta \rightarrow$ 90 deg [$\cotg(\theta) \rightarrow 0$]

Silicon @ 20KeV, $h=k=l=13$, $(\Delta E/E) \sim 10^{-8}$, $\Delta E \approx 0.5$ meV,

**ID16, ID28**
5. How high crystal quality and how to measure it?

Crystal quality - limit in high energy resolution, ...(?)

Influence on coherence preservation, image quality, focussing efficiency, ...
The Avogadro constant: Counting atoms in a single-crystal $^{28}\text{Si}$ sphere

Peter Becker
Physikalisch-Technische Bundesanstalt
Braunschweig

The new SI
Royal Society London, 24-25 Jan 2011
\[ N_A = \frac{M_{Si} \cdot V_{Kagel}}{\sqrt{8 \cdot d_{220}^3 \cdot m_{Kagel}}} \]

Lattice parameter

variations of the (220) lattice-plane spacing along the 50 mm, 5 mm from top

\[ \Delta d/d = 5 \times 10^{-9} \]

\[ u_r(d_{220}) = 3.6 \times 10^{-9} \]

No lattice strain within the \( \pm 5 \times 10^{-9} \) survey resolution.
Crystal interferometers

\[ N_A = \frac{M_{Si} \cdot V_{Kugel}}{\sqrt{8 \cdot d_{220}^3 \cdot m_{Kugel}}} \]

Lattice parameter

Analyzer displacement up to 50 mm
(~10^7 lattice planes)

Feedback loops provide:
- picometer positioning
- nanoradian alignment
- Sub-nanometer movements

Exact displacements sensed via
additional laser interferometry and via capacitors

\[^{28}\text{Si} \] interferometer, analyzer crystal 50mm
Diamonds at the ESRF

From the very beginning of the ESRF used as phase plates and monochromators.

Now locally dislocation- and stacking fault free material available.

Which is the level of local residual strains?
Future MX BL ID30A (MASSIVE)

Figure 2.7. Layout of the optical elements of ID30A in OH2. (1) represents a vertically focusing white beam Compound refractive lens (CRL), (2) (111)-cut diamond monochromators, (3) a Si(111) monochromator and (4) horizontal focusing multilayer. The high power primary slits are located in the first optical hutch (OH1).
X-ray diffraction topography

White beam topograph in transmission
(work with Fabio Masiello)
110-oriented plate
supplier: Paul Balog/Element Six

Dislocation free areas of 6x4mm² and more!!!

Locally crystal quality close to that of silicon, quantitatively confirmed by double crystal topography

But are we able to measure weak strains quantitatively?
Quantitative analysis strain analysis

110-oriented crystal plate
effective misorientation map basing on one topograph

20keV, Si [880] C* [660], detection limit: $\delta \Theta > 8 \cdot 10^{-9}$

The effective misorientation is of the order of $4 \times 10^{-8}$ for a region of interest of $0.5 \times 0.5$ mm$^2$ and $1 \times 10^{-7}$ in a region of $1 \times 1$ mm$^2$

Sample is slightly bent due to the non-homogeneous dislocation distribution!

work with Fabio Masiello
6. Plane or divergent, monochromatic or polychromatic waves

Two basic questions:

What are “plane” or “divergent” waves?

What are “monochromatic”, “polychromatic”, “white” beams/waves?

For our monochromator and/or single crystalline sample!

Reminder of basic physics:

Plane wave - infinite extend, wave front is plane, one wave vector perpendicular to it, delta-function in k-space.

Monochromatic wave - wave train of infinite length, infinitely sharp spectral line, delta-function in ω-space.

They do not exist in nature!!!
The full width at half maximum of the reflectivity curve, a good reference for our sample, monochromator, ... , to define the character of a wave.

