REFRACTIVE X-RAY LENSES

NEW DEVELOPMENTS

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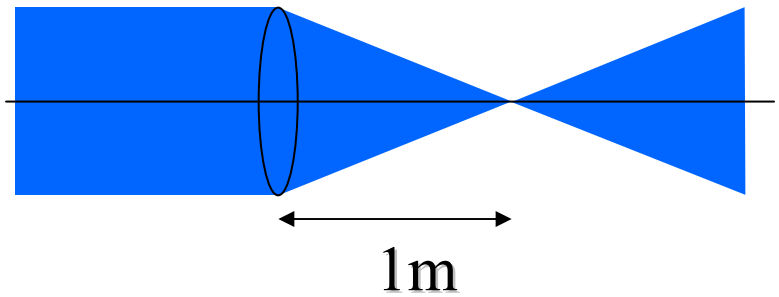
(Grenoble, July 2010)

A. Strategy for refractive x-ray lenses

> **have been considered as not feasible for a long time**

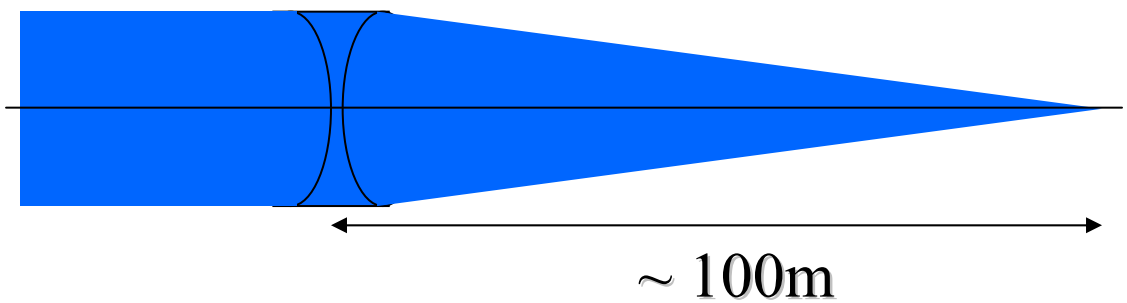
> **visible light**: index of refraction $n = 1 + \delta$ with $\delta \sim 0.5$ for glass

* refraction strong
* absorption weak
* focal length short
* focusing lens convex



> **x-rays**: $n = 1 - \delta + i\beta$ with $\delta, \beta \sim 10^{-6}$ and positive

* refraction weak
* absorption strong
* focal length long
* focusing lens concave



„There are no refractive lenses for x-rays!“ W.C.Roentgen

BUT: **refraction is not zero and absorption is not infinite!**

Design of refractive x-ray lenses

lensmaker formula: $\frac{1}{f} = (1-n)\frac{2}{R}$ or $f = \frac{R}{2\delta}$

$$\delta = 2.70(\lambda^2 \rho Z / A)10^{-6}$$

λ in Angstrom
ρ in g/cm³
Z atomic number
A atomic mass in g

To obtain a **small focal length**:

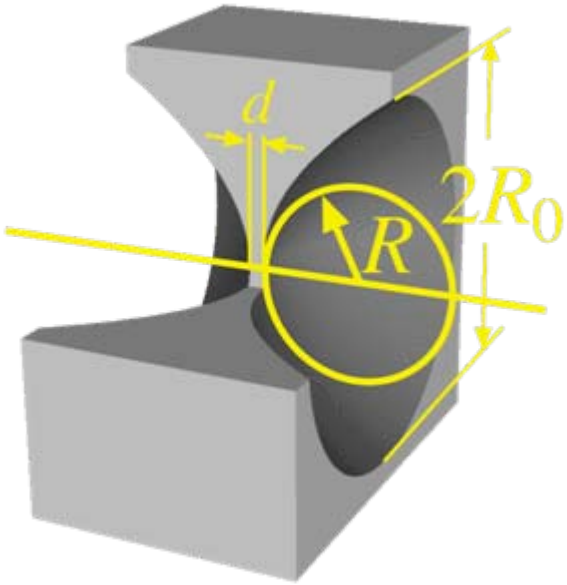
i) small radius of curvature R:
typical: R = 50 to 1500μm

ii) high density of lens material

iii) profile must be parabolic: no spherical aberration

Lens surfaces must be paraboloids of rotation

single lens



parameters for Be lenses:

R = 50 to 1500μm

$2R_0$ = 0.45 to 2.5mm

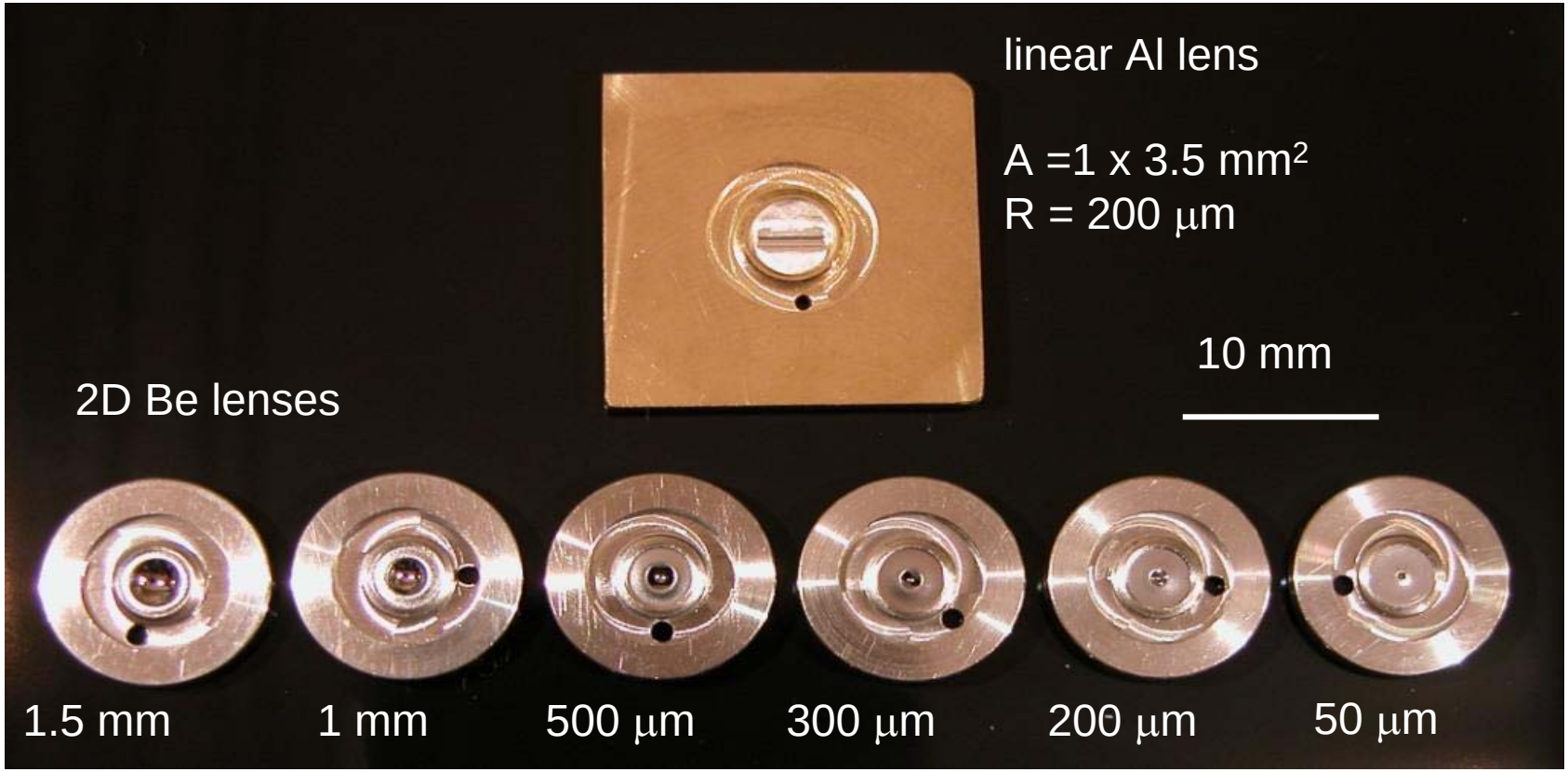
d below 30μm

parabolic profile: no spherical aberration

focusing in full plane

=> excellent imaging optics

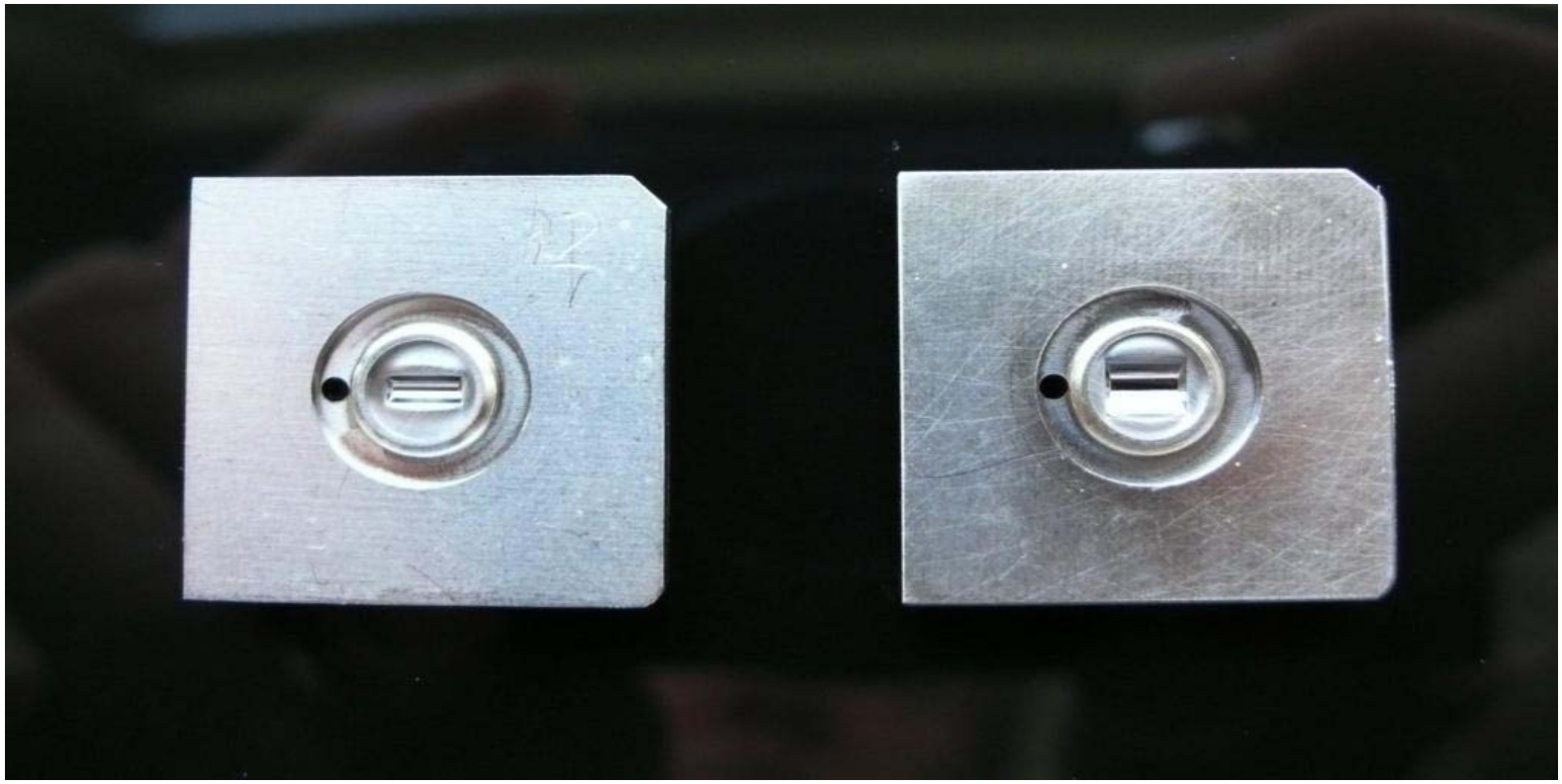
Rotational parabolic and linear parabolic lenses



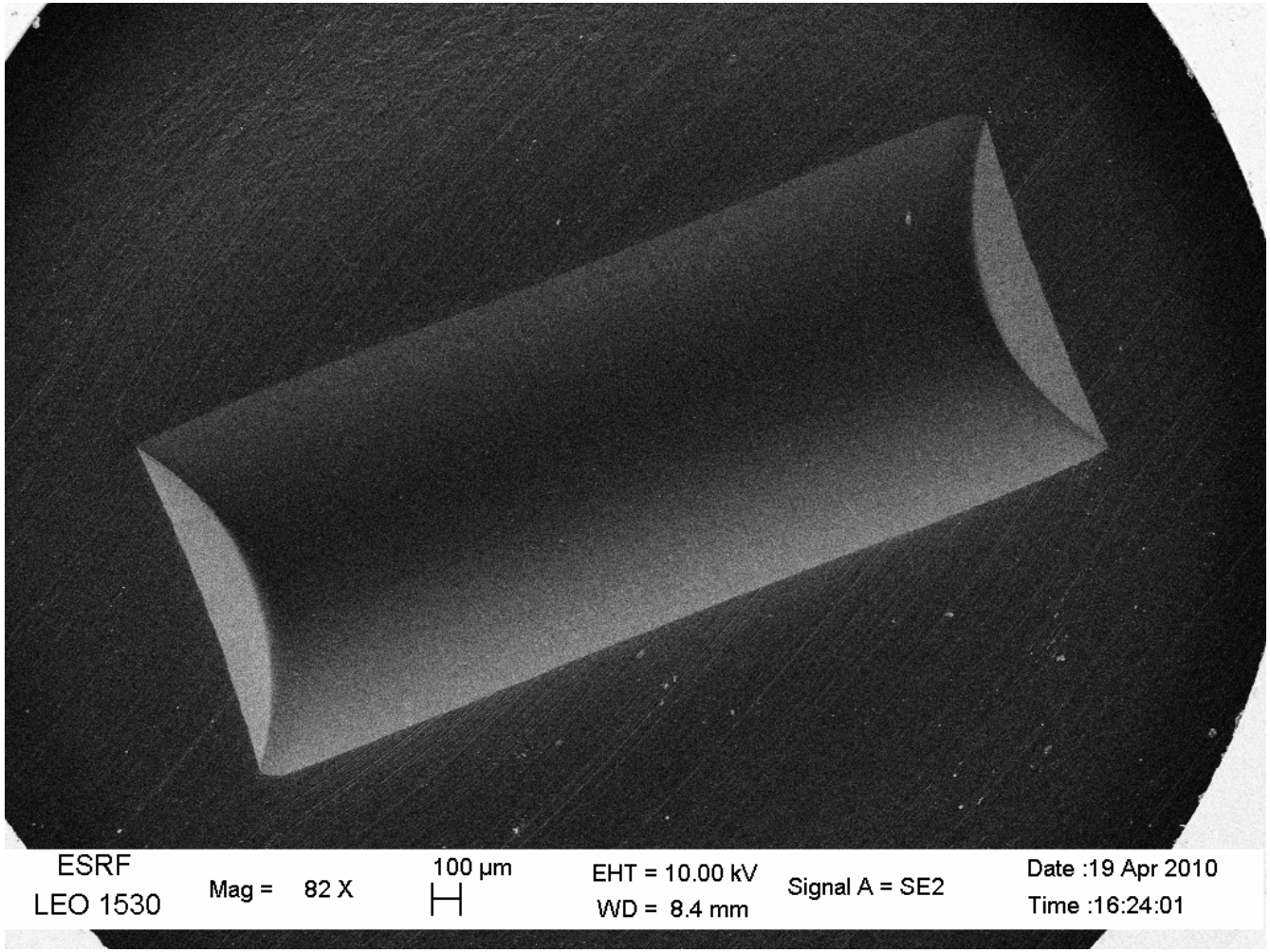
Linear Be lenses (cylinder paraboloids)

length 2.5mm

R=500μm R=1500μm



SEM image of linear Be lens (R=500μm)



Refractive x-ray lenses available at Aachen University

- material: Be 6 to 40 keV
  Al 40 to 80 keV
  Ni 80 to 150 keV

- profile: rotationally parabolic (2D)
  cylinder parabolic (1D)

- radii R at apex and geometric aperture $2R_0$

$$R = 50,\ 100,\ 200,\ 300,\ 500,\ 1000,\ 1500\mu m$$
$$2R_0 = 450,\ 632,\ 894,\ 1095,\ 1414,\ 2000,\ 2450\mu m$$

