Phase Problem and the Radon Transform

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Under the phase problem we understand the problem of reconstructing the complex amplitude of the coherent field from intensity measurements. Two different solutions to this problem are considered. Both solutions are linear and make the use of the Radon transform. The mathematics of the Radon transform is applied to formulate the solutions in terms of convolution integrals.

In the first type of solution, a 3D phase problem for a completely polarized coherent monochromatic field is linearized by using the Wigner transform in the parabolic approximation. It is shown that the Wigner distribution function (WDF) can be obtained from intensity measurements by applying the inverse Radon transform. After the WDF is obtained, the amplitude of the coherent field can be determined up to unknown multiplicative constant. The method presents a nice theoretical approach that gives us a possibility of writing down linear equations relating intensity measurements to the scalar field amplitude. However, in spite of its theoretical elegance, a practical implementation of the method is quite cumbersome because the intensity measurements are required in many planes along the beam. The number of planes needed is governed by the Nyquist criteria for the discrete Radon transform and can be relatively large for a sophisticated WDF [1].

A more practical solution requiring less measurements can be found for applications of phase-contrast tomographic imaging. The linearization of the phase problem is achieved here by using sufficiently small distances in the near field of the Fresnel region. It is also assumed that absorption is weak and change slowly. Under these conditions a fundamental theorem is proved [2]. The theorem plays the same role in phase-contrast tomography as the Fourier slice theorem does in conventional absorption-based tomography, but it is based on the Radon transform rather than the Fourier transform. The theorem gives us the way of reconstructing a 3D object function by covering its Radon space and applying the 3D inverse Radon transform afterwards. Because computation of the complete Radon space is a demanding task, another way of using the results of the theorem is considered. In a similar way as the filtered backprojection (FBP) algorithm is derived from the Fourier slice theorem, we derive a practical FBP algorithm using the fundamental Radon theorem for the phase-contrast data. The algorithm requires intensity measurements in a single plane of the near field in the case of the purely phase object. Reconstruction of the mixed amplitude and phase object will require two planes. The theorem and the method not only provide a nice theoretical insight on the phase problem of phase-contrast imaging, but also give us the means for fast practical inversion of the huge volumes of high-resolution phase-contrast data. The application of the approach is now investigated at several synchrotron sources.

References

[1] - A.V. Bronnikov, B.O. Maier, N.G. Preobrazhenskii, "Discrete Wigner Transform in the Phase Problem of Optics," Sov. Opt. Spectrosc., 70(4), pp. 512-516, 1991.

[2] - A.V. Bronnikov, “Theory of quantitative phase-contrast computed tomography,” J. Opt. Soc. Am. A, 19, pp. 472-480, 2002.