**FWHM in the angular space:**

\[
W_\theta = \frac{2 |P| \sqrt{\chi_h \chi_{\bar{h}}} \sqrt{\gamma_h}}{\sin 2\Theta_B \sqrt{\gamma_0}}
\]

**FWHM in the wavelength space:**

\[
W_\lambda = W_\theta \lambda \cot \Theta_B
\]
Example of a typical crystal/monochromator reflection:
silicon, 111 reflection, Bragg case, thick crystal

<table>
<thead>
<tr>
<th>energy</th>
<th>$w_h^\circ$</th>
<th>$w_h^{\lambda/\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 keV</td>
<td>7.6 arcsec</td>
<td>$1.5 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>20 keV</td>
<td>2.9 arcsec</td>
<td>$1.5 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

Those are to be compared with source properties:

- wave length spread of the source $\Delta \lambda$ ($\Delta \lambda/\lambda$)
- angular source size $\delta$ ($\delta = s/p$)
- not source divergence!!
relative wave length spread of the source $\Delta \lambda / \lambda$ ($\Delta E / E$)

$\ll$ FWHM in the wavelength space $w_\lambda$

“monochromatic” wave

angular source size $\delta = s/p$

$\ll$ FWHM in the angular space $w_\Theta$

“plane” wave
Practical examples:

relative wave length or energy spread of the source $\Delta \lambda / \lambda$

<table>
<thead>
<tr>
<th>source</th>
<th>$\Delta \lambda / \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si 111 double mono</td>
<td>$2 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>laboratory (e.g. CuK$_{\alpha 1}$)</td>
<td>$3 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>white beam</td>
<td>$1 \cdots 10$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>energy</th>
<th>$w_{h \lambda} / \lambda$</th>
</tr>
</thead>
<tbody>
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<td>$1.5 \cdot 10^{-4}$</td>
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<tr>
<td>20 keV</td>
<td>$1.5 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

SR-white beams: $\Delta \lambda / \lambda \gg w_{h \lambda}$ really polychromatic

"monochromatic" beam (SR or laboratory):

$\Delta \lambda / \lambda \ll w_{h \lambda}$ often not monochromatic

(for all reflections narrower than the $1.5 \cdot 10^{-4}$ for silicon 111)

special effort is necessary to approximate "monochromaticity"
Practical examples:

**angular source size** \( \delta \) \( (\delta = s/p) \)

<table>
<thead>
<tr>
<th>source</th>
<th>source size s</th>
<th>source dist. D</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>class. lab tube</td>
<td>400 ( \mu )m</td>
<td>0.4 m</td>
<td>( \approx 1 \cdot 10^{-3} \approx 3.5 ) arcmin</td>
</tr>
<tr>
<td>( \mu )-focus tube</td>
<td>5 ( \mu )m</td>
<td>1 m</td>
<td>( \approx 1 \cdot 10^{-5} \approx 1 ) arcsec</td>
</tr>
<tr>
<td>SR</td>
<td>100 ( \mu )m</td>
<td>150 m</td>
<td>( \approx 6.7 \cdot 10^{-7} \approx 0.14 ) arcsec</td>
</tr>
<tr>
<td>SR</td>
<td>50 ( \mu )m</td>
<td>75 m</td>
<td>( \approx 6.7 \cdot 10^{-7} \approx 0.14 ) arcsec</td>
</tr>
</tbody>
</table>

**energy** \( w_{h}^{\oplus} \)

<table>
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<td>8 keV</td>
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</table>

laboratory: \( \delta \ll w_{h}^{\oplus} \) divergent & quasi plane waves possible

SR - ESRF: \( \delta < w_{h}^{\oplus} \) quasi plane wave (mostly)

Source divergences > angular source sizes
For your curiosity:

First published (direct) measurements of reflectivity curves,

(Not rocking curves! Those were measured earlier),

the ones that we are able to calculate since Darwin’s first dynamical theory published in 1914, was in???

1962

nearly 50 years after Darwin’s results!!!

C. G. Darwin, The theory of X-ray reflection, Phil. Mag. 27, 315, 675 (1914)

Charles Galton Darwin (1887-1962) grandson of Charles Darwin
How can we measure a reflectivity curve???

Not trivial. One needs two ingredients.

1. Narrow instruments function
   in angular space - asymmetrical reflections
   in wavelength space - high-resolution mono.

2. Crystals of good quality.
   They became available with development
   of electronics, later micro-electronics and
   opto-electronics
Double (or triple) crystal monochromator-collimator with one asymmetrical Bragg reflection
111-reflections of Ge, different asymmetries

CuK$_{\alpha_1}$-radiation, triple crystal set-up, silicon double monochromator

By the way - the probably first published measurements in a double crystal set-up with an asymmetric monochromator/collimator crystal were about X-ray diffraction topography in 1952.


This technique was later developed further to the “Plane” Wave Topography, to detect extremely small strain fields in nearly perfect crystals.
Thank you for your attention!