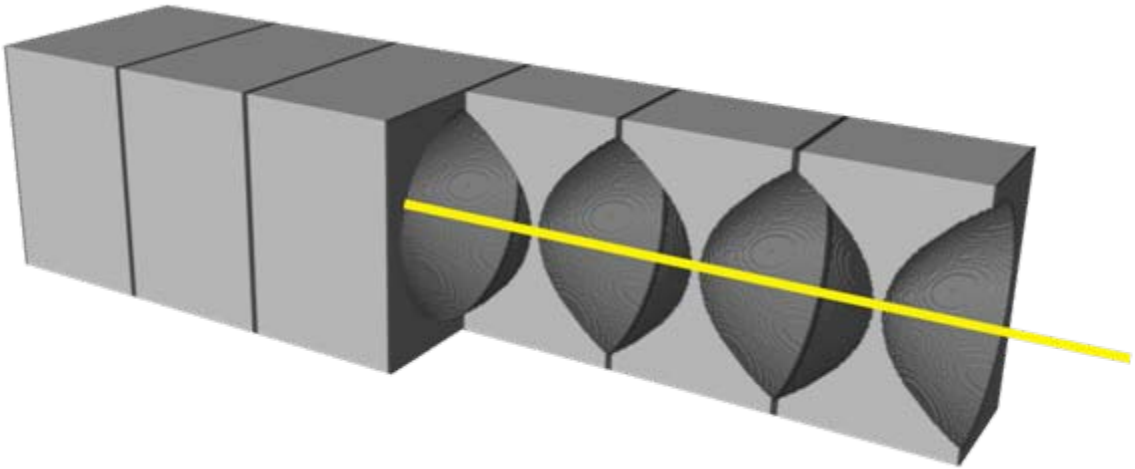
length of 1D-lenses: 2.5mm

- small radii for imaging and focusing
  large radii for prefocusing

iv). Stacking many lenses in a row

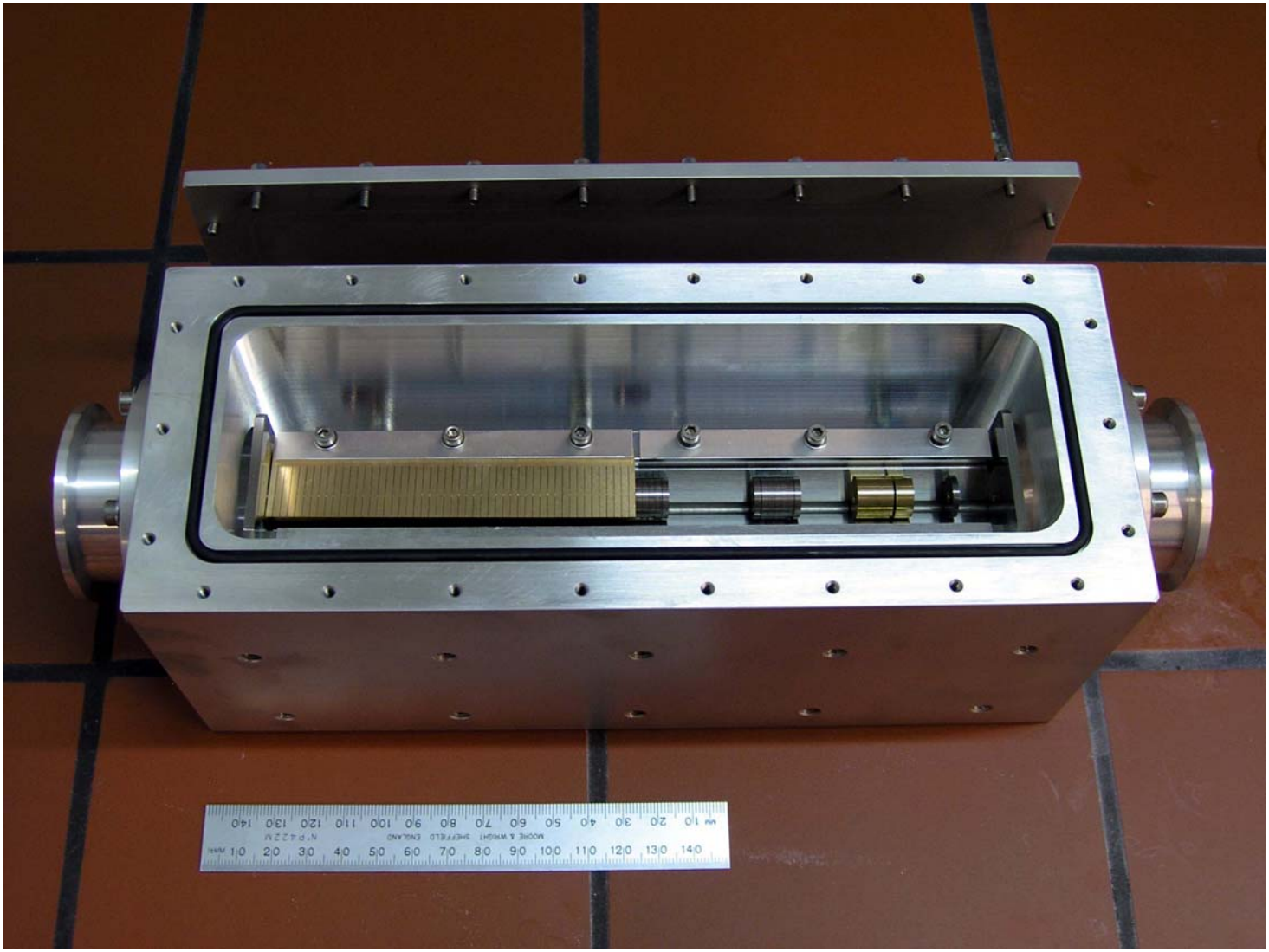
$$f = R / 2\delta N \qquad (\text{thin} - \text{lens})$$

variable number of lenses : N = 1 to about 300
Precision of stacking: better than 1 μm



typical: f = 0.2m - 10m

NEW LENS CASING (can be integrated in vacuum of beam line)



A few examples: **for 1m focal length by lenses with R=50μm**

E (keV)	material	2δ (10^{-6})	N	f (m)
12.4	Be	4.4341	11	1.025
17	Be	2.3591	21	1.009
40	Be	0.4261	117	1.003
40	Al	0.6746	74	1.002
80	Al	0.1687	296	1.002
80	Ni	0.5515	91	0.996

How close can you adjust the focal length f (e.g. at 10 keV) ?

N \ R	200μm	300μm	500μm	1000μm
4	7.334 m	11.001 m	18.334 m	36.668 m
3	9.778 m	14.667 m	24.446 m	48.891 m
2	14.667 m	22.001 m	36.668 m	73.336 m

stacking of different lenses $$\frac{1}{f} = \sum_j \frac{1}{f_j}$$

for f=8m : 3*R=200μm and 1*R=300μm : f=8.000
for f=9m : 3*R=200μm and 1*R=1000μm : f=9.167m

if possible and needed: choose E=9.908keV
then 3*R=200μm and 1*R=1000μm gives f=9.000m

More flexibility by lenses with larger R!

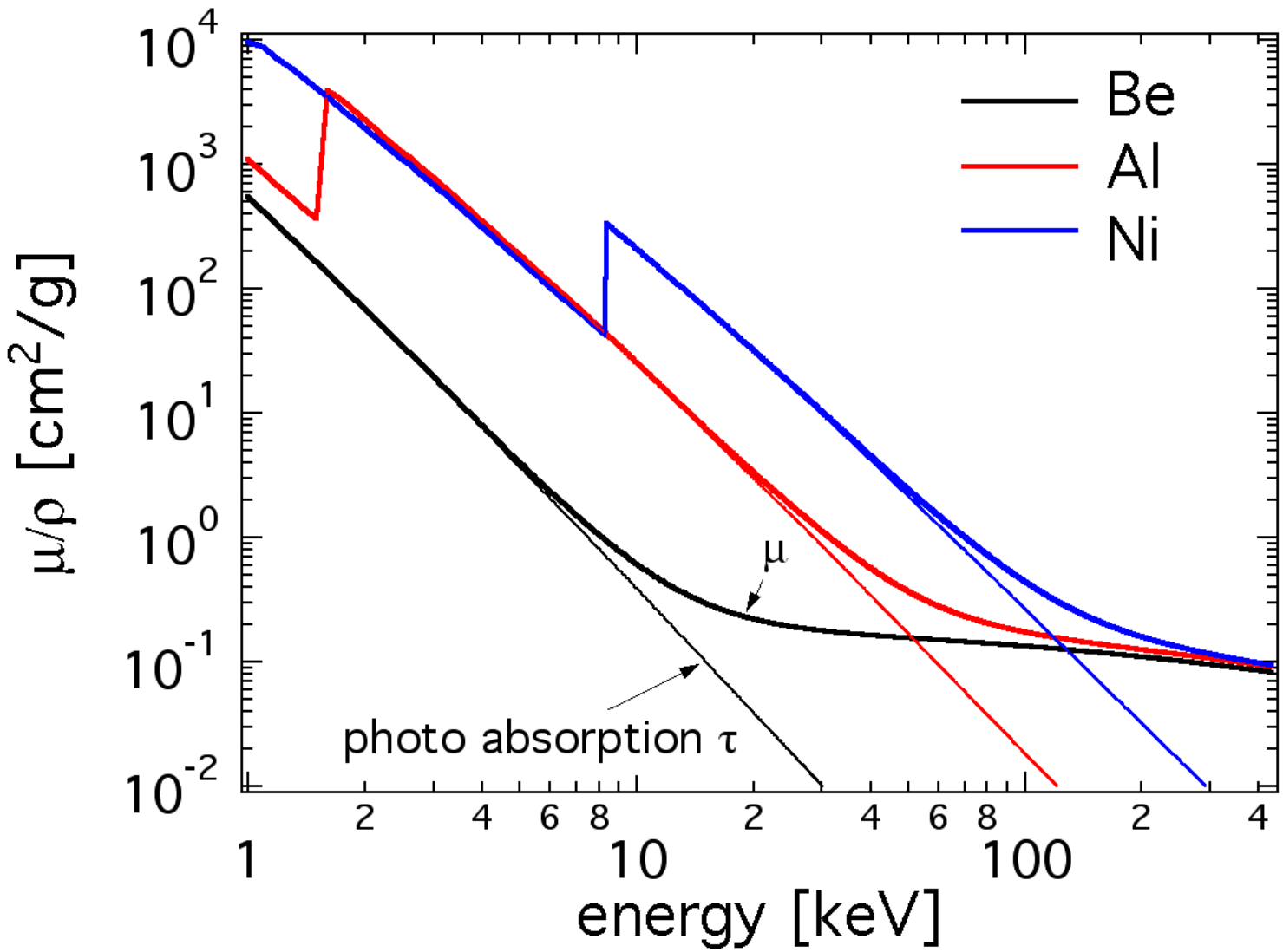
v). Lens material must be mechanically, thermally and chemically stable

vi). **low Z** lens material:

mass absorption cofficient $\mu / \rho \sim Z^3 / E^3$

candidates: **Be**, B, C, **Al**, **Si, Ni**

Attenuation of x-rays in typical lens materials



Ultimately, **Compton scattering** limits transmission at high x-ray energies!

Cylinder parabolic (1D) lenses from ESRF-Russia and from TU Dresden

material: Si

technology: microfabrication (e-beam lithography, etching)

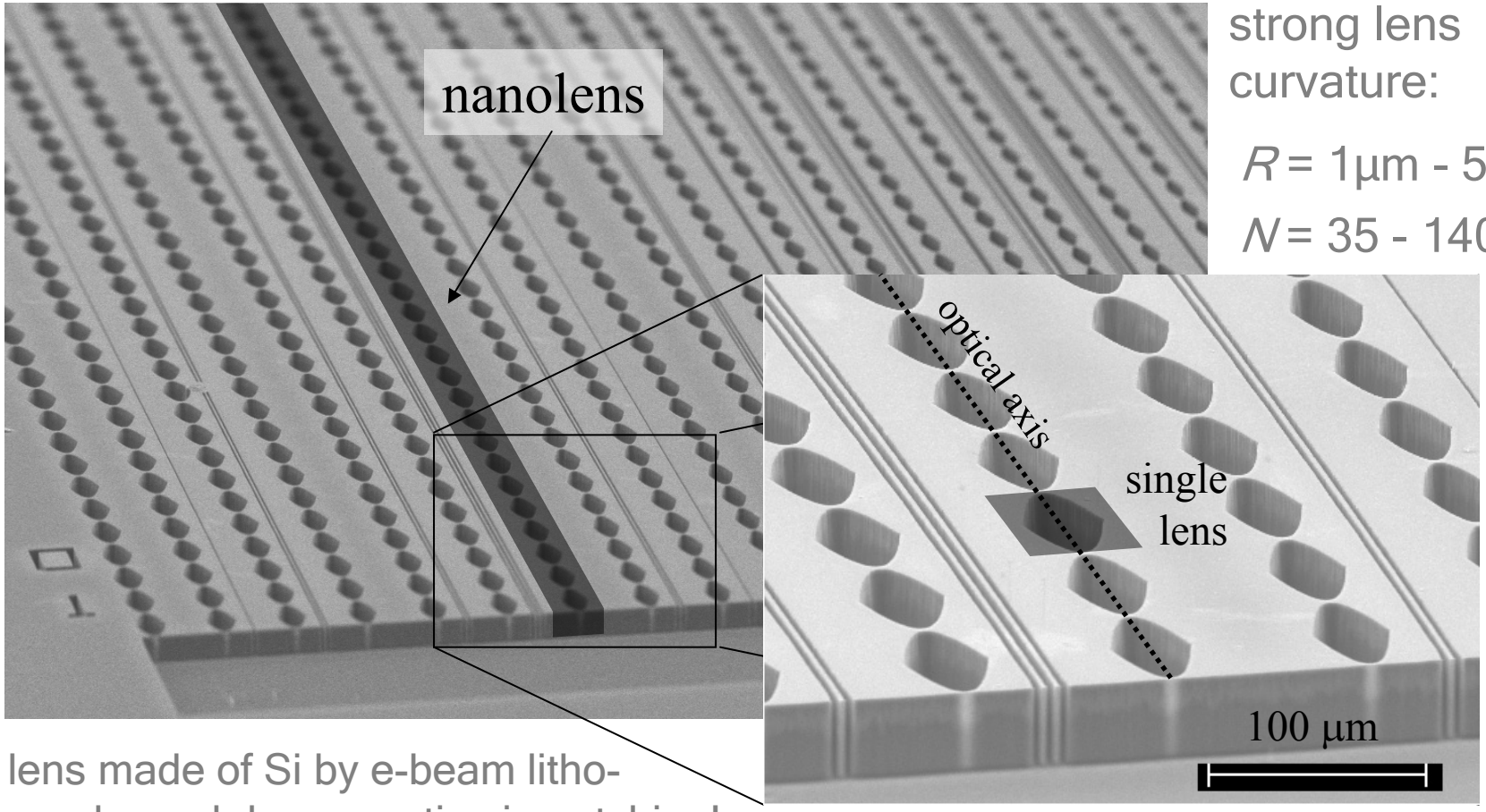
performance: stapling is done by microfabrication

bilenses possible

very small radii possible, (down to 1μm)
=> very small focal length

only 1D- lenses

Nanofocusing Lenses (NFL)



strong lens curvature:

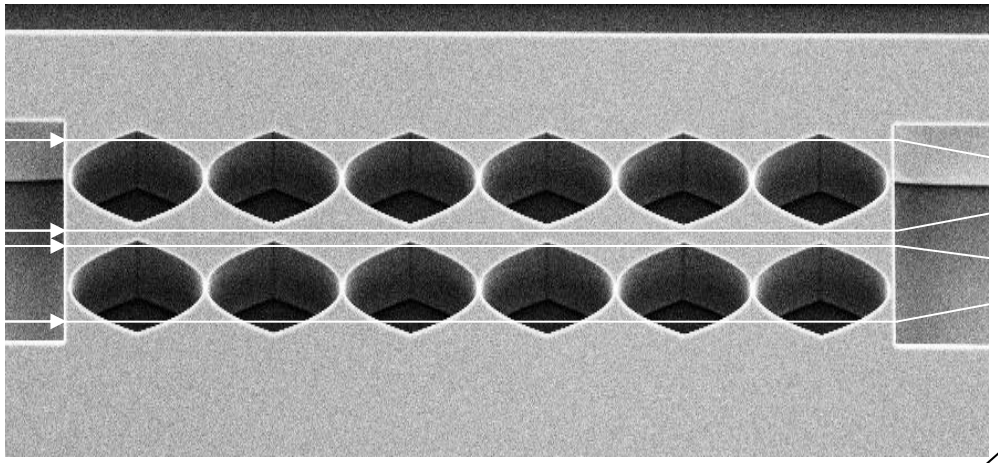
R = 1µm - 5µm

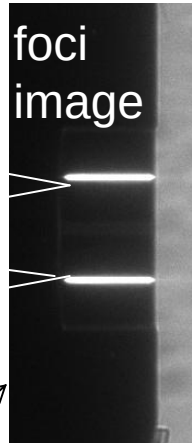
N = 35 - 140

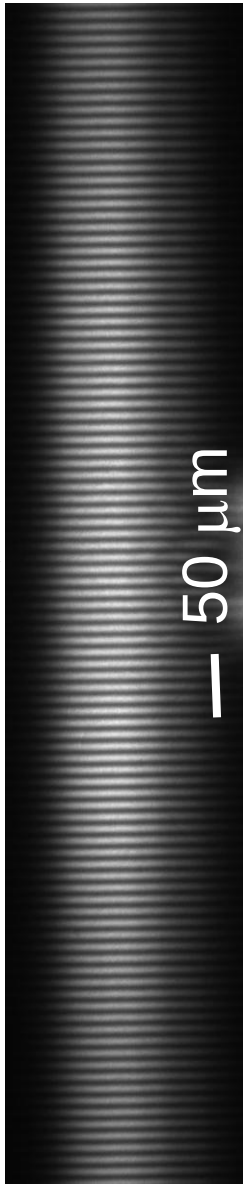
lens made of Si by e-beam litho-graphy and deep reactive ion etching!

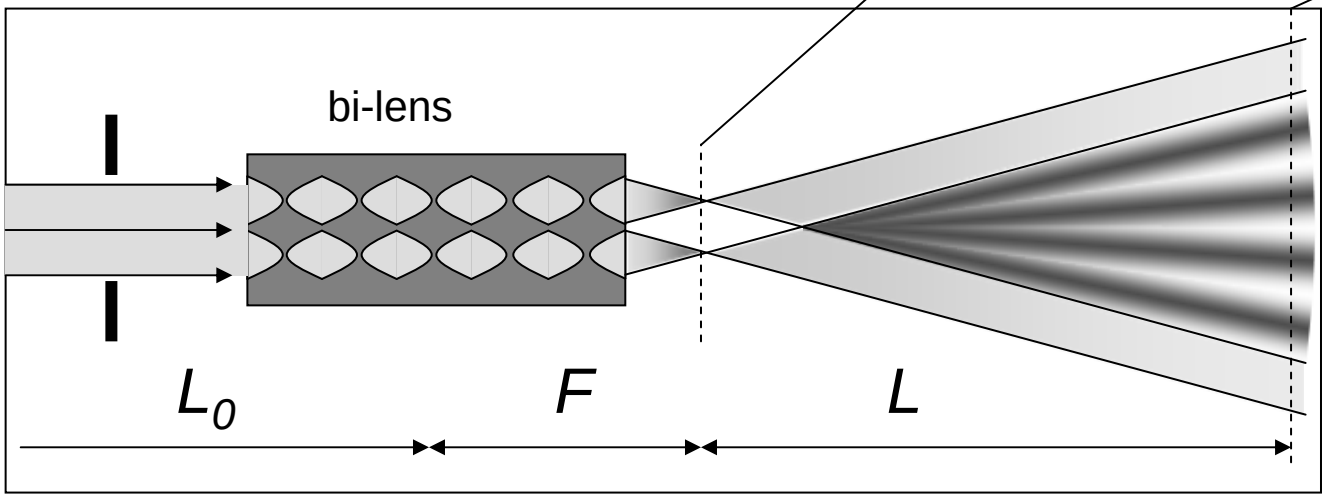
Schroer et al APL 82, 1485 (2003)

Silicon bilens (ESRF-Russia)









B. Properties of refractive x-ray lenses

In the following we consider mainly Be, Al and Ni

1. Energy range

Be : about 5 to 40keV

d guaranteed below 50μm, typically 30μm

Al : about 30 to 80 keV

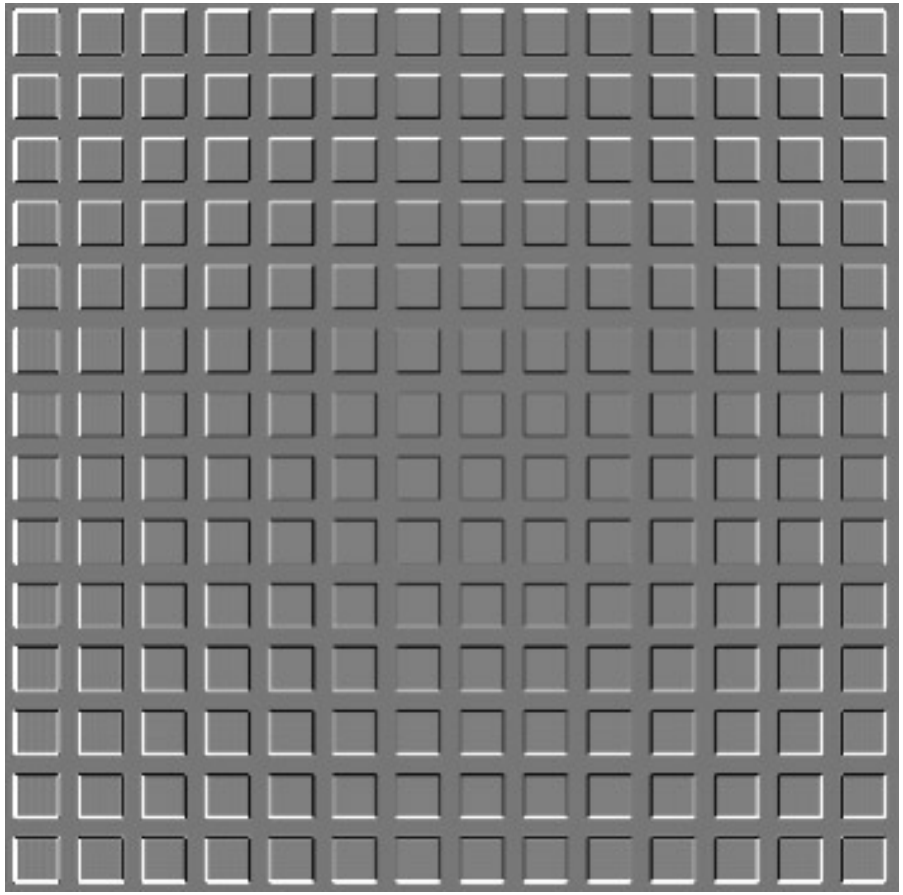
d guaranteed below 30μm, typically 22μm

Ni : about 80 to 150 keV

d guaranteed below 20μm, typically 10-16μm

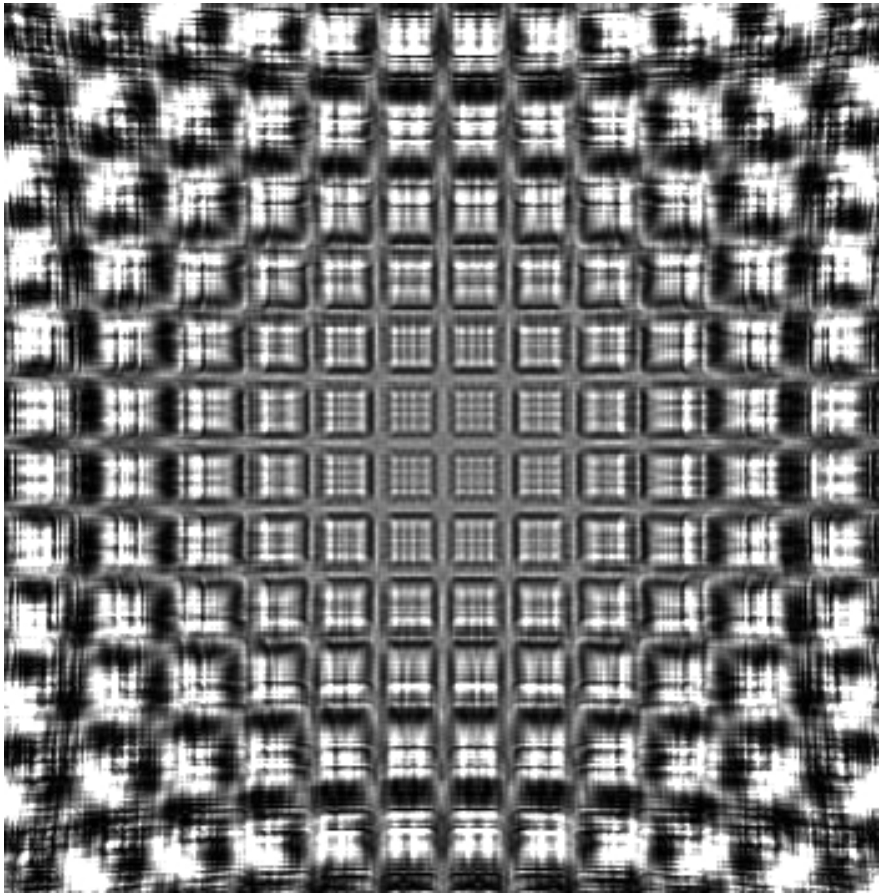
2. **Comparison parabolic versus spherical lens**

parabolic



25µm

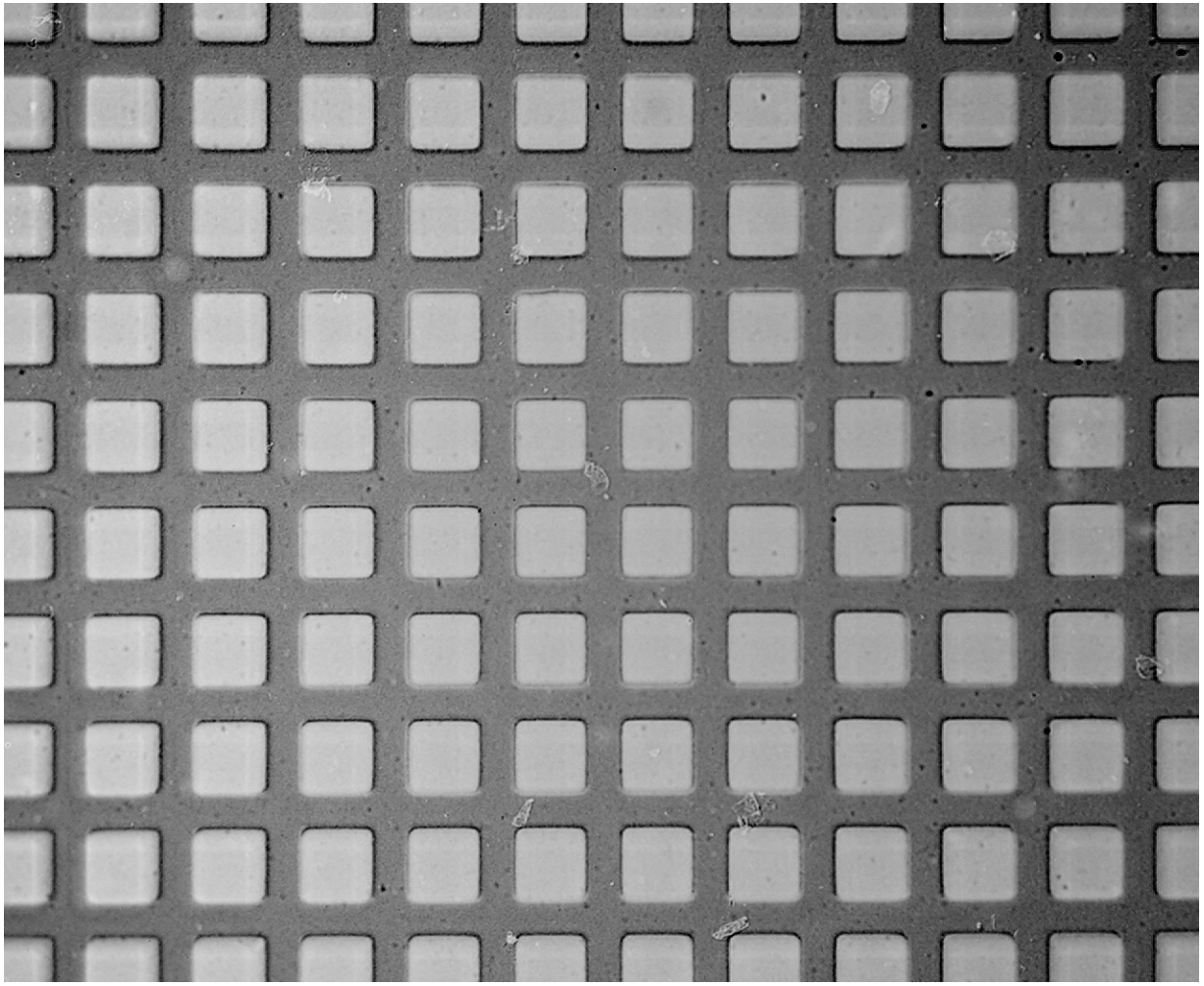
spherical



spherical lenses are inappropriate for imaging!

Example: Ni mesh 12.7μm period

parabolic refractive Be lens N = 91, R = 200μm
f = 495 mm at 12 keV



magnification: 10

detector: high resolution film

NO DISTORTION!

3. Material properties

Beryllium

manufactured by powder metallurgy
contains up to 1wt% of BeO
contains many grain boundaries
=> **small angle x-ray scattering**
results in background radiation

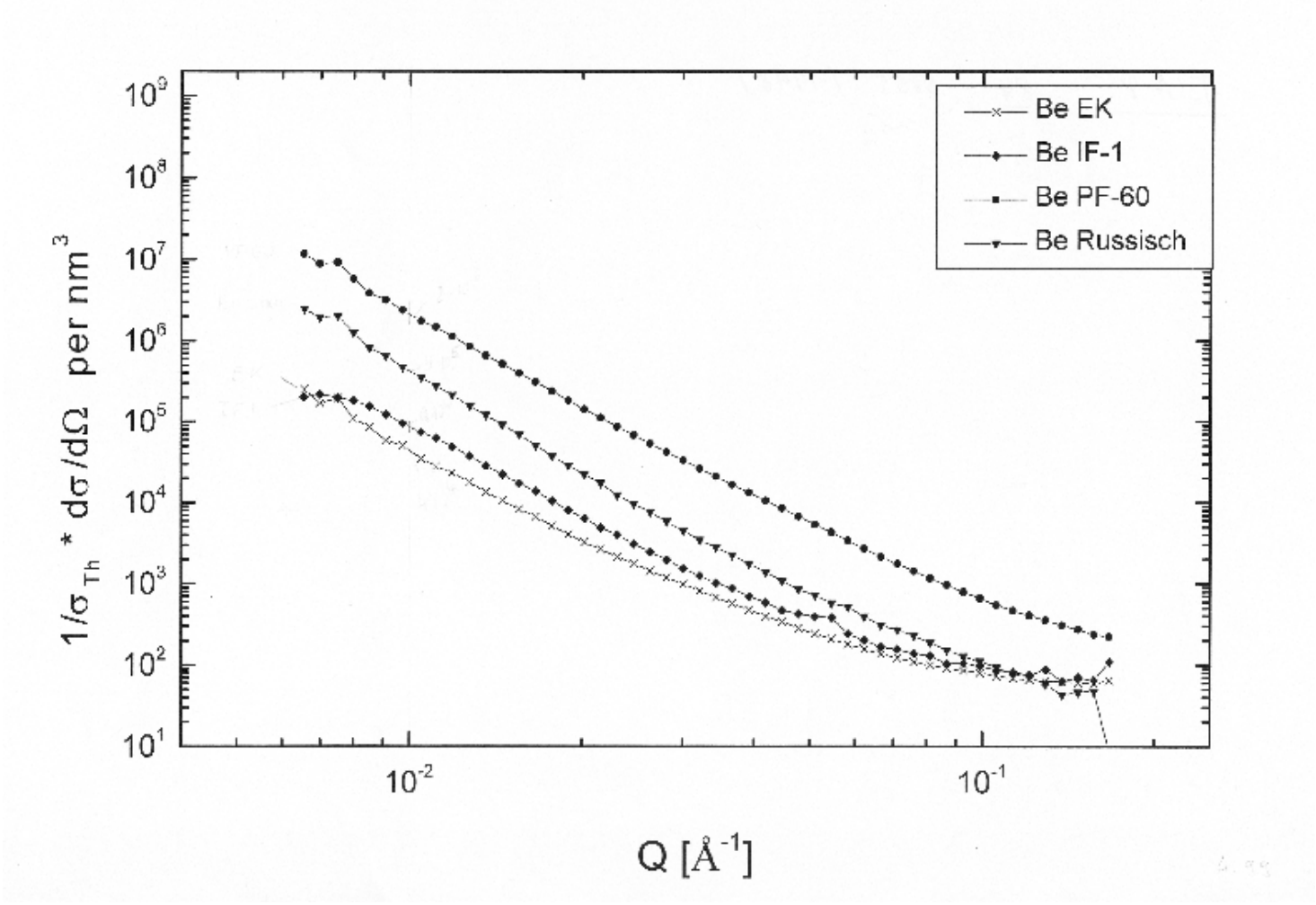
density : 1.85 g/cm³

melting point : 1287 °C

recrystallisation: about 600°C

main supplier: BRUSH-WELLMAN

Small-angle x-ray scattering in different types of Be



PF-60 is standard Be from BW

IF-1 has 20 times less SAXS than PF-60

only 2 times more SAXS than single crystal (EK)

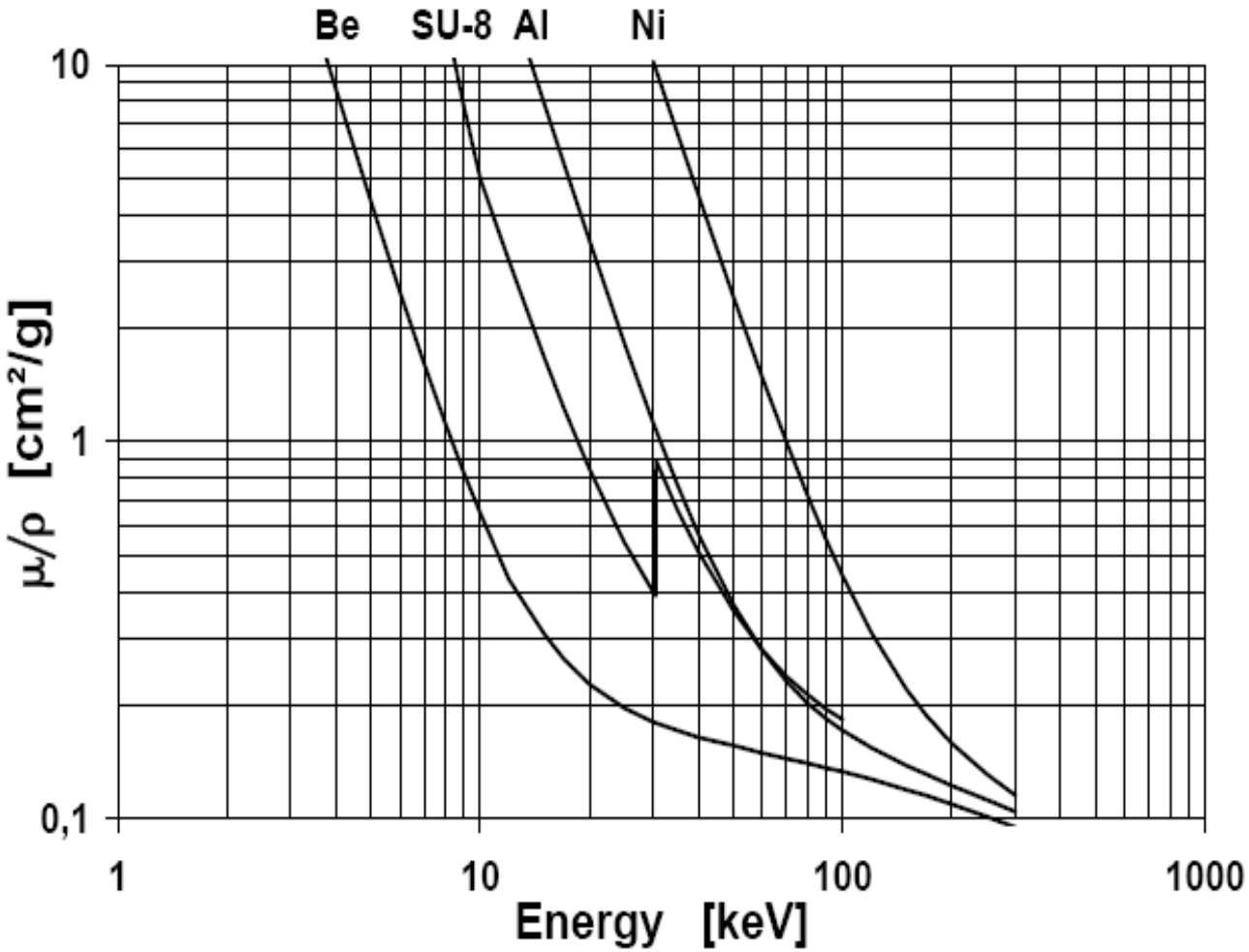
Small angle scattering of different lens materials

Be single crystal	5	*	10^4 σ_{Th}/nm^3	at 0.0565°
Be IF-1	10			or $Q=10^{-2}$ /A
Be PF-60	238			
Be russian	47			
Al 5N	90			
B HCStarck	20			
diamond	14			
PMMA	2			
Teflon CF_2	770			
Pyro-graphite	200			
glassy carbon	1000-10000			
sapphire Al_2O_3	2			

Lens material: metals versus resists

	metals Be	Al	Ni	resists PMMA, Kapton, SU-8,…
radiation damage	none			yes
heat conductivity (W/m.K)	200	237	91	ca 0.2
melting point (°C)	1277	660	1453	ca 200
SAXS	low to medium			low to high
density	1.85	2.7	8.9	ca 1.1
form	1D and 2D			only 1D
R_{min}	50µm			10µm
kinoform	no			yes

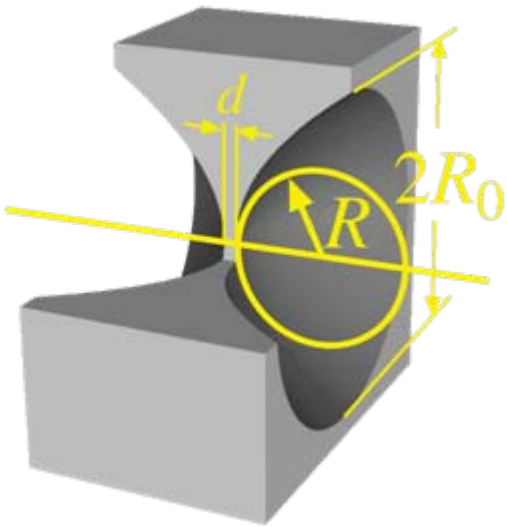
X-ray absorption in SU-8



SU-8 contains 1 atom of Sb per formula unit!

SU-8: no advantage compared to Be and Al !

4. Aperture of paraboloid of rotation:

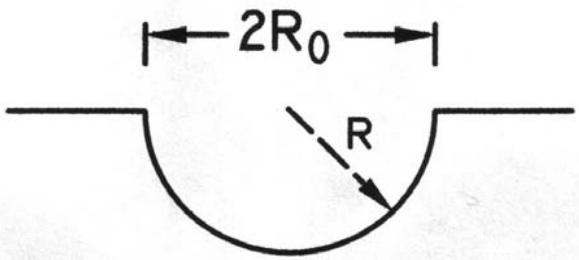


* no spherical aberration
* focusing in full plane

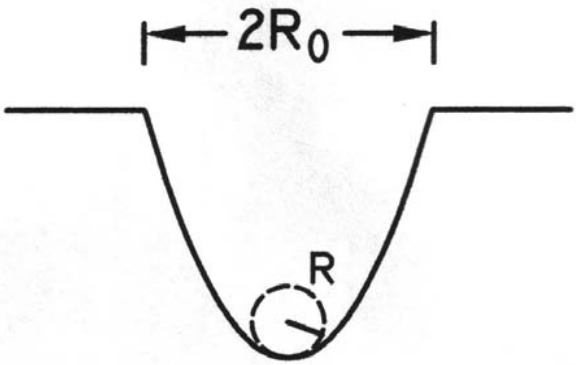
  => excellent imaging optics

* radius R and aperture $2R_0$ are decoupled

spherical lens:



parabolic lens:

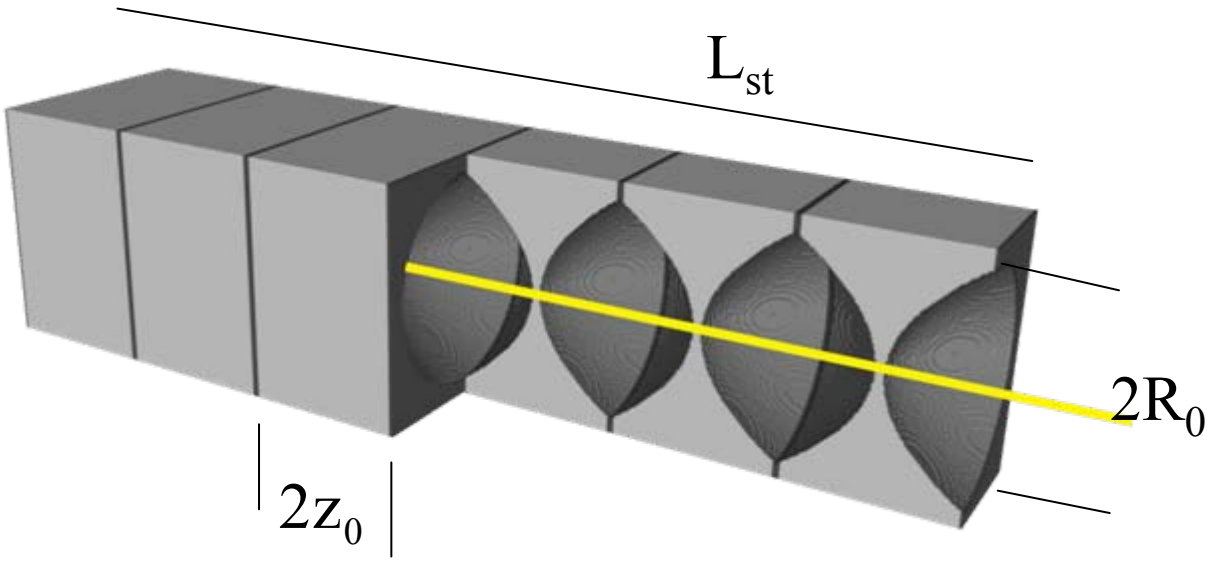


$R_0 \leq R$

R_0 and R independent

Effective lens aperture D_{eff}

Absorption reduces the effective aperture below the value of the geometric aperture $2R_0$



$$D_{eff} = 2R_0 \sqrt{\left[1 - \exp\left(-a_p\right)\right] / a_p}$$

$$a_p = \mu N z_0 = \frac{1}{2} \mu L_{st}$$

Transmission T versus effective aperture D_{eff} (A_{eff})

transmission T: fraction of transmitted intensity compared to intensity falling on geometric aperture πR_0^2

$$T = \frac{1}{\pi R_0^2}\int_0^{R_0} \exp(-\mu N 2z) = \frac{1}{2a_p}[1-\exp(-2a_p)]$$

$$a_p = \mu N R_0^2 / 2R = \mu N z_0$$

effective aperture D_{eff} reduced by absorption compared to geometric aperture $2R_0$

$$D_{eff} = 2R_0\sqrt{[1-\exp(-a_p)/a_p}$$

Example: Be stack with N = 50, R = 50μm at 17 keV

$2\delta = 2.359\ 10^{-6}$ and $\mu = 0.4903/cm$

f = 423.9mm

z_0 (μm)	$2R_0$ (μm)	$\mathbf{D_{eff}}$ (μm)	T
500	447.2	339.5	37.3%
1000	632.5	386.2	20.2%
	100	98.5	94.1%

The effective aperture is the relevant parameter for characterizing the transmission of refractive lenses!

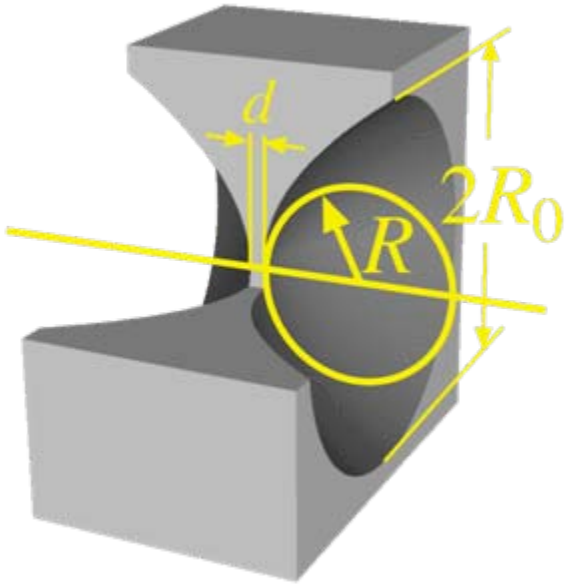
Influence of material (thickness d) between apices on transmission of lensstack

Transmission = exp(-μNd)

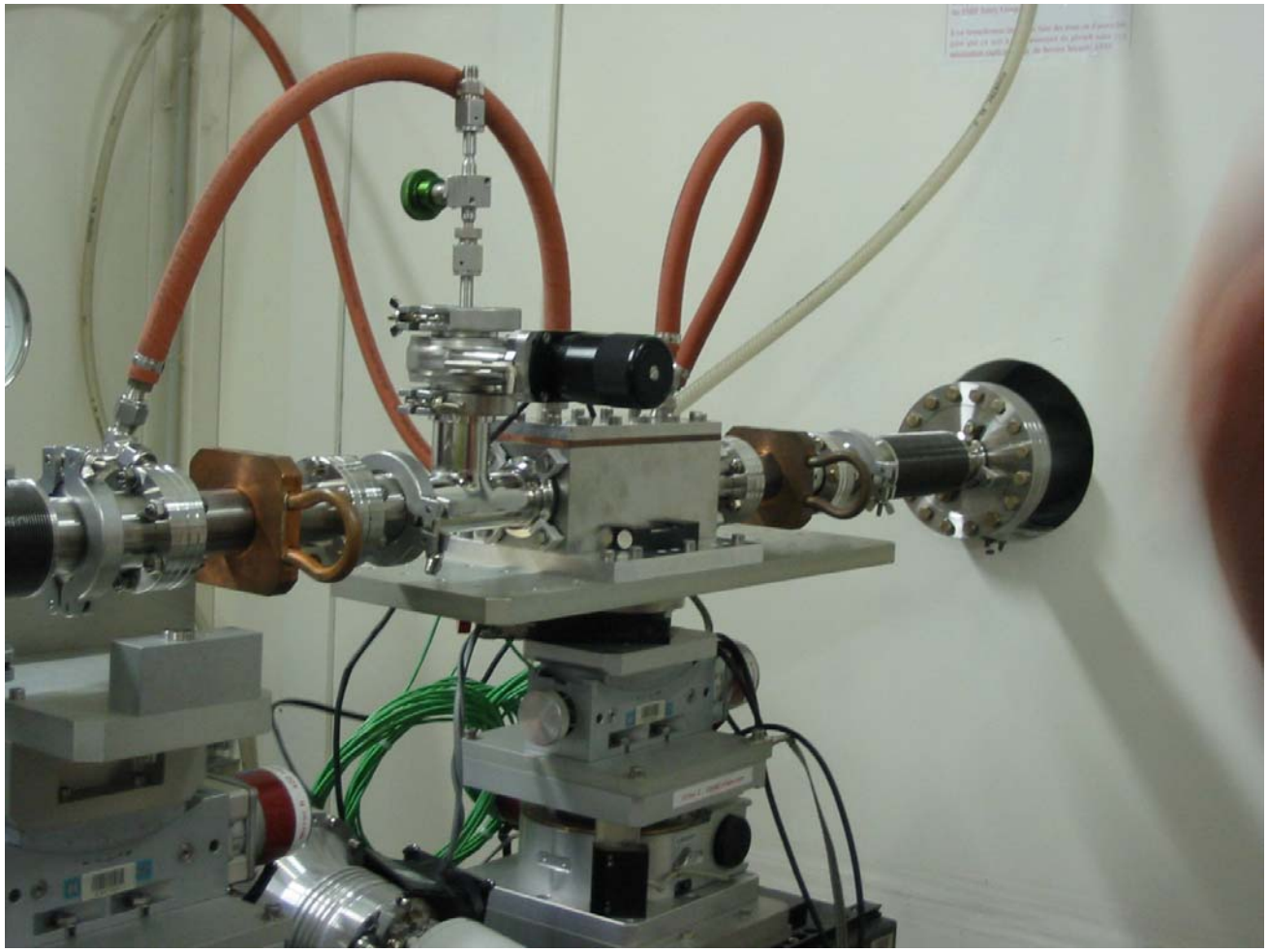
Example : Be lenses R=50μm, d=30μm

1. 12keV, μ=0.8196/cm,
   N=22, f=0.480m
   transmission: 94.7%

2. 17keV, μ=0.4903/cm
   N=42, f=0.505m
   transmission: 94.0%



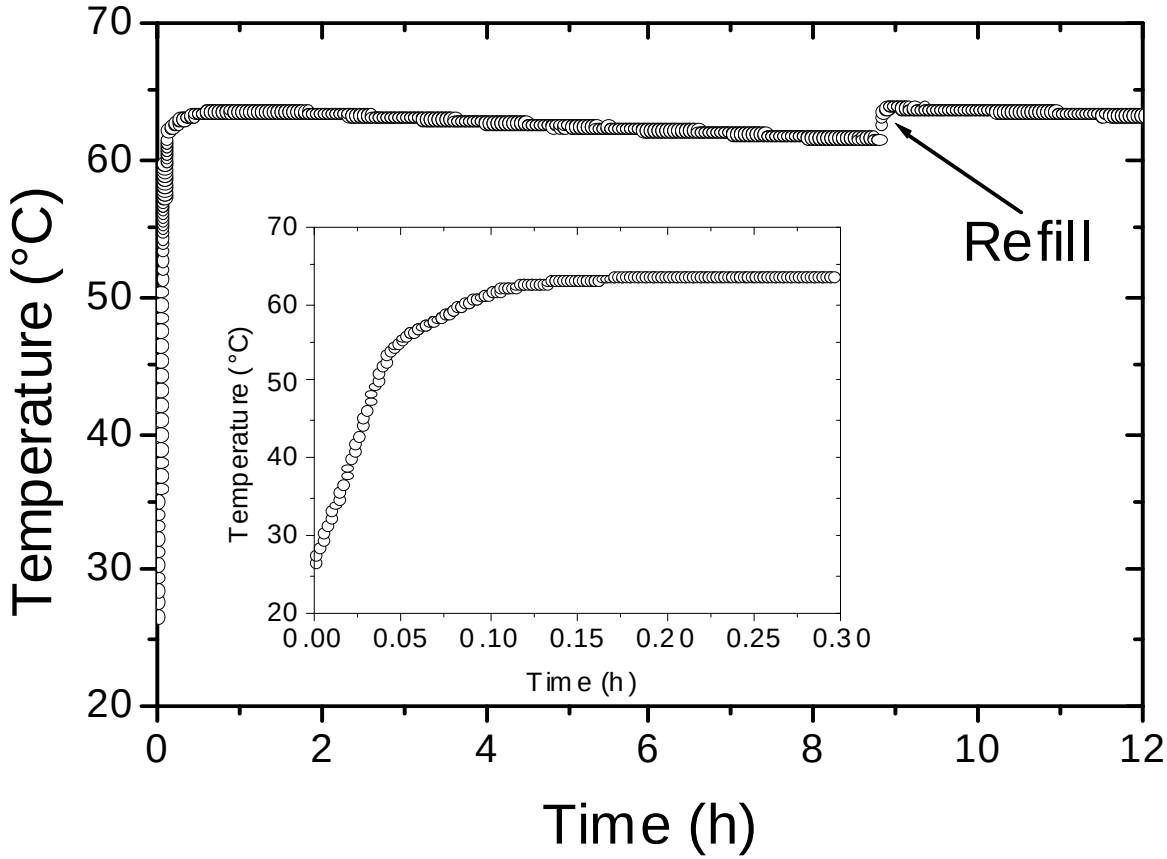
5. Thermal stability in the beam



Water cooled beryllium lens at ESRF (ID10)

Temperature - time profile in white beam at ID10 ESRF

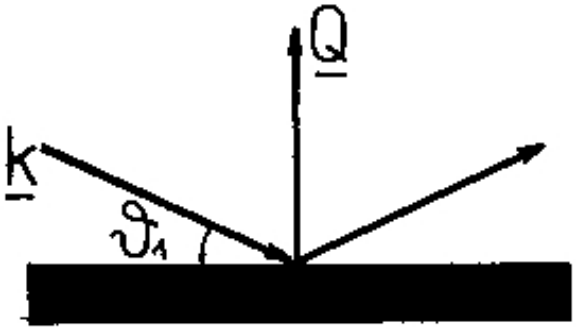
ca. 100 W/mm² & total 40 W **(Be lens)**



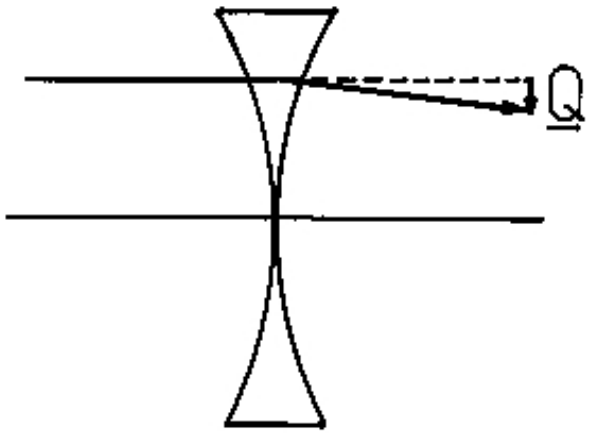
In Be lenses the temperature should not exceed about 300°C!

6. Insensitivity of lenses to surface roughness and contamination

(compared to mirrors)



mirror



lens stack

Damping of intensity due to surface roughness $\sigma \sim \exp[-Q^2 \sigma^2]$

with momentum transfer $Q = 2k \sin\theta_1 \cong 2k\, \theta_1$

mirror $Q = 1.4\ 10^{-1}\ A^{-1}$ at $\theta_1 = 0.6°$ and $\lambda = 1A$

lens stack $Q = N^{1/2}\, k\, \delta = 1.4\ 10^{-4}\ A^{-1}$ at $N = 100$ and $\lambda = 1A$

A lens is about 1000 times less sensitive to σ than a mirror!

Typical value of surface roughness of our lenses: $0.1\mu m$

For $l = 1A$

$N = 100$

$Q = 1.4\ 10\text{-}4\ /A$

$\exp(-Q^2 s^2) = 0.981$

This is tolerable!

7. Chromatic aberration

refractive x-ray lenses show strong chromatic aberration

$$f = R/2\delta N$$

$$\delta = 2.70 * 10^{-6} * \lambda^2 \rho Z/A$$

Changing the energy at fixed focal length
implies changing the number of lenses in the stack!

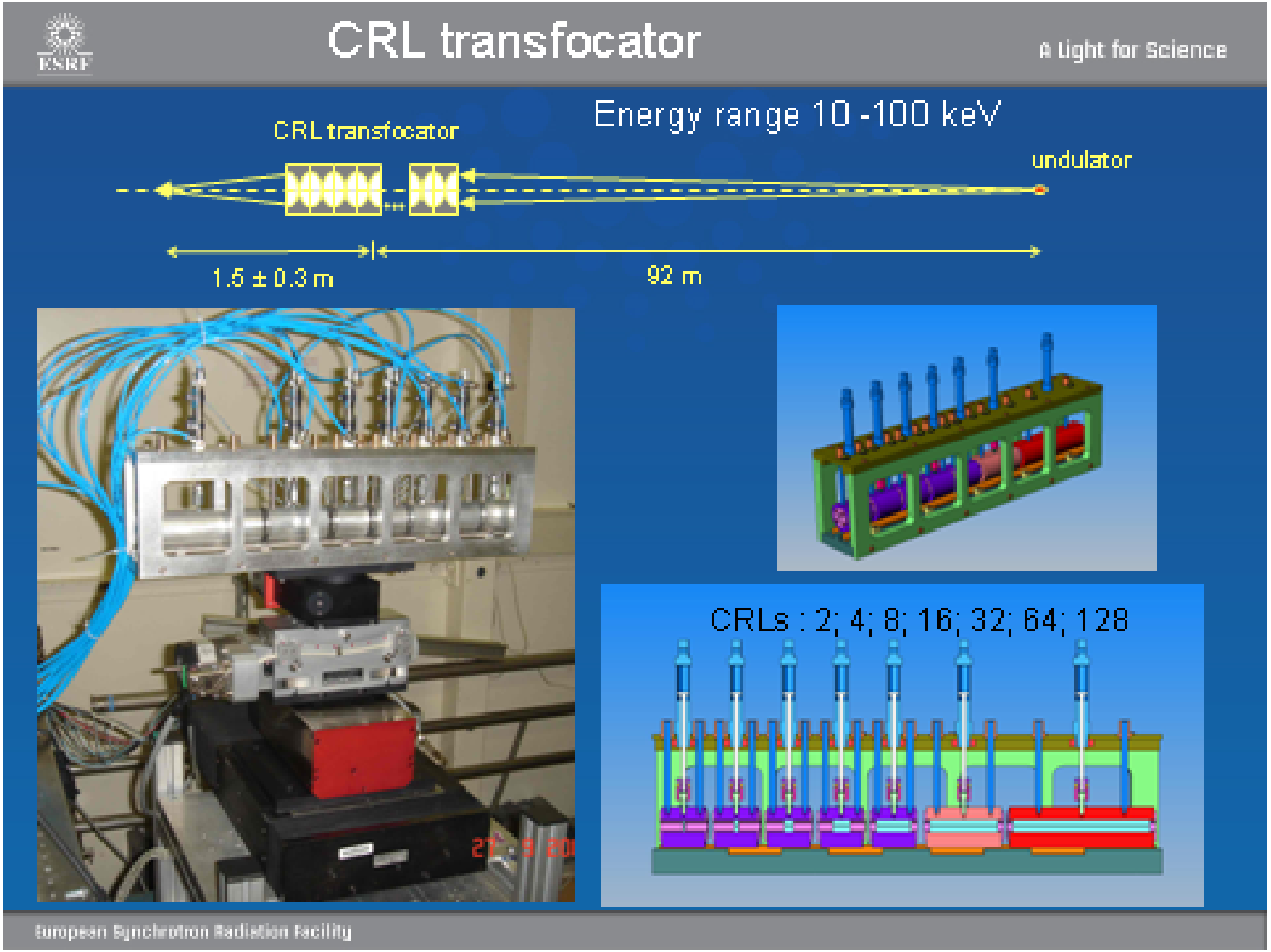
solution: **TRANSFOCATOR** developed at ESRF

flexible change of f

in air and in vacuum

new type of monochromator

TRANSFOCATOR (ESRF development)



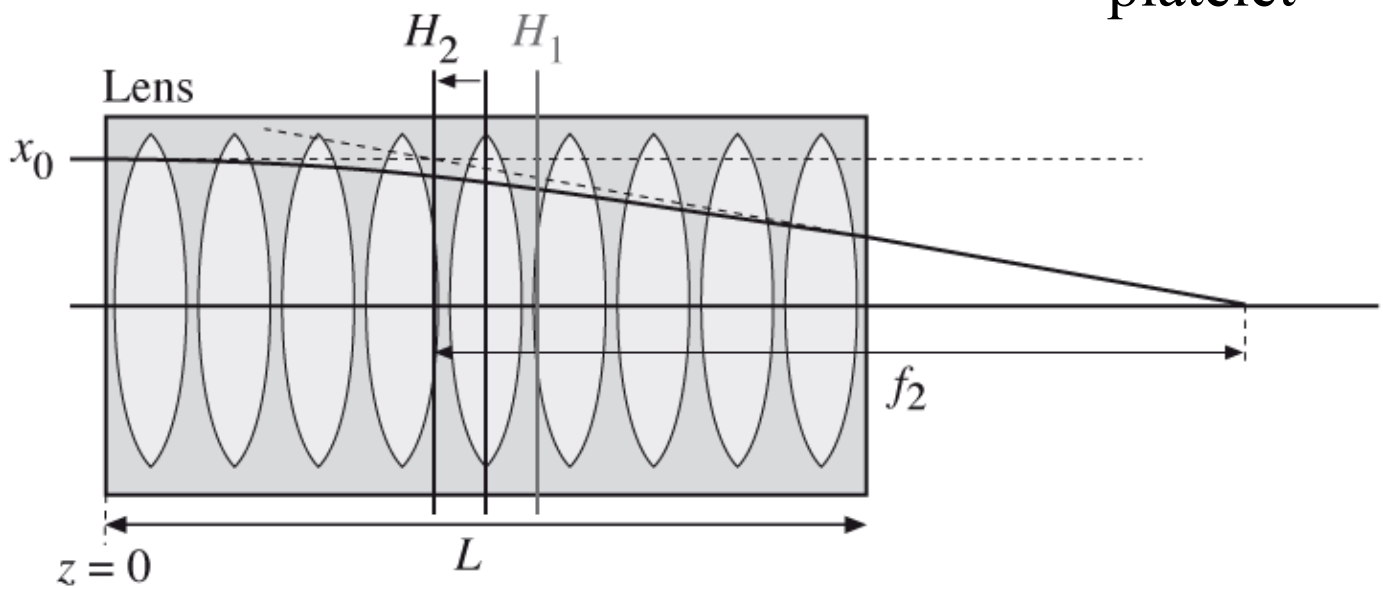
8. Thick lenses

* if L << f (thin lens): $f_0 = R / 2\delta N$

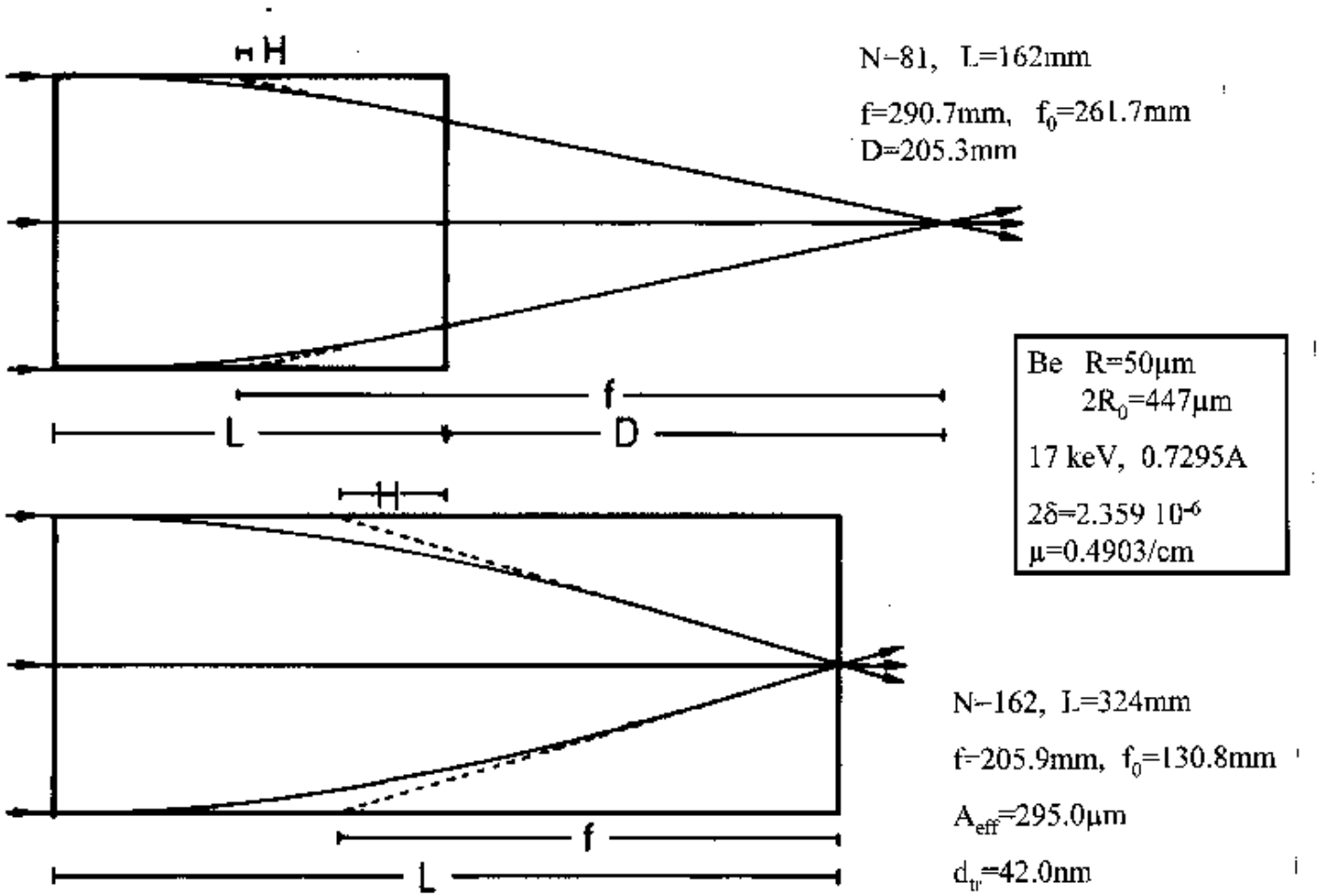
* if L comparable to f : rays are bent towards optical axis inside lens

$$r(z) = R_0 \cos \kappa z \qquad \kappa = \sqrt{\frac{2\delta}{RF}}$$

Refracting power/length
F: thickness of lens platelet



Minimal focal length achievable with Be, R = 50µm at 17 keV

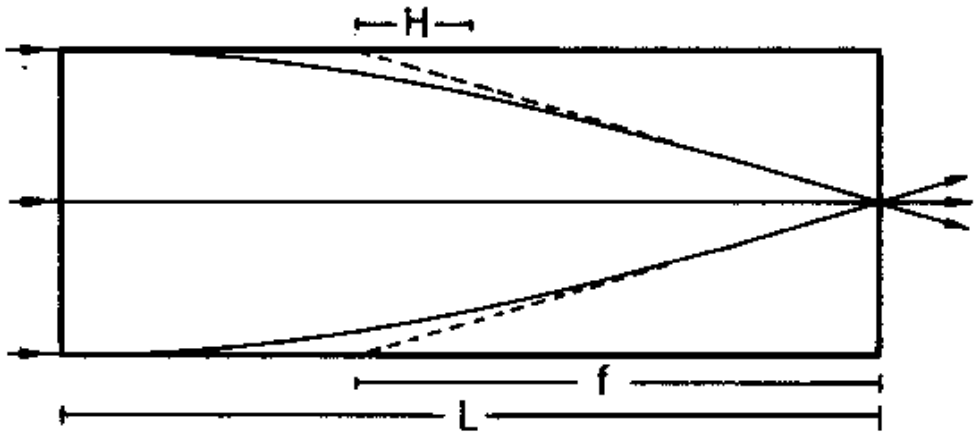


=> **effective aperture : 295µm**

=> **best lateral resolution: 42nm (diffraction limit)**

For lenses with constant refracting power:

number of lenses in the stack can be reduced slightly without loss of performance (the last lenses do not refract any more)



N=162, L=324mm

f=205.9mm, f_0=130.8mm
D=0

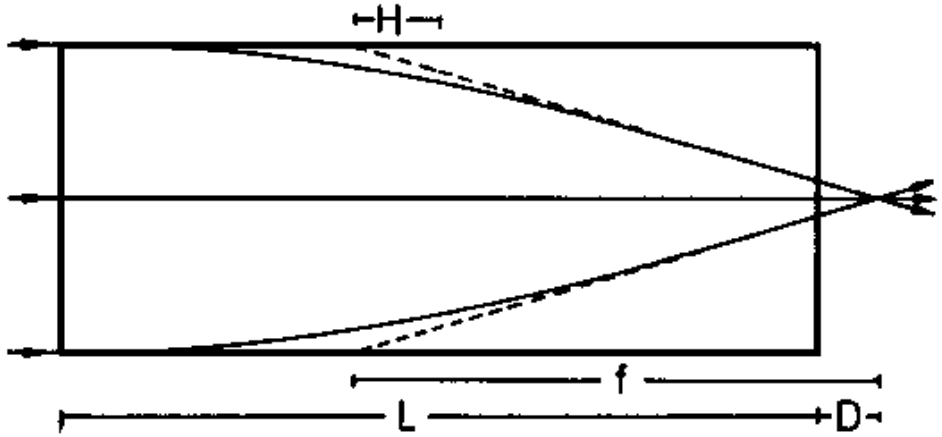
A_{eff}=295.0μm

d_{tr}=42.0nm

Be R=50μm
$2R_0$=447μm

17 keV, 0.7295A

$2\delta=2.359\ 10^{-6}$
μ=0.4903/cm



N=150, L=300mm

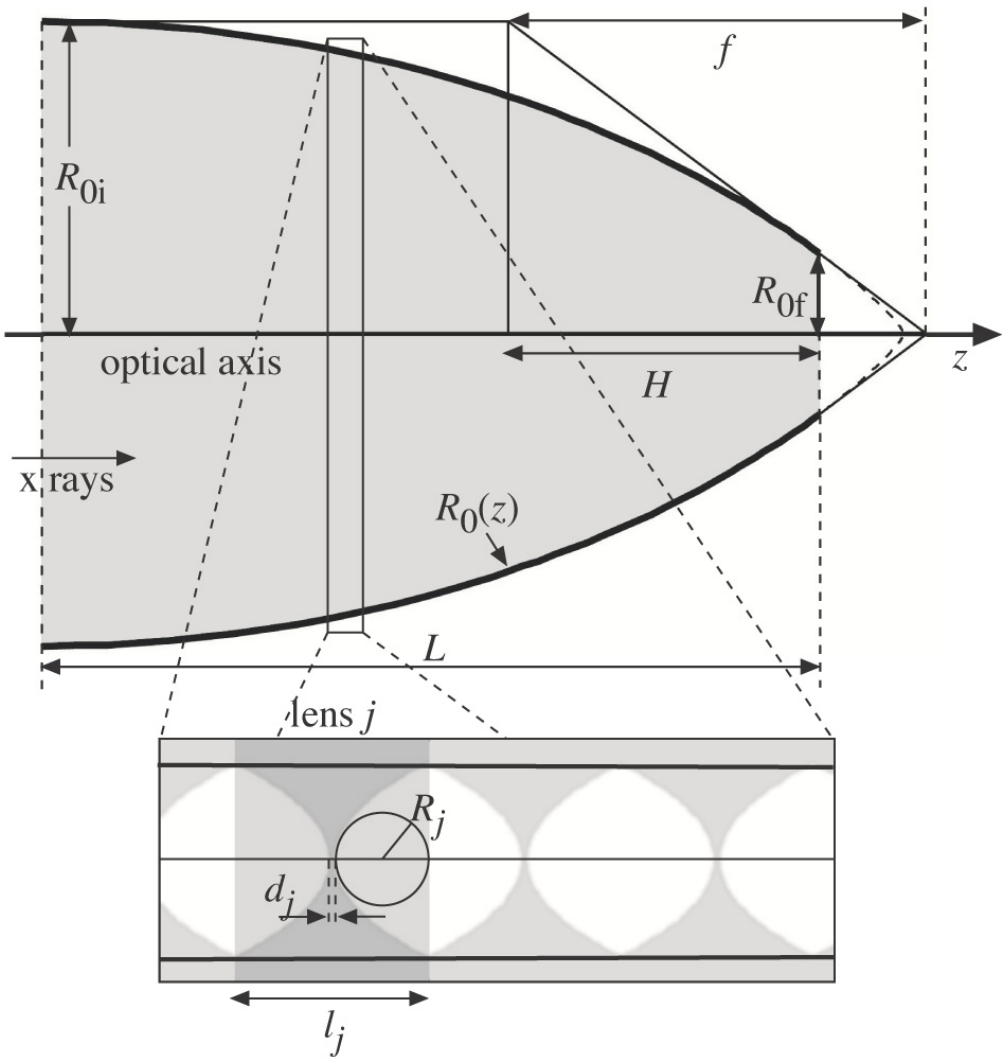
f=207.2mm, f_0=141.3mm
D=23.5mm

A_{eff}=287.7μm

d_{tr}=43.3nm

Adiabatically focusing lenses (PRL **94**, 054802(2005))

the refractive power per unit length increases along the lens!



$$f = \frac{R_{0i}}{\sqrt{4\delta \ln \frac{R_{0i}}{R_{0f}}}}$$

Adiabatically focusing lenses

$2R_{0i}$ and $2R_{0f}$ entrance and exit diameter

focal length

$$f = \frac{R_{0i}}{\sqrt{4\delta \ln \frac{R_{0i}}{R_{0f}}}}$$

length of stack

$$L = \frac{\sqrt{\pi} R_{0i}}{\sqrt{4\delta}} \Phi(\ln \frac{R_{0i}}{R_{0f}})$$

effective aperture

$$D_{eff} = 2R_{0i} \sqrt{\frac{1}{a_p}(1 - \exp(-a_p)}$$

$$a_p = \mu L / 2$$

Two examples of adiabatically focusing lenses

1. rotationally parabolic Be lenses at 17 keV ($2\delta = 2.359\ 10^{-6}$)

$R_i = 1500\mu m$, $R_{0i} = 1224.7\mu m$
$R_f = 50\mu m$, $R_{0f} = 223.6\mu m$

=> f = 432.4mm
L = 983.2mm
$D_{eff} = 498.9\mu m$
$\mathbf{d_{tr} = 52.2nm}$

compared to $\mathbf{d_{tr} = 42.0nm}$ for thick lens
with maximum length:

$R = 50\mu m$, N=162, L=324mm, f=205.9mm, $D_{eff} = 295.0\mu m$

not worth the effort!

2. **Cylinder paraboloids (C.Schroer TU Dresden)**

a. adiabatically focusing **diamond** lenses at 27.6keV

$R_{0i} = 9.43\mu m$
$R_{0f} = 50nm$ ($R_f = 25nm$)
$N = 1166$

d_{tr} = 4.7nm (11.6μm behind stack) **diffraction limit**

compared to

b. nanofocusing **diamond** lens with constant aperture

$N = 200$
$R = 1.31\mu m$
$R_0 = 7.2\mu m$

d_{tr} = 14.2nm **diffraction limit**

9. Handling and adjustment

a. refractive lenses are **robust and compact:**

easily installed and removed
in its own lens casing or in the vacuum of the beam line

b. **focus stays on axis**:

fast adjustment (typically in 15 minutes)
relatively insensitive to misorientation
to vibrations
no need for readjusment of the beam-line components
downstream

c. **comfortable working distance between optics and sample**

REFRACTIVE LENSES: NO RACING HORSES
BUT EXCELLENT WORKING HORSES !

C. Applications of refractive x-ray lenses

refractive x-ray lenses can be used like glass lenses are used for visible light

but

the numerical aperture N.A. is very small

typically 10^{-4} to 10^{-3}

New and improved x-ray techniques

1. **Imaging**: x-ray microscopy: 2D image
   x-ray tomography: 3D reconstruction

   in absorption and phase contrast

2. **Focusing**: diffraction,
   spectroscopy…..
   with high lateral resolution
   in the sub 100 nm range (50 nm were reached)

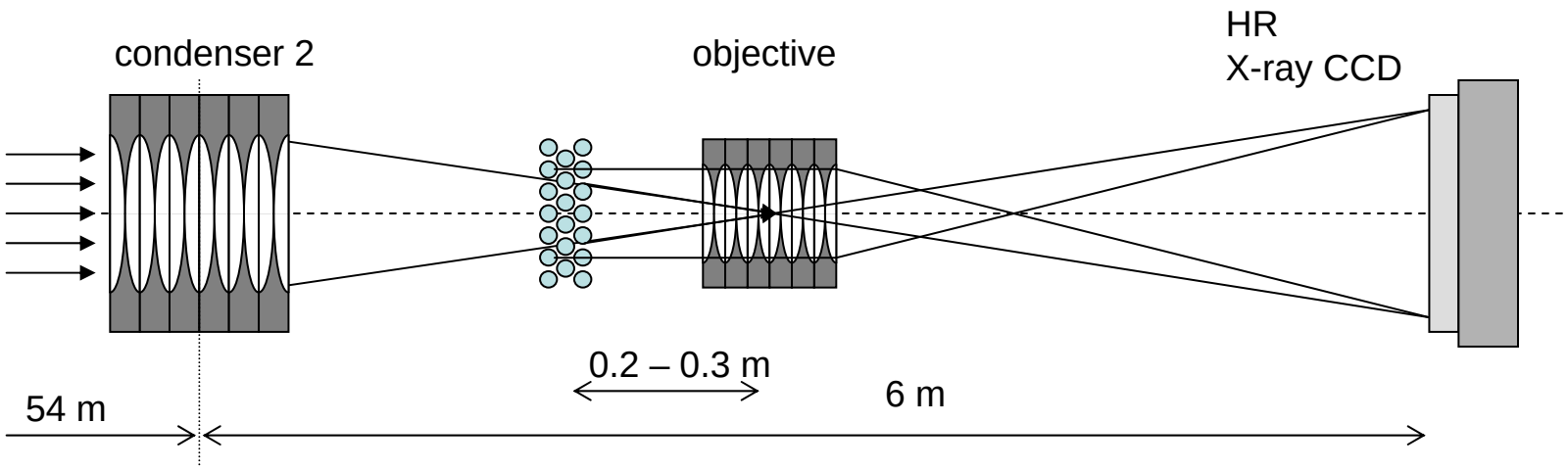
3. **Coherent photon flux**:
   X-ray diffraction
   speckle spectroscopy

1. High-resolution x-ray microscopy

illumination of object from behind via **prefocusing lens** (condenser 2) in order to adjust beam size on sample

objective with small focal length and low distortion (rotationally parabolic) see below: d_{tr} about 50nm

large magnification in order to relieve requirements on CCD camera (object slightly outside focus)



A. Snigirev et al

PREFOCUSING with

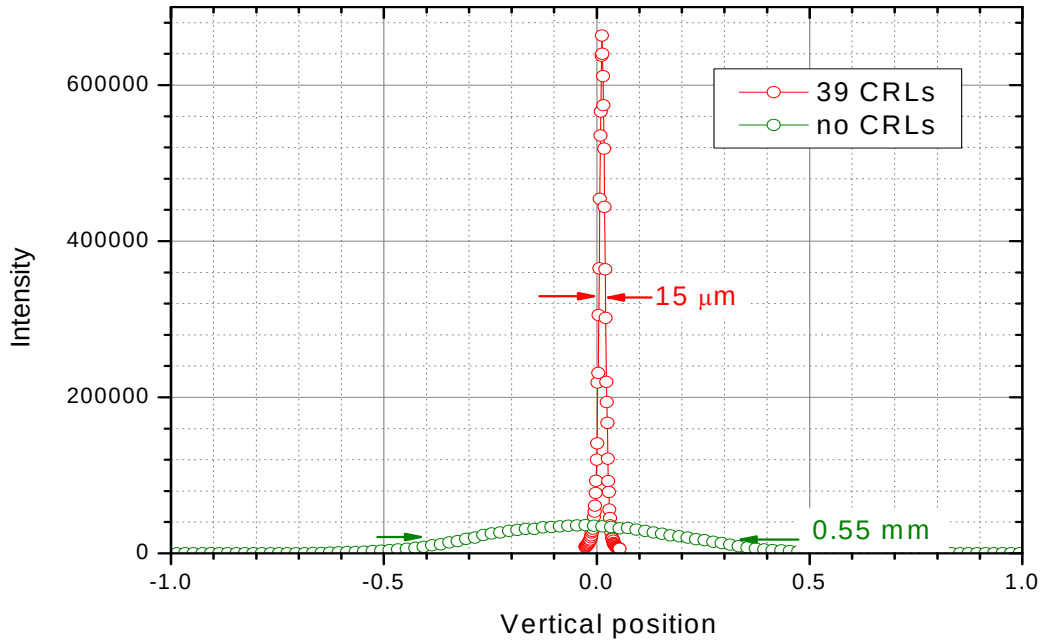
rotationally parabolic Be lenses
(R = 1500µm)

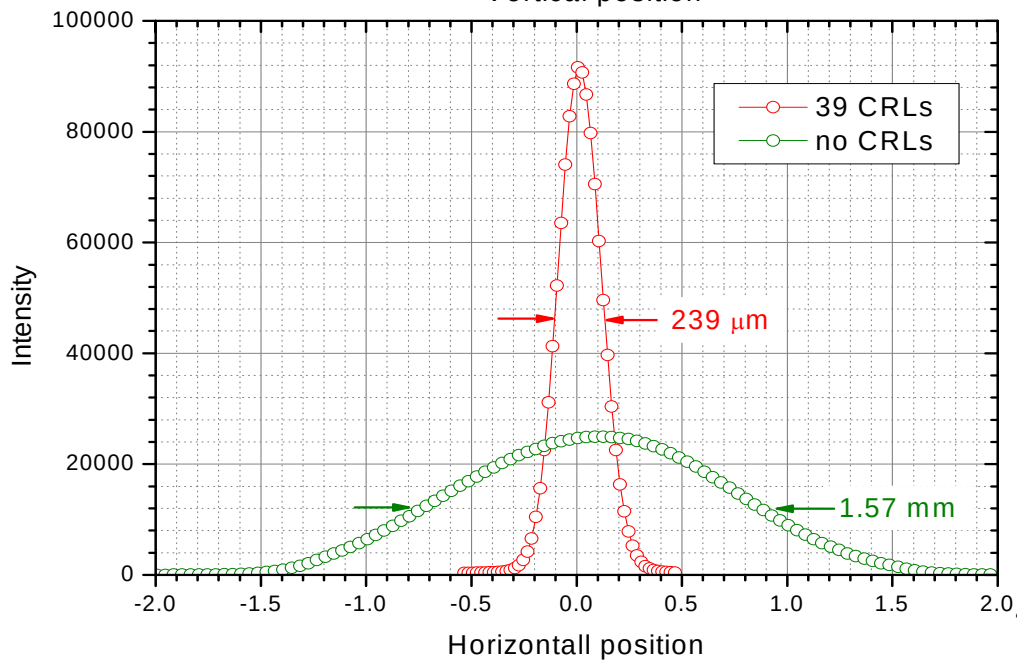
Image of the ID18 source at ESRF

14.4125eV
39 Be lenses
R = 1500µm

f = 11.718m
geometric aperture: 2.5mm

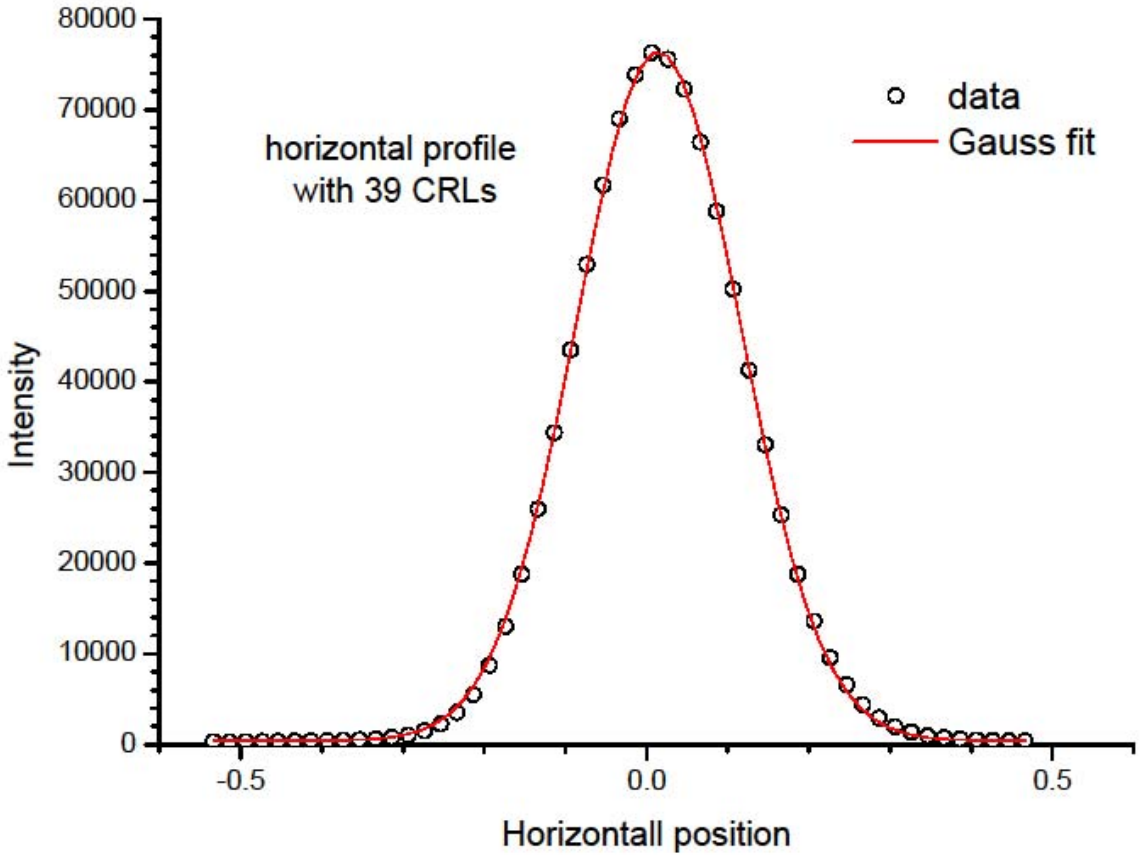
(A. Chumakov ESRF)





Intensity profile in the horizontal: ID18

well fitted by a Gaussian with 239 µm FWHM

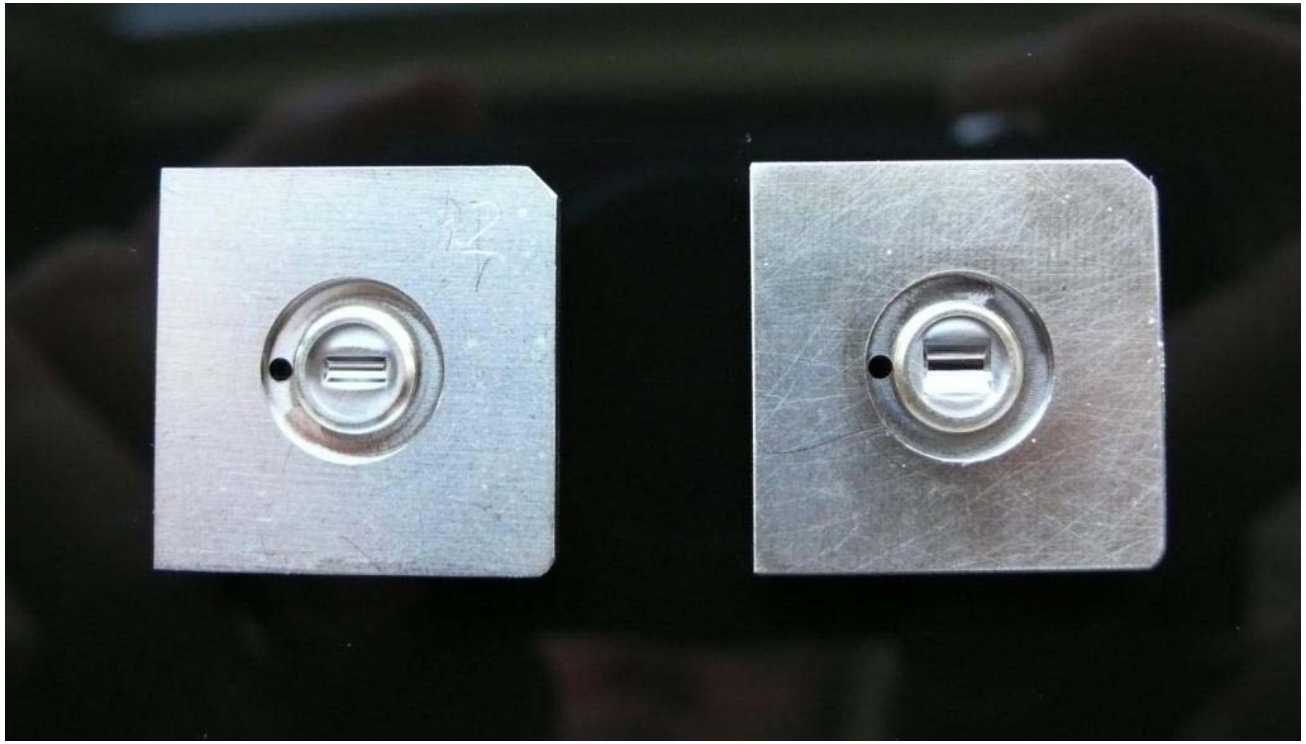


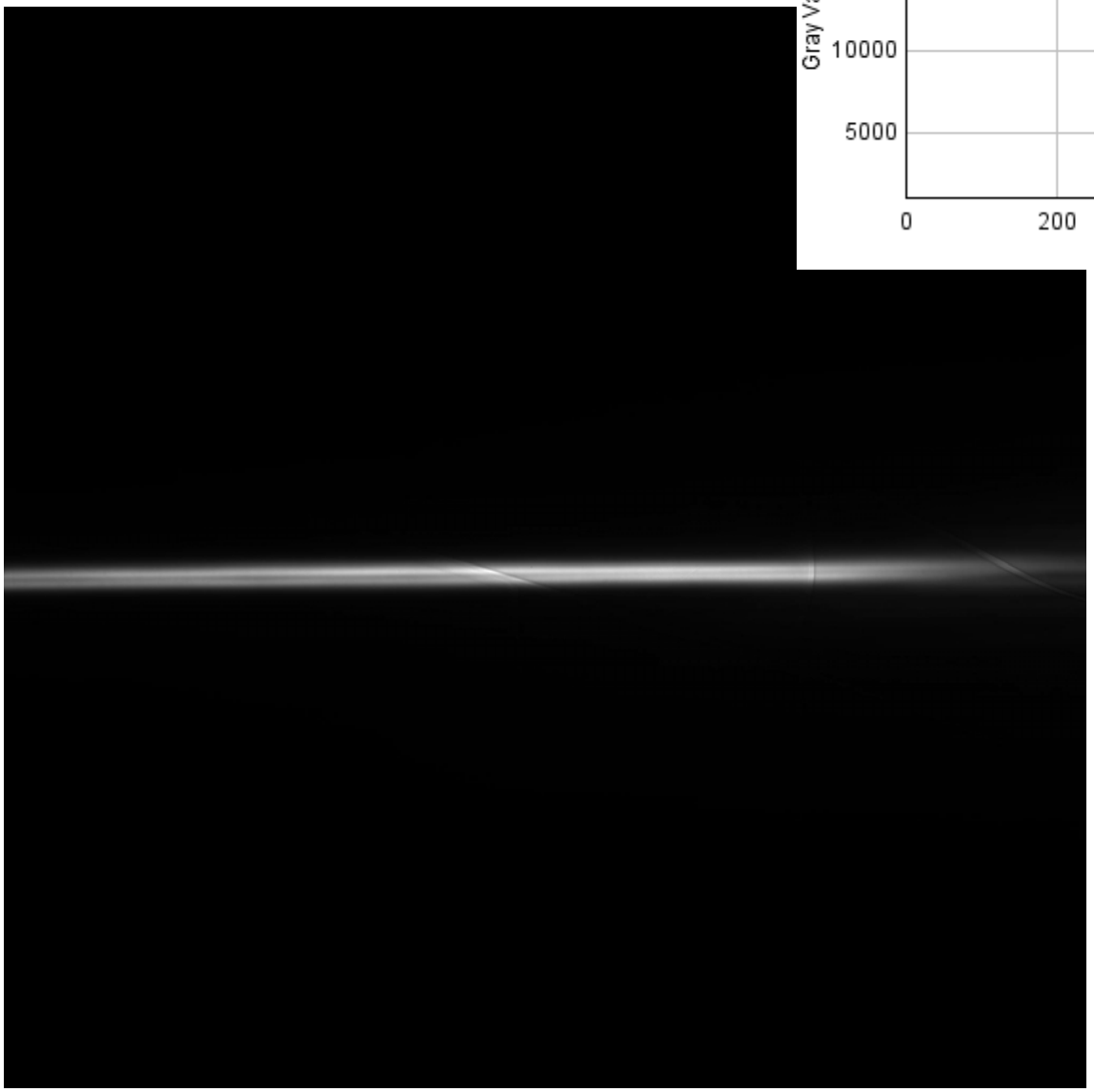
Prefocusing with linear lenses Be, Al and Ni

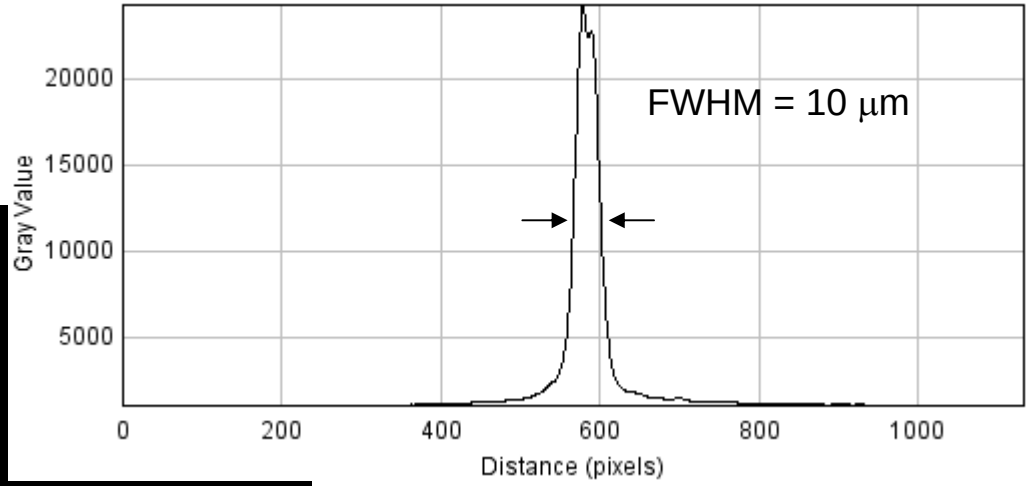
R = 50 to 1500mm
length 2.5 mm

* collecting more intensity

* for making spot on sample more circular (on storage rings)



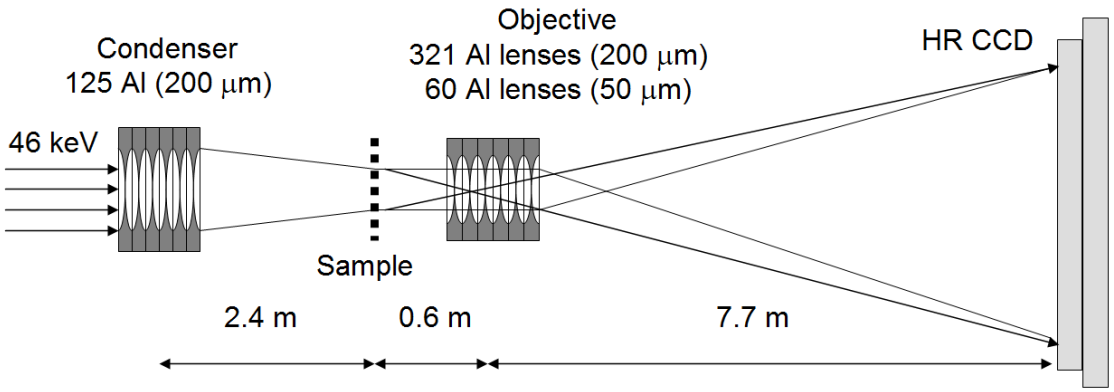


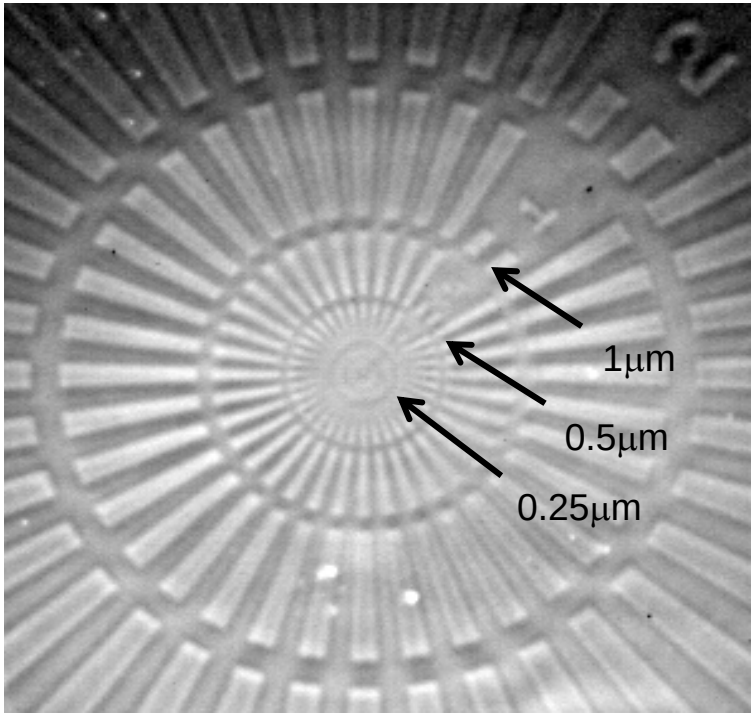


1D- Be lenses
R = 500μm
2.7 mm long

E = 10 keV
N = 20
R = 500 mm
L_1 = 56.5 m
L_2 = 3.91 m
A_{eff} = 690 mm
Snigirev …

High Energy X-ray Microscopy at ID15 Al lenses



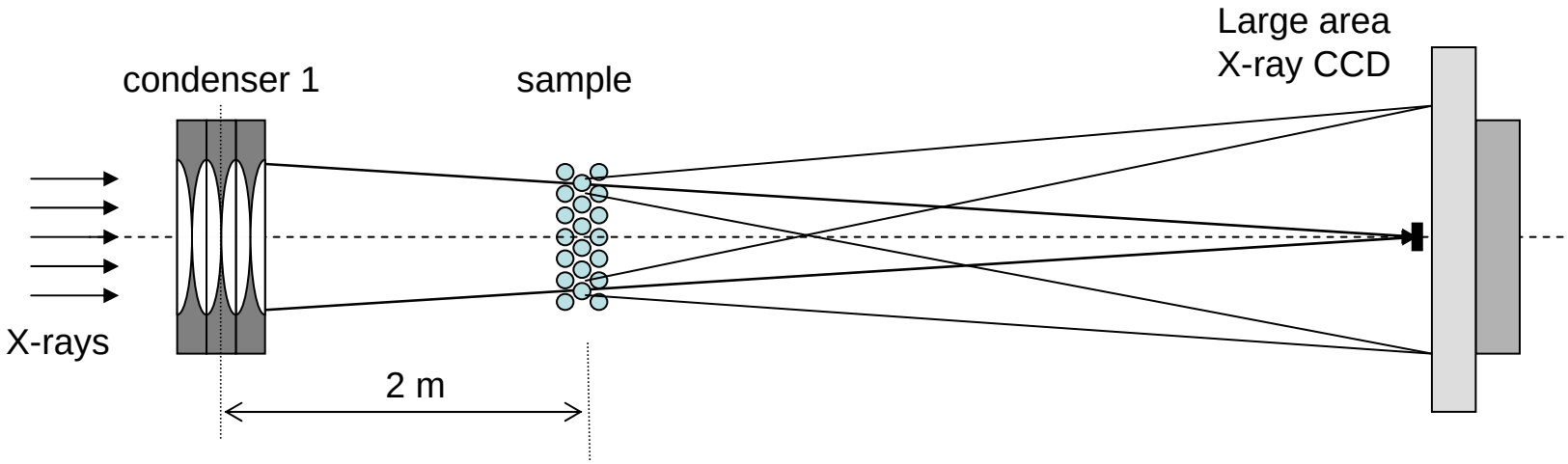


Siemens star
Ta 0.5 μm

E = 46 keV

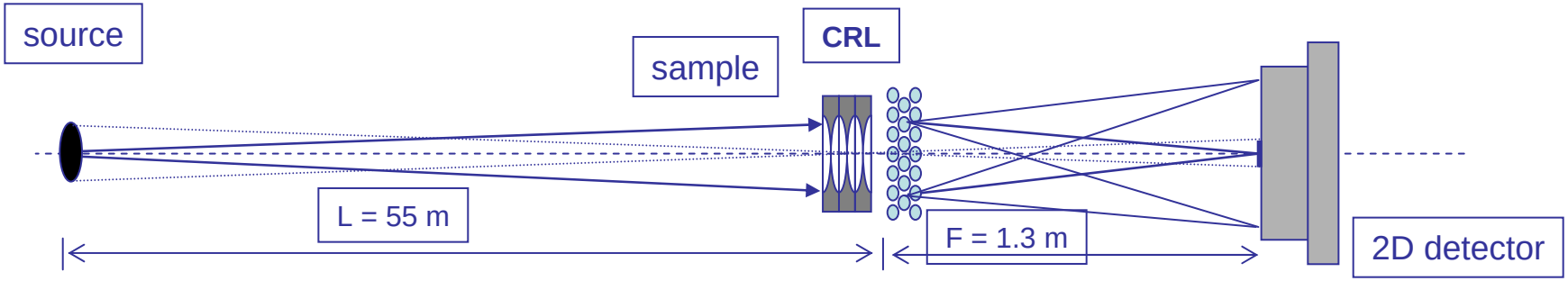
M. Di Michiel
M. Scheel
A. Snigirev
I. Snigireva

Microscopy in diffraction mode



Be N = 19 , R = 300μm

X-ray High Resolution Diffraction Using Refractive Lenses



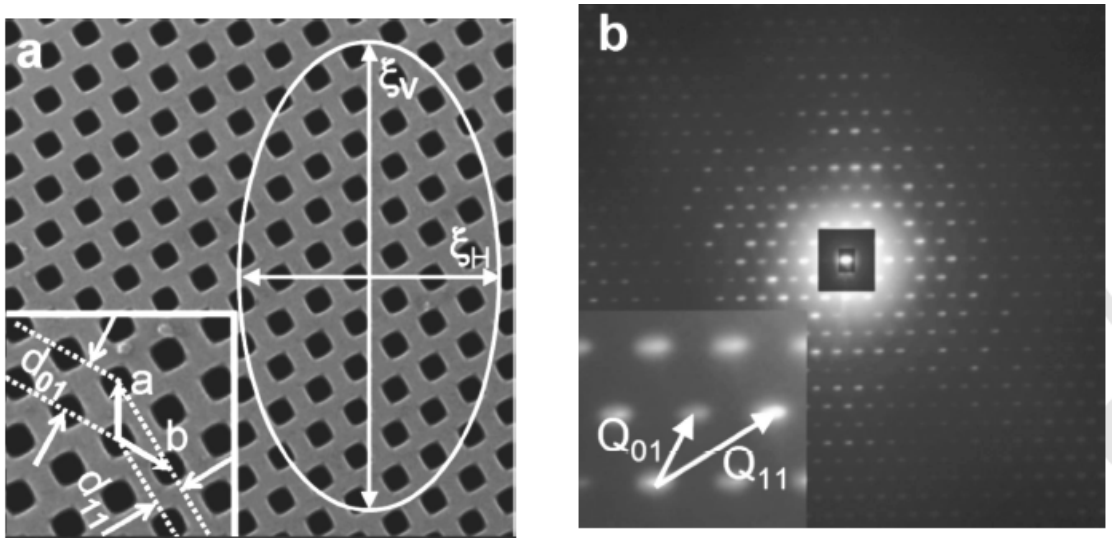
E = 28 keV
Al CRL, N = 112, F = 1.3 m

CCD resolution 2 μm
pixel / Θ = d

Resolution is limited
by angular source size:
s/L ~ 1 μrad

Momentum transfer
Resolution: 10^{-4} nm^{-1}

Si photonic crystal
a=b=4.2 μm d_{01}=3.6 μm d_{11}=2.1 μm



Lattice vectors g_{01} =1.75·10^{-3} nm^{-1} g_{11} =3·10^{-3} nm^{-1}

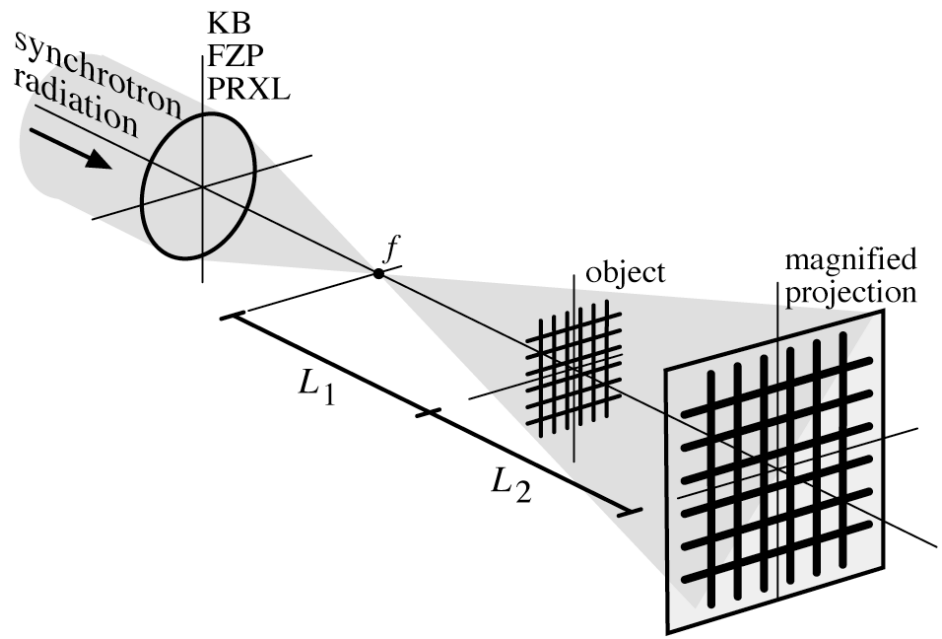
M. Drakopoulos, A. Snigirev, I. Snigireva, J. Schilling, Applied Physics Letters, 86, 014102, 2005.

2. Focusing

Microscopy

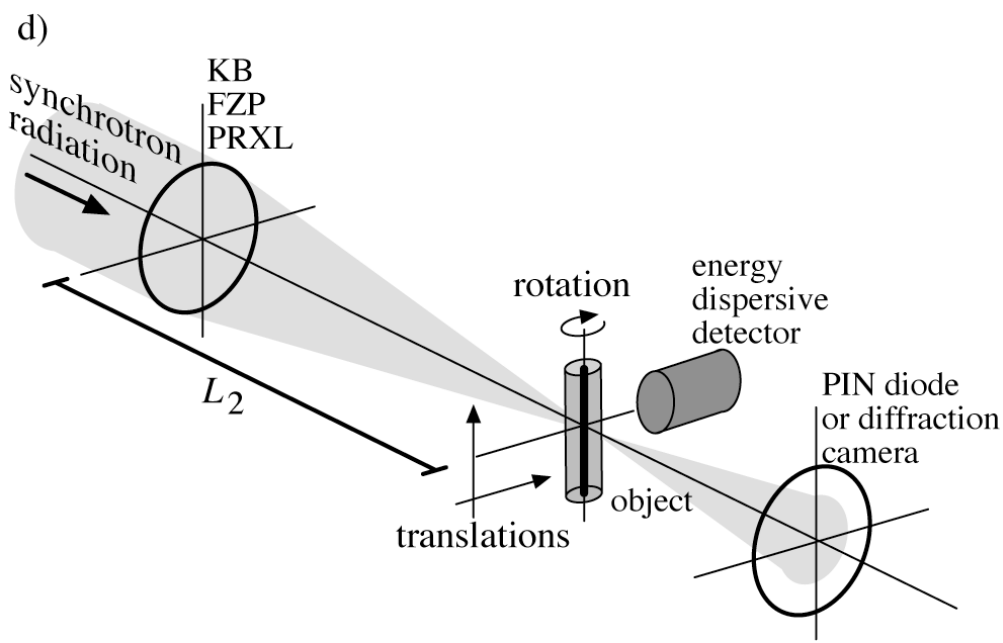
Object placed close to secondary source:
=> strong magnification

The smaller the focus, the sharper the image!



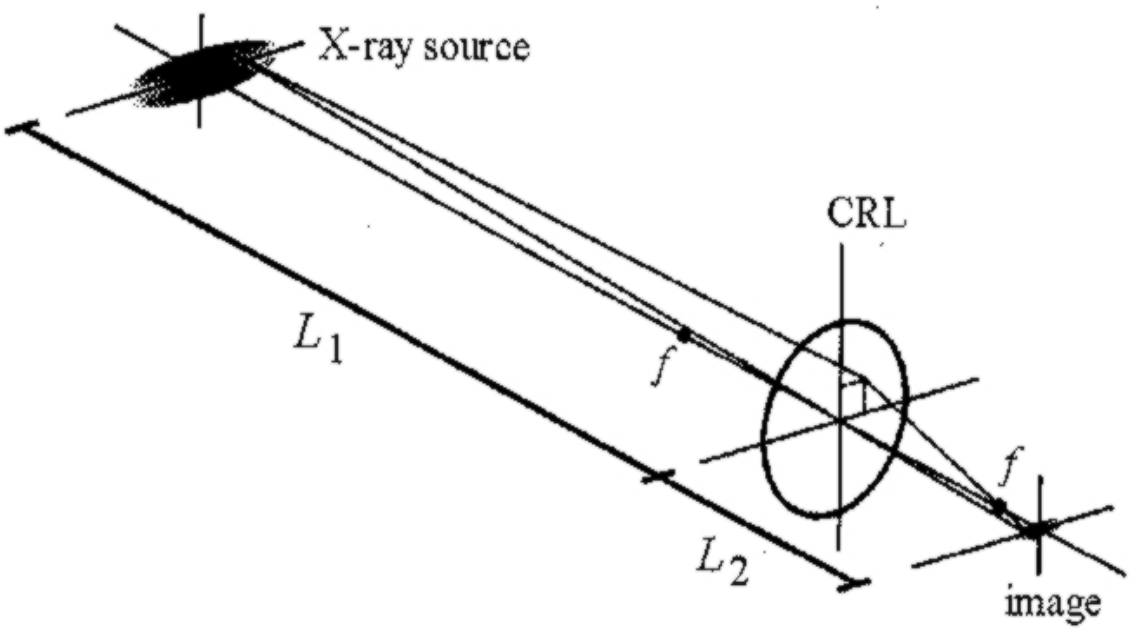
Spectroscopy, tomography

large depth of field
scanning beam over sample
(diffraction, SAXS, XAS, fluorescence…)



Small focus requires

1. small source
2. long distance L_1 source-lens
3. small focal length and large effective aperture of lens



Example: **ID13 at ESRF**

Be lens: $R = 50\mu m$, $N = 162$, $f = 205.9mm$,
$D_{eff} = 295\mu m$, $d_{tr} = 42nm$

$L_1 = 100m$, $L_2 = 206.3mm$

geometric image of source $S' = S\frac{L_2}{L_1}$

FWHM	S (μm)	S‘ geom (nm)	S‘ incl diffr (nm)
horizontal	120	248	251
vertical	20	41	59

diffraction limited in the vertical !

3. Coherent flux

* diffraction of individual large molecules, nanoparticles

* speckle spectroscopy

Illuminated area on sample must be smaller than the lateral coherence area at the sample position. Then all monochromatic photons are undistinguishable, i.e. they are in the same mode!

* coherent photon flux is a property of the brillance B of the source and of the degree of monochromaticity

$$F_c = B\lambda^2 \frac{\Delta\lambda}{\lambda}$$

* the coherent flux can at best be conserved, it cannot be increased by a focusing optic.

Example: **low-betha undulator at ESRF**

1. Be lenses, 17 keV, N = 162, f = 205.9mm, $\mathbf{d_{tr}}$ **= 42nm**

L_1 = 100 m, L_2 = 0.2063 m

2.

	Source size FWHM	Geometric image FWHM
horizontal	120μm	248 nm
vertical	20μm	**41nm**

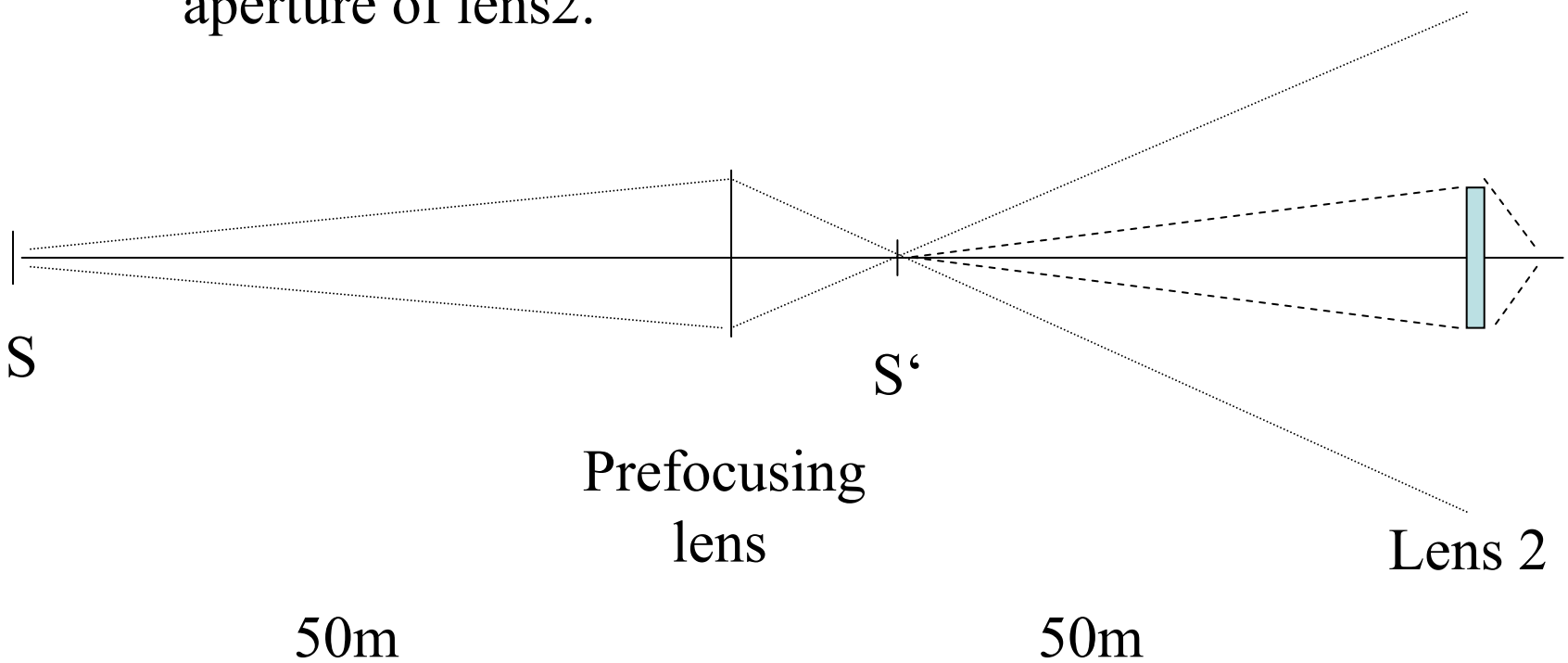
Image is diffraction limited in the vertical:
=> coherent illumination in the vertical

Not so in the horizontal!

3. **remedy for horizontal direction**

* insert a linear lens (prefocussing lens) which focuses only in the horizontal

* the secondary source S‘ must have a lateral coherence length at the postion of lens 2 which is equal to the effective aperture of lens2.



Prefocusing lens

Be linear: $R = 500\mu m$, $N = 55$,
$f = 3.854m$, $D_{eff} = 1048\mu m$

Image S‘ at $b_1 = 4.168m$ behind horizontal lens

lateral (horizontal) coherence length at position of lens 2: 295µm

this is equal to D_{eff} of lens 2: only the coherent flux passes through lens 2, the rest is peeled off.

gain in flux (compared to no prefocusing): about factor 10.

MANY THANKS

to

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Herbert Schloesser from our workshop

for many years of efficient and pleasant collaboration